PHYSICS 211 **MIDTERM II** **12 May 2004**

- Exam is closed book, closed notes. Use only your formula sheet.
- Write all work and answers in exam booklets.
- The backs of pages will not be graded unless you so request on the front of the page.
- Show all your work and explain your reasoning (except on #1).
- Partial credit will be given (not on #1). No credit will be given if no work is shown (not on #1).
- If you have a question, raise your hand or come to the front.

1. (20 points) For each of these multiple choice questions, indicate the correct response (A, B, C, or D (where needed)) on the page for problem 1 in your exam booklet.

i) A block is on a ramp that is at an angle θ to the horizontal. As the angle θ is increased, the block remains stationary (due to the static frictional force on it from the ramp) until a maximum angle θ_{max} when the block begins to slide down the ramp. If we double the mass of the block, and repeat the experiment, is the new maximum angle greater than, less than, or the same as the original maximum angle?

A) Greater than. B) Less than. C) The same as.

ii) A ball is tied to the end of a string and swung in a vertical circle on earth. The tension in the string is constant. Is the speed of the ball larger at the top of the circle, the bottom of the circle, or the same at top and bottom?

A) Larger at top. B) Larger at bottom. C) Same at top and bottom.

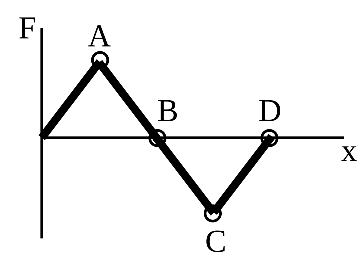
iii) An apple falls from a tree and hits Sir Isaac on the head. The apple falls due to the force of gravity that the earth exerts on the apple. Is the magnitude of the force of gravity that the apple exerts on the earth greater than, less than, or the same as the magnitude of the force of gravity that the earth exerts on the apple?

A) Greater than. B) Less than. C) The same as.

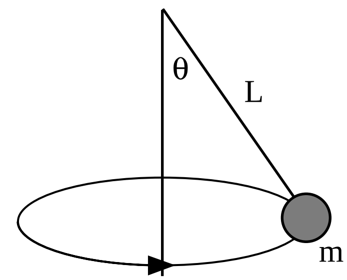
iv) Kepler's 2nd law states that a line that connects a planet to the Sun sweeps out equal areas in equal times. This implies that the planet has its largest speed when it is in which relative position in its orbit?

A) Closest to the Sun. B) Farthest from the Sun. C) Same speed at all points in orbit.

v) The plot at right shows the values of a force ***F*** that will act on a particle at the corresponding values of *x*. The force is along the *x*-axis and the particle starts at *x* = 0 with a positive velocity. At which one of the four labeled points is the kinetic energy of the particle the greatest?



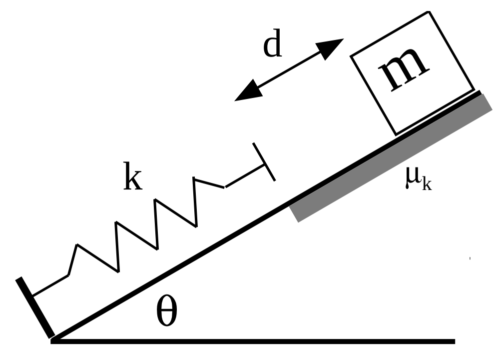
2. (20 points) A small ball of mass m hangs by a cord of length L from the ceiling . The ball swings in a horizontal circle such that the cord makes an angle θ with the vertical, as shown at right. The time for the ball to make one revolution is τ. Express answers below in terms of m, L, g, τ, and other constants as needed.



   a) Find an expression for the angle θ.
   b) Find an expression for the tension in the cord.
   c) If the ball revolves faster, does the angle θ increase or decrease?

3. (20 points) A box of mass 5.0 kg is thrown down an inclined plane with an initial speed $v_0 = 3\ m/s$. After traveling a distance $d = 4\ m$ down the incline, the box hits a spring ($k = 108\ N/m$) whose other end is anchored to the incline. There is friction ($\mu_k = 0.5$) between the box and incline during the first part of the motion, but not once the spring is contacted. The incline is at an angle θ above the horizontal, where $\sin\theta = 3/5$. Assume $g = 10\ m/s^2$. Ignore the size of the box.

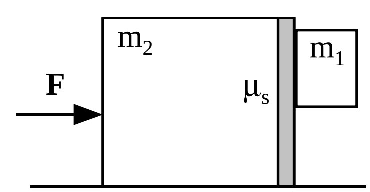
   a) What is the speed of the box as it contacts the spring?
   b) How far is the spring compressed before the box comes to rest?



