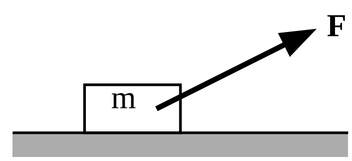
- Exam is closed book, closed notes. Use only your formula sheet.
- Write all work and answers in exam booklets.
- The backs of pages will not be graded unless you so request on the front of the page.
- Show all your work and explain your reasoning (except on #1).
- Partial credit will be given (not on #1). No credit will be given if no work is shown (not on #1).
- If you have a question, raise your hand or come to the front.

1. (20 points) For each of these multiple choice questions, indicate the correct response (A, B, C, or D (where needed)) on the page for problem 1 in your exam booklet.

i) An apple and an orange are thrown straight up into the air. Each has the same initial velocity, but the orange is thrown a short time after the apple is thrown. While both are moving upwards, does the distance between them increase, decrease, or stay the same?

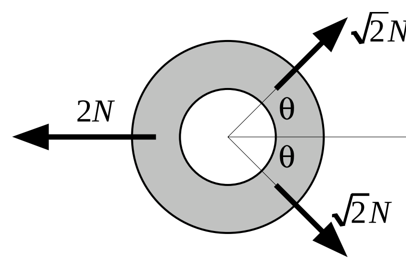
A) Increase. B) Decrease. C) Stay the same.

ii) The force **F** is applied to the box of mass *m* as shown. What is true about the magnitude of the normal force on the box (due to the floor)?



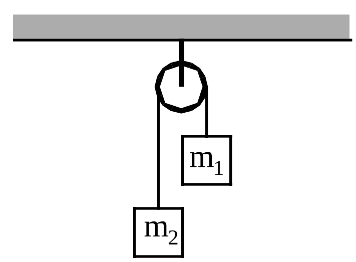
A) It is larger than mg.
B) It is smaller than mg.
C) It is equal to mg.
D) It is independent of the mass of the block.

iii) Three forces are applied to the tire as shown at right. As the angle θ is decreased from its initial value of 45˚, does the magnitude of the acceleration of the tire increase, decrease, or stay the same? There are no forces other than those shown.



A) Increase. B) Decrease. C) Stay the same.

iv) Two blocks, with masses m_1 and m_2, are connected by a string that passes over a massless, frictionless pulley. Block 1 has a greater mass than block 2. Which statement is true regarding the tension T in the string?



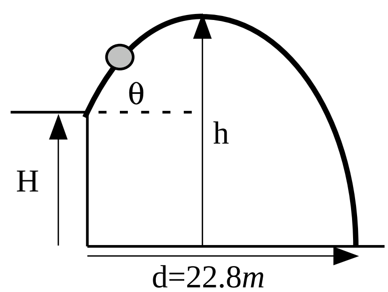
A) $T > m_1 g$.
B) $T < m_2 g$.
C) $T = m_1 g$.
D) $m_2 g < T < m_1 g$.

v) A child sits in a wagon that is moving at constant speed. The child throws an apple straight upward (from her point of view), while the wagon continues to travel forward at constant speed. Where will the apple land?

A) Behind the wagon.
B) In front of the wagon.
C) In the wagon.
D) Need more information.

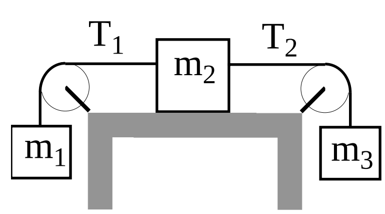
2. (20 points) A speeding car (constant velocity v_1) passes an initially stationary motorcycle policeman. At the instant the car passes the motorcycle, the policemen accelerates (constant acceleration a_2) in pursuit of the lawbreaker. Express the answers below in terms of the parameters v_1, a_2, and other constants as needed.
   a) How much time does it take for the policeman to catch up to the speeder?
   b) How far does the policeman have to travel to catch up to the speeder?
   c) How fast is the policeman traveling when he catches up to the speeder?
   d) What is the policeman's average speed during the chase?
   e) Draw a single graph depicting the positions of the speeder and policeman as a function of time.

