

- Exam is closed book, closed notes. Use only your formula sheet.
- Write all work and answers in exam booklets.
- The backs of pages will not be graded unless you so request on the front of the page.
- Show all your work and explain your reasoning (except on #1).
- Partial credit will be given (not on #1). No credit will be given if no work is shown (not on #1).
- If you have a question, raise your hand or come to the front.

1. (35 points) For each of these multiple choice questions, indicate the correct response (A, B, C, or D (where needed)) on the page for problem 1 in your exam booklet.

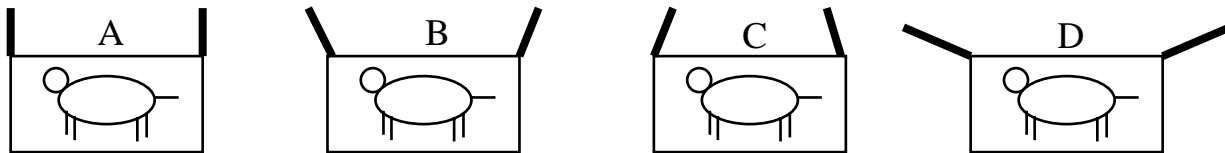
i) A penguin stands at one end of a stationary sled that lies on frictionless ice. The penguin then walks to the other end of the sled. When it reaches the other end, its friends notice that the sled has also moved with respect to the ice. If the penguin were heavier, would its friends observe the distance the sled moves to be greater than, less than, or the same as the original distance the sled moved?

- A) Greater than. B) Less than. C) The same as.

ii) You throw a ball into the air. During the flight of the ball, which statement is true? Ignore air resistance and assume you are on earth.

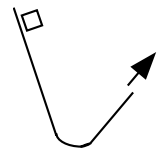
- A) The momentum of the ball is conserved in the vertical direction.
 B) The momentum of the ball is conserved in the horizontal direction.
 C) The momentum of the ball is conserved in the vertical and horizontal directions.
 D) The momentum of the ball is conserved in neither direction.

iii) Four possibilities for hanging a picture are shown below. In each case, two identical lengths of wire are connected to the upper two corners of the picture and then to the wall. In which arrangement is the tension in the wires the largest?

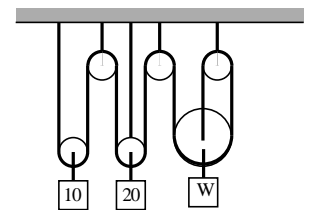


iv) A block slides down a frictionless ramp as shown at right after being released from rest. The end of the ramp is angled upwards, causing the block to become airborne, as the arrow indicates. Is the maximum height reached by the block in the air larger, smaller, or the same as the original height of the block on the ramp.

- A) Larger. B) Smaller. C) The same as.



v) Three blocks hang from the stationary assembly of massless pulleys and ropes shown at right. One long rope runs from the ceiling at the left to the lower pulley at the right. Several shorter ropes suspend pulleys from the ceiling or blocks from the pulleys. The weights (in Newtons) of two blocks are given. Is the weight W of the third block greater than 20 N , less than 10 N , or between 10 N and 20 N inclusive?



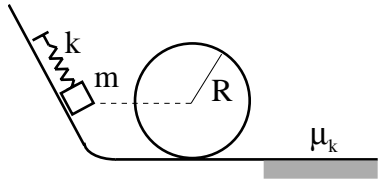
- A) $W > 20\text{ N}$ B) $W < 10\text{ N}$ C) $10\text{ N} \leq W \leq 20\text{ N}$

vi) Two balls approach each other at equal speeds. The ball moving to the right has a larger mass than the ball moving to the left. After the two balls collide head on, is the center-of-mass of the two-ball system moving to the right, to the left, or is it stationary?

- A) To the right. B) To the left. C) Stationary.

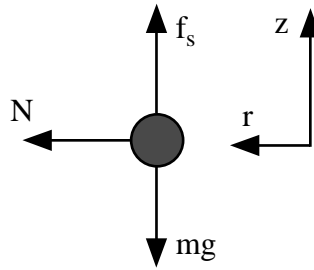
vii) A block slides up a ramp, momentarily comes to rest, and slides back down. There is friction between the block and the surface of the ramp. Assume that the ramp is so steep that the block cannot remain at rest on the ramp. Which statement is true regarding the time that the block takes to go up the ramp compared to the time to come back down?

