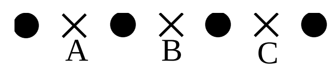
PHYSICS 211 **MIDTERM II** **12 November 2003**

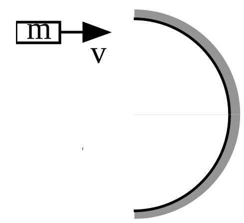
- Exam is closed book, closed notes. Use only your formula sheet.
- Write all work and answers in exam booklets.
- The backs of pages will not be graded unless you so request on the front of the page.
- Show all your work and explain your reasoning (except on #1).
- Partial credit will be given (not on #1). No credit will be given if no work is shown (not on #1).
- If you have a question, raise your hand or come to the front.

1. (20 points) For each of these multiple choice questions, indicate the correct response (A, B, C, or D (where needed)) on the page for problem 1 in your exam booklet.

i) Four equal masses are spaced equally along a line as shown at right. The masses do not move. A test mass can be placed at one of the three positions labeled with an x, each of which is midway between the neighboring masses. At which position or positions will the gravitational force on the test mass be zero?



ii) A block of mass m is sliding on a horizontal, frictionless table and then encounters a fixed semi-circular wall as shown at right (looking down on the table). There is friction between the wall and the block, so that the kinetic energy of the block is reduced by ΔK during its contact with the wall. If the initial velocity v is increased, does the magnitude of the kinetic energy loss of the block increase, decrease, or stay the same?



A) Increase. B) Decrease. C) Stay the same.

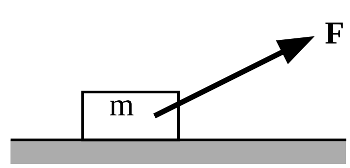
iii) A ball is tied to the end of a string and swung in a horizontal circle. If the string is made longer, but the time to swing the ball once around is kept fixed, does the tension in the string increase, decrease, or remain the same?

A) Increase. B) Decrease. C) Remain the same.

iv) Spring A is stiffer than spring B, that is, $k_A > k_B$. The same work is done to compress each spring. Is the compression of spring A greater than, less than, or the same as the compression of spring B?

A) Greater than. B) Less than. C) The same as.

v) The force **F** is applied to the box of mass m as shown. What is true about the magnitude of the normal force on the box (due to the floor)?



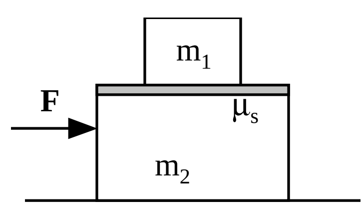
A) It is larger than mg.
B) It is smaller than mg.
C) It is equal to mg.
D) It is independent of the mass of the block.

2. (20 points) A spider of mass *m* hangs by a single strand of silk from the ceiling of a car. The car is traveling on a flat road in a circle of radius *r* with a constant speed *v*. The resulting acceleration tends to make the spider hang not quite vertically.
   a) Does the hanging spider tend to tilt toward the center of the circle or away from it?
   b) Find an expression (in terms of *m*, *r*, *g* and *v*) for the angle that the strand of silk makes with the vertical.
   c) Find an expression (in terms of *m*, *r*, *g* and *v*) for the tension in the strand of silk.

3. (20 points) A box of mass 4.0 *kg* is pushed up a frictionless incline by a worker who applies a horizontal force of 150 *N*. The incline is at an angle θ above the horizontal, where sinθ = 3/5. The box moves 3.0 *m* along the incline. Assume $g = 10\ m/s^2$.
   a) How much work is done on the box by the worker?
   b) How much work is done on the box by gravity?
   c) How much work is done on the box by the normal force of the incline?
   d) What is the final speed of the box, assuming it started at rest?