3. (20 points) A ball is thrown from the edge of a cliff into the air as shown at right. The initial speed of the ball is 10 *m/s* and the initial upward angle θ is such that sinθ = 4/5. The ball hits the flat ground at a horizontal distance of d = 22.8 *m* from the vertical cliff. Assume g = 10 m/s^2.
   a) How long does the ball remain in the air?
   b) What is the maximum height *h* of the ball above the ground?
   c) What is the height *H* of the cliff?



4. (20 points) A diver of mass 40 *kg* hits the water with an initial vertical speed of 5 *m/s*. After penetrating vertically into the water a distance of 5 *m* she finally comes to rest.
   a) What is the acceleration of the diver as she moves through the water, assuming it remains constant?
   b) What is the force *F exerted by the water* on the diver as she comes to rest? Consider this force as a generic force (*i.e.*, you don't need to know what kind of force it is).
   c) How long does it take the diver to come to rest after first hitting the water?

5. (20 points) Consider a pulley system whose initial state is shown at right. Ignore the mass of the strings and pulleys. Ignore friction. Gravity (g) points down. Express the answers below in terms of the masses, g, and other constants as needed.
   (a) Draw a free-body diagram for each block.
   (b) Find the acceleration of the masses.
   (c) Find the tensions T_1 and T_2 in the strings.



1. i) B As the apple and orange rise, the speed of the apple is always less than the speed of the orange at any given time, since the apple started decelerating sooner. This speed differential causes the apple's lead over the orange to decrease.

   ii) B If the box is not moving normal to the floor, the vertical forces must sum to zero. The normal force plus the vertical component of **F** must then equal *mg*, which means that the normal force will be smaller than *mg*. If **F** were strong enough to cause the box to rise, then the normal force would be zero, which is also less than *mg*.

   iii) A At the initial angle of $\theta = 45^\circ$, the diagonal forces each have horizontal components of 1 *N*, which add up to exactly cancel the 2 *N* force. The vertical components of the diagonal forces cancel each other for all θ. The initial acceleration is thus zero, which means that the magnitude cannot decrease. As θ is decreased, the horizontal components of the diagonal forces get larger and their sum becomes bigger than the 2 *N* force, giving the tire a net acceleration to the right. Thus the magnitude of the tire's acceleration increases.

   iv) D Since $m_1 > m_2$, mass 1 will fall and mass 2 will rise. For this to happen, the tension must be greater than the weight of mass 2 and less than the weight of mass 1.

   v) C With respect to the road or sidewalk, the wagon and the apple have the same horizontal velocities when the apple is thrown up. They maintain these same horizontal velocities while the apple is in the air, so that the apple falls back into the wagon.

2. Define a coordinate system with the *x*-axis along the direction of the speeder and the origin at the initial location of the policeman.

a) Write the kinematic equations and equate the positions when the policeman catches up to the speeder:

$$x_1 = v_1 t$$

$$x_2 = \tfrac{1}{2} a_2 t^2$$

$$x_{1f} = x_{2f} \quad \Rightarrow \quad v_1 t_f = \tfrac{1}{2} a_2 t_f^2$$

$$t_f\left(v_1 - \tfrac{1}{2} a_2 t_f\right) = 0 \quad \Rightarrow \quad t_f = 0, \frac{2v_1}{a_2}$$