4. (20 points) Planet Claire orbits around the star Antares in a circular orbit at a distance of $r_{orb} = 3.0 \times 10^{11}\ m$. Planet Claire has a radius of $R_C = 8.0 \times 10^6\ m$ and a mass of $M_C = 6.0 \times 10^{24}\ kg$ and Antares has a mass of $M_A = 6.0 \times 10^{30}\ kg$. Assume $G = 6.0 \times 10^{-11}\ Nm^2/kg^2$.

   a) What is the period of Planet Claire's orbit?
   b) What is the gravitational acceleration on Planet Claire?
   c) Where is the net gravitational force due to Planet Claire and Antares zero?

5. (20 points) A block of mass m_1 is in contact with a block of mass m_2 as shown at right. There is no friction between block 2 and the floor, but there is friction between block 1 and block 2, with a static coefficient of μ_s. An external force F is applied to the back of block 2 and the two blocks move to the right, without block 1 slipping down the front of block 2.



   a) What is the acceleration of the two-block system?
   b) What minimum force F_{min} must be applied to block 2 to prevent block 1 from slipping with respect to block 2?
   c) What is the normal force of the floor on block 2?

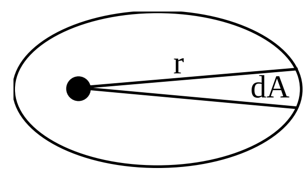
PHYSICS 211 MIDTERM II SOLUTIONS 12 May 2004

1. i) C The angle θ_{max} is the angle where the maximum possible frictional force ($f_{s,\max} = \mu_s N = \mu_s mg\cos\theta$) equals the weight component ($mg\sin\theta$) down the ramp. Doubling the mass will double both of these together, leaving the equality ($mg\sin\theta = \mu_s mg\cos\theta \Rightarrow \mu_s = \tan\theta$) unchanged. Hence the angle is unchanged.

   ii) A At the top of the circle T and mg both point down, in the same direction as the centripetal acceleration. At the bottom of the circle T points up and mg points down, while the centripetal acceleration points up. Applying $F = ma$ to each situation gives $T + mg = mv^2/r$ at the top and $T - mg = mv^2/r$ at the bottom. Since T is constant, the speed will be larger at the top of the circle.

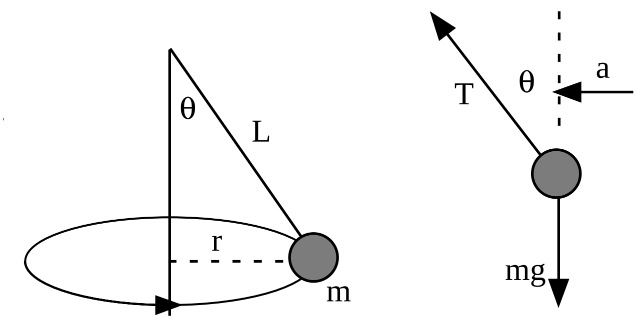
   iii) C The forces are equal and opposite according to Newton's third law.

   iv) A An elliptical orbit is shown at right. Kepler's 2nd law of equal areas in equal times means that the area dA swept out in a time dt is a constant, or that dA/dt is constant. For small times, we can approximate the area dA as an isosceles triangle with height r and base $v_\perp dt$, where $v_\perp$ is the component of velocity perpendicular to the position vector. We thus find that $rv_\perp$ is a constant during the orbital motion. This implies that the largest speed occurs when the planet is closest to the sun.



   v) B The change in kinetic energy is given by the integral of the force, or the area under the curve. If that area increases as x increases, then the kinetic energy will increase. As we integrate from $x = 0$, the area increases until we reach point B, after which it decreases, so the kinetic energy will increase until point B and then decrease.

2. As the ball travels in a circle it experiences an acceleration toward the center of the circle. Hence there must be a force toward the center of the circle acting on the ball. Since gravity points downward, only the tension in the cord can provide this centripetal force. Hence the ball must hang away from the center of the circle as shown below, with the horizontal component of the tension providing the needed centripetal force.



a) From the force diagram above, we can write down the two equations of motion for the vertical and horizontal directions. Note that the radius of the circle is $r = L\sin\theta$.

$$T\sin\theta = ma = m\frac{v^2}{r}$$

$$T\cos\theta - mg = 0 \quad \Rightarrow \quad T\cos\theta = mg$$

Now divide these two equations and write the velocity in terms of the period:

$$\frac{\sin\theta}{\cos\theta} = \frac{v^2}{rg} = \frac{1}{rg}\left(\frac{2\pi r}{\tau}\right)^2 = \frac{4\pi^2 r}{g\tau^2} = \frac{4\pi^2 L\sin\theta}{g\tau^2}$$