4. (20 points) A small spaceship orbits around Planet Claire in an elliptical orbit with a maximum distance from the planet of $r_{max} = 8.0 \times 10^6\ m$ and a minimum distances of $r_{min} = 1.0 \times 10^6\ m$. The maximum speed of the spaceship during the orbit is $v_{max} = 800\ m/s$. Planet Claire has a mass of $M = 6.0 \times 10^{21}\ kg$. Assume $G = 6.0 \times 10^{-11}\ Nm^2/kg^2$.
   a) What is the minimum speed of the spaceship in its elliptical orbit about Planet Claire?
   b) When the spaceship is closest to Planet Claire it fires its rocket engines to instantaneously change its speed and change to a circular orbit around the planet. What change in velocity is required for this new orbit?

5. (20 points) A block of mass $m_1 = 3\ kg$ sits atop a block of mass $m_2 = 7\ kg$ as shown at right. There is no friction between block 2 and the floor, but there is friction between block 1 and block 2, with a static coefficient of $\mu_s = 0.5$. An external force $F = 30\ N$ is applied to block 2 as shown and the two blocks move to the right, without block 1 slipping on block 2. Assume $g = 10\ m/s^2$.



   a) What is the acceleration of the two-block system?
   b) What maximum force F_{max} could be applied to block 2 before block 1 slipped with respect to block 2?

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1. i) B At each possible position of the test mass, the gravitational forces of the nearest neighbor masses cancel out, so the resultant force is that due to the remaining two masses. At position B the two remaining masses will produce opposite forces that cancel. At positions A and C the two remaining masses are both on the same side and so do not cancel.

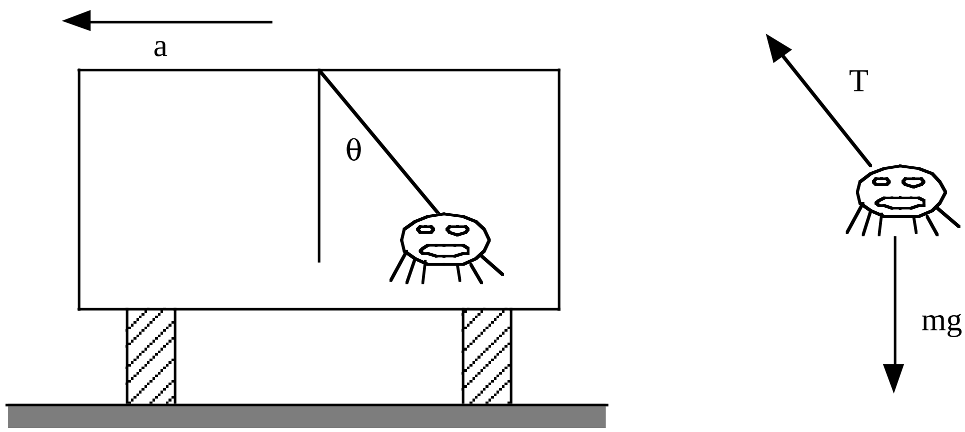
   ii) A The loss of kinetic energy is given by the work done by the friction force $W = -f_k d$, where $f_k = \mu_k N$ and d is the arc length of the semicircle. Since the normal force N is the only horizontal force, it must produce the centripetal acceleration mv^2/r. As v increases, the normal force will increase, so the frictional force will increase. Thus, for larger v, the kinetic energy loss will increase.

   iii) A The centripetal acceleration toward the center is provided by the tension T. Newton's second law gives $T = mv^2 / r$. The velocity for a period τ is given by $v = 2\pi r/\tau$. The tension is then $T = 4\pi^2 mr / \tau^2$, giving a larger tension for the case of larger r and the same τ.

   iv) B The work done in each case is $\frac{1}{2}kx^2$, so a larger k requires less compression for the same work.

   v) B If the box is not moving normal to the floor, the vertical forces must sum to zero. The normal force plus the vertical component of **F** must then equal mg, which means that the normal force will be smaller than mg. If **F** were strong enough to cause the box to rise, then the normal force would be zero, which is also less than mg.