The first solution corresponds to the initial time, so we want the second solution:

$$\boxed{t_f = \frac{2v_1}{a_2}}$$

b) The distance can be found from either position equation:

$$x_{1f} = v_1 t_f = v_1 \frac{2v_1}{a_2}$$

$$\boxed{x_1 = \frac{2v_1^2}{a_2}}$$

c) Since the initial speed of the policeman is zero, his final speed is

$$v_{2f} = v_{2i} + a_2 t_f = a_2 t_f = a_2 \frac{2v_1}{a_2}$$

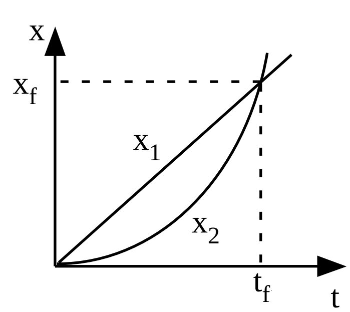
$$\boxed{v_{2f} = 2v_1}$$

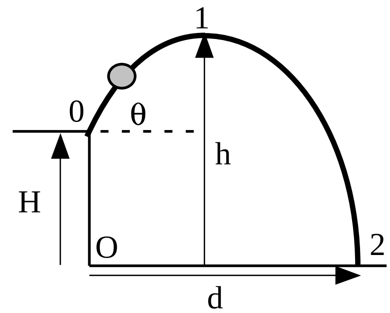
d) The average speed of the policeman is

$$v_{2ave} = \frac{x_{2f} - x_{2i}}{t_f - t_i} = \frac{x_{2f}}{t_f} = \frac{\frac{2v_1^2}{a_2}}{\frac{2v_1}{a_2}} = \frac{2v_1^2}{a_2}\frac{a_2}{2v_1}$$

$\boxed{v_{2ave} = v_1}$ as it should since both travel the same distance in the same time

e) The graph for the chase and catch is:





3. Put the origin at the base of the cliff. Let $t_0 = 0$ be the time when the ball was thrown, t_1 be the time when the maximum height is reached, and t_2 be the time when the ball returns to the ground.

a) First determine the components of the initial velocity and use the *x*-component to find the time in the air, since the horizontal speed is constant.

$$v_{0y} = v_0 \sin\theta = 10\,m/s(4/5) = 8\,m/s$$
$$v_{0x} = v_0 \cos\theta = 10\,m/s(3/5) = 6\,m/s$$
$$d = v_{0x} t_2$$
$$t_2 = d/v_{0x} = 22.8m/(6\,m/s)$$
$$\boxed{t_2 = 3.8s}$$

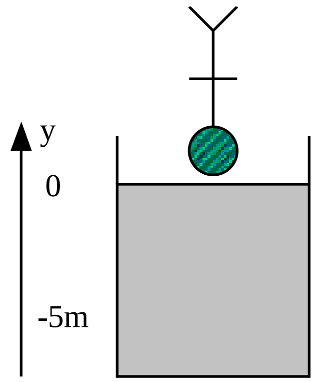
b) To find the maximum height $h = y_1$, find the time to reach that height (where $v_{1y} = 0$), then find how far the ball falls in the remaining time.

$$v_{1y} = v_{0y} - gt_1 \quad \Rightarrow \quad t_1 = \frac{v_{0y}}{g} = \frac{8\,m/s}{10\,m/s^2} = 0.8s$$
$$y_2 = y_1 + v_{1y}(t_2 - t_1) - \tfrac{1}{2} g (t_2 - t_1)^2$$
$$0 = h + 0 - \tfrac{1}{2} g (t_2 - t_1)^2$$
$$h = \tfrac{1}{2} g (t_2 - t_1)^2 = \tfrac{1}{2}(10\,m/s^2)(3.8 - 0.8)^2$$
$$\boxed{h = 45m}$$

c) The height of the cliff $H = y_0$, can be found by considering the first part of the trip:

$$v_{1y}^2 = v_{0y}^2 + 2a_y(y_1 - y_0)$$
$$0 = v_{0y}^2 + 2(-g)(h - H)$$
$$H = h - \frac{v_{0y}^2}{2g} = 45m - \frac{(8\,m/s)^2}{2(10\,m/s^2)} = 45m - 3.2m$$
$$\boxed{H = 41.8m}$$