$$\cos\theta = \frac{g\tau^2}{4\pi^2 L}$$

$$\boxed{\theta = \arccos\left(\frac{g\tau^2}{4\pi^2 L}\right)}$$

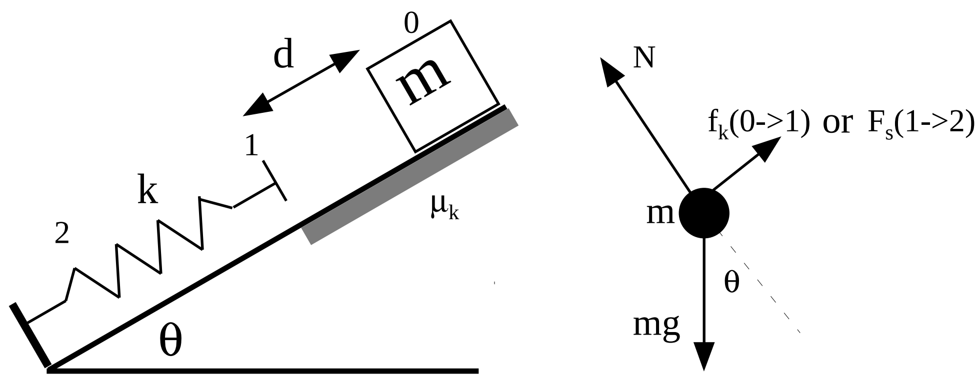
b) Use the vertical equation of motion to find the tension:

$$T\cos\theta = mg \quad \Rightarrow \quad T\frac{g\tau^2}{4\pi^2 L} = mg$$

$$\boxed{T = \frac{4\pi^2 mL}{\tau^2}}$$

c) If the ball revolves faster, then the period τ decreases. The equation for θ tells us that if τ decreases, then cosθ also decreases. The function cosθ between 0 and π/2 decreases as θ increases. $\boxed{\text{Thus as } \tau \text{ decreases, } \theta \text{ increases.}}$

3. Consider the motion in 2 steps: 0->1 gravity and friction act, 1->2 gravity and spring act.



a) The speed of the box at spring contact (point 1) is found by considering the work done by gravity and friction:

$$W_g = m\vec{\mathbf{g}} \bullet \vec{\mathbf{d}} = mgd\cos\left(\tfrac{\pi}{2} - \theta\right) = mgd\sin\theta$$

$$W_f = \vec{\mathbf{f}}_k \bullet \vec{\mathbf{d}} = -f_k d = -\mu_k mgd\cos\theta$$

$$\Delta K = W_g + W_f$$

$$K_1 - K_0 = \tfrac{1}{2}mv_1^2 - \tfrac{1}{2}mv_0^2 = mgd\sin\theta - \mu_k mgd\cos\theta = mgd(\sin\theta - \mu_k\cos\theta)$$

$$v_1^2 = v_0^2 + 2gd(\sin\theta - \mu_k\cos\theta)$$

$$v_1 = \sqrt{v_0^2 + 2gd(\sin\theta - \mu_k\cos\theta)} = \sqrt{(3\,m/s)^2 + 2(10\,m/s^2)4m(3/5 - 0.5(4/5))}$$

$$\boxed{v_1 = 5\,m/s}$$

b) During compression of the spring the block travels from $x_1 = 0$ to x_2. Gravity also acts during this travel, giving:

$$W_g = mgx_2\sin\theta \quad , \qquad W_s = \tfrac{1}{2}kx_1^2 - \tfrac{1}{2}kx_2^2 = -\tfrac{1}{2}kx_2^2$$

$$\Delta K = W_g + W_s$$

$$K_2 - K_1 = 0 - \tfrac{1}{2}mv_1^2 = mgx_2\sin\theta - \tfrac{1}{2}kx_2^2$$

$$\tfrac{1}{2}kx_2^2 - mgx_2\sin\theta - \tfrac{1}{2}mv_1^2 = 0$$

$$x_2 = \frac{mg\sin\theta \pm \sqrt{(mg\sin\theta)^2 + kmv_1^2}}{k} = \frac{mg\sin\theta}{k}\left[1 \pm \sqrt{1 + \frac{kv_1^2}{mg^2\sin^2\theta}}\right]$$

$$x_2 = \frac{5kg(10\,m/s^2)3/5}{108\,N/m}\left[1 \pm \sqrt{1 + \frac{108\,N/m\,(5\,m/s)^2}{5kg(10\,m/s^2)^2(3/5)^2}}\right] = \frac{5}{18}m\left[1 + \sqrt{1+15}\right]$$

$$\boxed{x_2 = 25/18\,m = 1.389\,m}$$

4. a) The force of gravity provides the centripetal force to keep the planet moving in a circle. Combine Newton's second law with his law of gravity to find the circular orbital speed.