2. a) As the car and the spider travel in a circle they each experience an acceleration toward the center of the circle. Hence there must be a force toward the center of the circle acting on the spider. Since gravity points downward, only the tension in the web can provide this centripetal force. Hence the spider must hang away from the center of the circle as shown below, with the horizontal component of the tension providing the needed centripetal force.



b) From the force diagram above, we can write down the two equations of motion for the vertical and horizontal directions.

$$T\sin\theta = ma = m\frac{v^2}{r}$$

$$T\cos\theta - mg = 0 \quad \Rightarrow \quad T\cos\theta = mg$$

Now divide these two equations to get:

$$\tan\theta = \frac{v^2}{rg}$$

$$\boxed{\theta = \arctan\left(\frac{v^2}{rg}\right)}$$

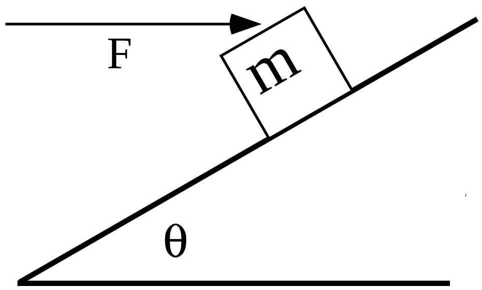
c) From the two equations of motion, we have the two components of the tension. Now just square and add and take the square root to find the tension:

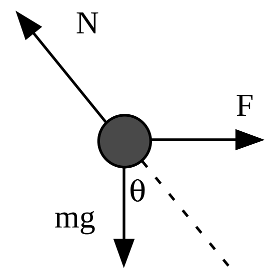
$$T^2 = (T\sin\theta)^2 + (T\cos\theta)^2 = \left(m\frac{v^2}{r}\right)^2 + (mg)^2$$

$$T^2 = (mg)^2\left(1 + \left(\frac{v^2}{rg}\right)^2\right)$$

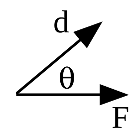
$$\boxed{T = mg\sqrt{1 + \frac{v^4}{r^2g^2}}}$$

3. The situation and the free body diagram are shown below:





a) The work done by the worker is:

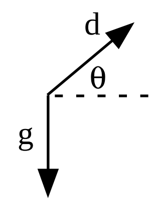


$$W_1 = \vec{\mathbf{F}} \bullet \vec{\mathbf{d}} = Fd\cos\phi$$

$$W_1 = Fd\cos\theta = 150N(3.0m)\tfrac{4}{5}$$

$$\boxed{W_1 = 360J}$$

b) The work done by gravity is:



$$W_2 = m\vec{\mathbf{g}} \bullet \vec{\mathbf{d}} = mgd\cos\phi$$

$$W_2 = mgd\cos\left(\theta + \tfrac{\pi}{2}\right) = -mgd\sin\theta = -4.0kg(10\,m/s^2)(3.0m)\tfrac{3}{5}$$

$$\boxed{W_2 = -72J}$$

c) The work done by the normal force is zero since it is perpendicular to the motion.

$$W_3 = \vec{\mathbf{N}} \bullet \vec{\mathbf{d}} = Nd\cos\phi$$

$$\boxed{W_3 = 0J}$$

d) The final speed of the block is found using the Work-Kinetic energy theorem:

$$\Delta K = W_1 + W_2 + W_3$$

$$K_f - K_i = W_1 + W_2$$

$$\tfrac{1}{2}mv^2 = W_1 + W_2$$

$$v = \sqrt{\tfrac{2}{m}\left(W_1 + W_2\right)} = \sqrt{\tfrac{2}{4.0kg}(360J - 72J)}$$

$$\boxed{v = 12\,m/s}$$

4. a) Kepler's 2nd law of equal areas in equal times means that the area *dA* swept out in a time *dt* is a constant, or that *dA/dt* is constant. For small times, we can approximate the area *dA* as an isosceles triangle with height *r* and base $v_\perp dt$, where $v_\perp$ is the component of velocity perpendicular to the position vector. We thus find that $rv_\perp$ is a constant during the orbital motion. Since the velocities at the maximum and minimum distances for an elliptical orbit are perpendicular to the respective radius vectors from the sun, the conserved quantity is simply *vr*. Hence, the speed will be smallest at the largest distance from the planet, and largest at the minimum distance.