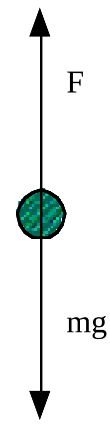
4. Let the 1-dimensional coordinate system have its origin at the surface of the water so that $y_0 = 0$, $v_0 = -5\ m/s$. The diver comes to rest at $y_1 = -d = -5\ m$, so we know that $v_1 = 0$.



a) To find the acceleration use:

$$v_1^2 = v_0^2 + 2a(y_1 - y_0)$$
$$0 = v_0^2 + 2a(-d - 0)$$
$$a = \frac{v_0^2}{2d} = \frac{(-5\,m/s)^2}{2(5m)} = \frac{25m^2/s^2}{10m}$$
$$\boxed{a = 2.5\,m/s^2}$$

b) First draw a free body diagram for the diver, as at right. Then write down the equation of motion.

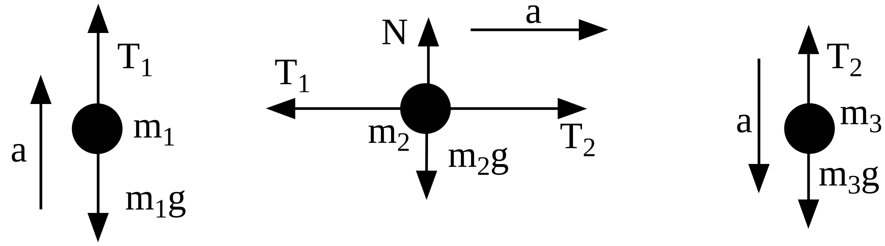


$$F - mg = ma$$
$$F = ma + mg = m(a + g)$$
$$F = 40kg(2.5\,m/s^2 + 10\,m/s^2)$$
$$\boxed{F = 500N}$$

c) The time for the diver to come to rest is found from:

$$v_1 = v_0 + at_1$$
$$0 = v_0 + at_1$$
$$t_1 = -\frac{v_0}{a} = -\frac{(-5\,m/s)}{2.5\,m/s^2}$$
$$\boxed{t_1 = 2s}$$

5. a) The three free body diagrams are shown below. Since the masses are tied together, they all accelerate with the same acceleration a. Since we don't know the masses, we don't know which way they will go, so we choose a direction and call that positive.



b) The equations of motion for the masses are (neglecting the vertical motion of m_2)

$$T_1 - m_1 g = m_1 a$$

$$T_2 - T_1 = m_2 a$$

$$m_3 g - T_2 = m_3 a$$

Solve these to find a:

$$T_1 = m_1 g + m_1 a$$

$$T_2 = m_3 g - m_3 a$$

$$m_3 g - m_3 a - (m_1 g + m_1 a) = m_2 a$$

$$m_3 g - m_1 g = (m_1 + m_2 + m_3) a$$

$$\boxed{a = \frac{m_3 - m_1}{m_1 + m_2 + m_3} g}$$

c) Now find the tensions:

$$T_1 = m_1 g + m_1 a = m_1 g + m_1 \frac{m_3 - m_1}{m_1 + m_2 + m_3} g$$

$$T_1 = \frac{m_1(m_1 + m_2 + m_3) + m_1(m_3 - m_1)}{m_1 + m_2 + m_3} g$$

$$\boxed{T_1 = \frac{(m_2 + 2m_3)}{m_1 + m_2 + m_3} m_1 g}$$

$$T_2 = m_3 g - m_3 a = m_3 g - m_3 \frac{m_3 - m_1}{m_1 + m_2 + m_3} g$$

$$T_2 = \frac{m_3(m_1 + m_2 + m_3) - m_3(m_3 - m_1)}{m_1 + m_2 + m_3} g$$

$$\boxed{T_2 = \frac{(2m_1 + m_2)}{m_1 + m_2 + m_3} m_3 g}$$