$$F = M_C a$$

$$\frac{GM_A M_C}{r_{orb}^2} = M_C \frac{v^2}{r_{orb}} = \frac{M_c}{r_{orb}}\left(\frac{2\pi r_{orb}}{\tau}\right)^2$$

$$\tau^2 = \frac{4\pi^2}{GM_A} r_{orb}^3$$

$$\tau = \sqrt{\frac{4\pi^2}{GM_A} r_{orb}^3} = \sqrt{\frac{4\pi^2\left(3.0\times10^{11}\,m\right)^3}{6.0\times10^{-11}\,N\,m^2/kg^2\,(6.0\times10^{30}\,kg)}}$$

$$\boxed{\tau = 5.44\times10^7\,s = 1.72\,yrs}$$

b) The gravitational acceleration is found by equating gravity to mg_C:

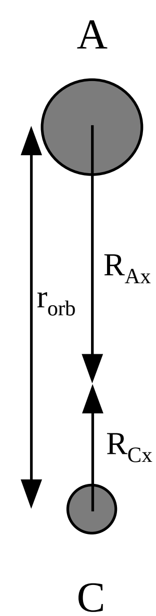
$$F_g = mg_C$$

$$\frac{GM_C m}{R_C^2} = mg_C$$

$$g_C = \frac{GM_C}{R_C^2} = \frac{6.0\times10^{-11}\,N\,m^2/kg^2\,(6.0\times10^{24}\,kg)}{\left(8.0\times10^6\,m\right)^2}$$

$$\boxed{g_C = 5.625\,m/s^2}$$

c) The net gravitational force will be zero somewhere between Claire and Antares, where both pull on an object and the forces cancel (see diagram below):



$$F_C - F_A = 0 \quad \Rightarrow \quad F_C = F_A$$

$$\frac{GM_C m}{R_{Cx}^2} = \frac{GM_A m}{R_{Ax}^2} = \frac{GM_A m}{\left(r_{orb} - R_{Cx}\right)^2}$$

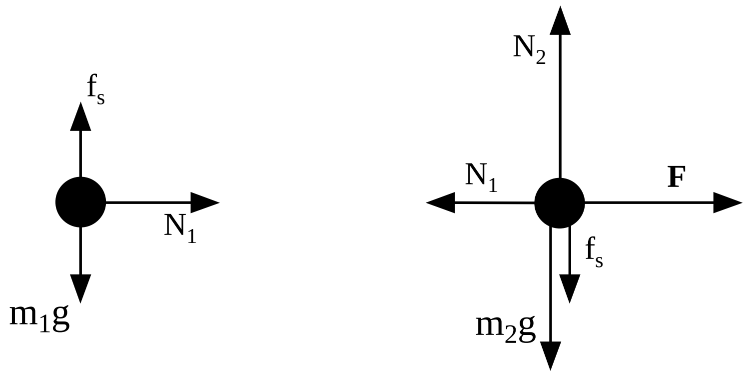
$$\frac{\left(r_{orb} - R_{Cx}\right)^2}{R_{Cx}^2} = \frac{M_A}{M_C} \quad \Rightarrow \quad \frac{r_{orb}}{R_{Cx}} - 1 = \sqrt{\frac{M_A}{M_C}}$$

$$R_{Cx} = \frac{r_{orb}}{1+\sqrt{\frac{M_A}{M_C}}} = \frac{r_{orb}}{1+\sqrt{10^6}} = \frac{r_{orb}}{1+1000}$$

$$\boxed{R_{Cx} = 3.0\times10^8\,m}$$

$$\boxed{R_{Ax} = \frac{r_{orb}}{1+\sqrt{\frac{M_C}{M_A}}} = \frac{r_{orb}}{1+\frac{1}{1000}} = 2.997\times10^{11}\,m}$$

5. The free body diagrams for each mass are shown below.



a) Neither block accelerates vertically, and they both accelerate to the right with acceleration *a*, giving the equations of motion

$$N_1 = m_1 a$$
$$f_s - m_1 g = 0$$
$$F - N_1 = m_2 a$$
$$N_2 - f_s - m_2 g = 0$$

Plugging the first equation into the third allows us to find the acceleration of the system:

$$F - m_1 a = m_2 a$$
$$F = m_1 a + m_2 a = (m_1 + m_2)a$$
$$\boxed{a = \frac{F}{m_1 + m_2}}$$

b) To find the minimum possible force F_{min} before block 2 slips, use the equation describing the limitation of static friction:

$$f_s \le \mu_s N_1$$
$$m_1 g \le \mu_s m_1 a$$
$$m_1 g \le \mu_s m_1 \frac{F}{m_1 + m_2}$$
$$F \ge \frac{(m_1 + m_2)g}{\mu_s}$$
$$\boxed{F_{\text{min}} = \frac{(m_1 + m_2)g}{\mu_s}}$$

c) The normal force is found from the fourth equation of motion

$$N_2 - f_s - m_2 g = 0$$
$$N_2 = m_2 g + f_s = m_2 g + m_1 g$$
$$\boxed{N_2 = (m_1 + m_2)g}$$