- A) The time up is longer than the time down.
 B) The time up is shorter than the time down.
 C) The time up is equal to the time down.

2. (30 points) Joe Bob is eager to ride the Rotor at the state fair. The Rotor is a large hollow cylinder that is rotated around its vertical axis. Joe Bob enters the Rotor through a door on the side and stands on the floor, up against a canvas covered wall. After the door is closed, the Rotor begins to spin. After a while the floor of the Rotor falls away, but Joe Bob is pinned to the wall, spinning with the same velocity as the wall of the Rotor. Joe Bob has a mass of 80 kg and the radius of the Rotor cylinder is 4 m . The coefficient of static friction between Joe Bob's clothes and the Rotor wall is $5/8$. Assume $g = 10\text{ m/s}^2$.
- Draw a free body diagram for Joe Bob.
 - What is the condition on the velocity of Joe Bob (and the wall of the Rotor) such that he doesn't slide down the wall when the floor falls away?
 - Joe Bob has a weak stomach and cannot keep his lunch down if he experiences an acceleration greater than 2 times g . Will Joe Bob lose his lunch on the Rotor? Assume that the Rotor spins with the minimum velocity needed to satisfy the condition found in part (b).
3. (25 points) Two 40 kg ice sleds are at rest a short distance apart on a frictionless frozen pond. A 10 kg cat, standing on one sled, jumps to the other sled at a speed of 8 m/s with respect to the ice. Find the final velocities of the two sleds.
4. (30 points) A small block of mass m can slide on the frictionless loop-the-loop track (radius R) shown at right. The block is initially at rest against a compressed spring (spring constant k) on the incline at a height R . Express your results in terms of the given quantities and the gravitational acceleration g .
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- What minimum compression of the spring is required to ensure that the block does not leave the track during its travel through the loop-the-loop? Assume that the block starts at height R no matter the compression.
 - After going through the loop-the-loop, the block slides across a rough horizontal surface with coefficient of kinetic friction μ_k . Find the distance the block travels on the rough surface before coming to rest, assuming the spring compression is the minimum value calculated in (a).
5. (30 points) Junior is riding in the shopping cart while shopping with her dad. While at rest in the cart, Junior notices a candy bar that is just out of her reach on a nearby shelf. In disgust, Junior tosses a can of spinach from the front of the cart to the back of the cart. Junior then notices that she has moved 5.0 cm closer to the candy bar, which she can now reach. The cart, with Junior and all groceries included, has a total mass of 30 kg and a length of 1.5 m , and there is no friction between the cart and the floor.
- What is the mass of the can of spinach?
 - As Junior's dad returns to the cart, she throws him a can of soup. He catches the soup and notes its speed as 19 m/s with respect to the floor. Junior races away at 1.0 m/s with respect to the floor (Junior was at rest before she threw the can of soup). What is the mass of the can of soup?
6. (25 points) A 50 kg girl and a 30 kg sled are on the surface of a frozen lake 16 m apart and initially at rest. By means of a rope, the girl exerts a constant horizontal force of 150 N on the sled, pulling it toward her. Ignore friction.
- What is the acceleration of the sled?
 - What is the acceleration of the girl?
 - How long does it take for the girl to meet the sled?
7. (25 points) A bungee-cord jumper of mass 40 kg is standing on a bridge that is 60 m above the river below. Her bungee cord behaves like a spring and has a relaxed length of 20 m . Assume $g = 10\text{ m/s}^2$ and that the bungee can only stretch, not compress.
- What must the spring constant k of the bungee cord be if she wishes to just barely touch the surface of the water after jumping off the bridge? Ignore her height in this problem.
 - What is the force acting on her at the lowest point in her fall?