$$v_{\max} r_{\min} = v_{\min} r_{\max}$$
$$v_{\min} = v_{\max} \frac{r_{\min}}{r_{\max}} = v_{\max} \frac{1.0\times10^6\,m}{8.0\times10^6\,m} = \frac{v_{\max}}{8} = \frac{800\,m/s}{8}$$
$$\boxed{v_{\min} = 100\,m/s}$$

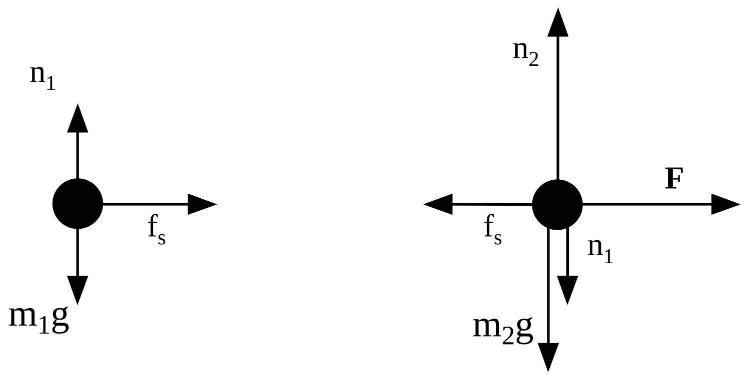
b) The force of gravity provides the centripetal force to keep the spaceship moving in a circle. Combine Newton's second law with his law of gravity to find the circular orbital speed. The new orbital radius is $R = r_{\text{miin}}$.

$$F = ma$$
$$\frac{GMm}{R^2} = m\frac{v^2}{R}$$
$$v^2 = \frac{GM}{R}$$
$$v = \sqrt{\frac{GM}{R}} = \sqrt{\frac{6.0\times10^{-11}N\,m^2/kg^2\,(6.0\times10^{21}kg)}{1.0\times10^6\,m}}$$
$$v = 600\,m/s$$

The required velocity change can now be found:

$$\Delta v = v_f - v_i = v_{\text{circle}} - v_{\max} = 600\,m/s - 800\,m/s$$
$$\boxed{\Delta v = -200\,m/s}$$

5. The free body diagrams for each mass are shown below.



a) Neither block accelerates vertically, and they both accelerate to the right with acceleration *a*, giving the equations of motion

$$f_s = m_1 a$$
$$n_1 - m_1 g = 0$$
$$F - f_s = m_2 a$$
$$n_2 - m_2 g - n_1 = 0$$

Plugging the first equation into the third allows us to find the acceleration of the system:

$$F - m_1 a = m_2 a$$
$$F = m_1 a + m_2 a = (m_1 + m_2)a$$
$$a = \frac{F}{m_1 + m_2} = \frac{30N}{3kg + 7kg} = \frac{30N}{10kg}$$
$$\boxed{a = 3 m/s^2}$$

b) To find the maximum possible force $F_{\max}$ before block 1 slips, use the equation describing the limitation of static friction:

$$f_s \le \mu_s n$$
$$m_1 a \le \mu_s m_1 g$$
$$m_1 \frac{F}{m_1 + m_2} \le \mu_s m_1 g$$
$$F \le \mu_s (m_1 + m_2) g$$
$$F_{\max} = \mu_s (m_1 + m_2) g = 0.5(10kg)(10\, m/s^2)$$
$$\boxed{F_{\max} = 50N}$$