1. i) A Since the sled and penguin are initially at rest, the center of mass of the system will remain at rest. Individual motion of the constituents of the system must then satisfy $m_1\Delta x_1 + m_2\Delta x_2 = 0$ to keep the center of mass fixed. Thus the motion of each object is inversely proportional to its mass, and proportional to the other mass. A heavier penguin will mean that the penguin moves a smaller distance and the sled a larger distance.
- ii) B The external force on the ball is gravity, which is in the vertical direction. Momentum in a given direction is conserved when there is no external force in that direction. Hence only the horizontal momentum is conserved here.
- iii) D In each case the vertical component of the tension must be half the weight of the picture. Hence the wires that are close to horizontal will have larger tension.
- iv) B When the block reaches the top of its parabolic path, it will still have the same horizontal velocity it had when it left the ramp. Since the kinetic energy at the top of the path is not zero, the gravitational potential energy, and hence the height, will be less than at the top of the ramp when it started at rest.
- v) C Since the pulley holding the 10 N block is hanging on two pieces of the long rope, the tension in the long rope is 5 N. The pulley holding the unknown block is supported by three pieces of the long rope, so its weight must be 15 N.
- vi) A Since the ball moving to the right has more mass, the center-of-mass of the system moves to the right; it is unaffected by the collision.
- vii) B Since kinetic friction dissipates energy, the speed of the returning block at the bottom of the ramp will be less than the initial speed up the ramp. This means that the average speed on the way up is larger than on the way down. Since the distances are the same, the block takes longer to get down.

2. a) The free body diagram is as shown, with coordinate axes vertically (z) and radially (r).



- b) Joe Bob is moving in a horizontal circle, so the equations of motion are:

$$N = ma_r = m \frac{v^2}{r}$$

$$f_s - mg = 0$$

Static friction must obey the equation:

$$f_s \leq \mu_s N$$

Combining these equations gives:

$$mg \leq \mu_s m \frac{v^2}{r}$$

$$v^2 \geq \frac{gr}{\mu_s}$$

$$v \geq \sqrt{\frac{gr}{\mu_s}} = \sqrt{\frac{10 \text{ m/s}^2 (4 \text{ m})}{5/8}}$$

$$\boxed{v \geq 8 \text{ m/s}}$$

- c) To see if Joe Bob can keep his lunch, calculate his acceleration

$$a_r = \frac{v^2}{r} = \frac{gr}{r\mu_s} = \frac{g}{\mu_s} = \frac{10 \text{ m/s}^2}{5/8}$$

$$\boxed{a_r = 16 \text{ m/s}^2 = 1.6g}$$

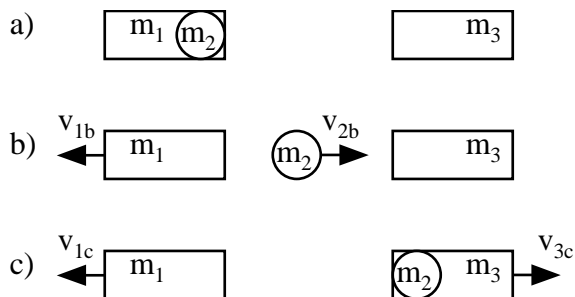
So Joe Bob holds onto his lunch!

3. Let $m_1 = m_3 = 40 \text{ kg}$ be the sleds and $m_2 = 10 \text{ kg}$ be the cat. This problem has three situations, which we will label a, b, and c as shown.

The velocity of each mass is labeled with the mass # and the letter for the time. We know that

$$v_{1a} = v_{2a} = v_{3a} = 0 \text{ m/s}, v_{1b} = v_{1c}, \text{ and } v_{2b} = 8 \text{ m/s} = V.$$

Momentum is conserved throughout since there are no external horizontal forces, but energy is not conserved since the cat needs friction to jump and stop.



Since everything is at rest to begin, there is no momentum in the system. Thus the momentum in each case must equal zero. The equations are given below (all velocities are assumed positive, so the signs will be explicit in the solution) along with their simplified versions

$$m_1 v_{1b} + m_2 v_{2b} = 0$$

$$m_1 v_{1c} + m_2 V = 0$$

$$m_1 v_{1c} + (m_2 + m_3) v_{3c} = 0$$

$$m_1 v_{1c} + (m_2 + m_1) v_{3c} = 0$$

Solve the 1st equation:

$$m_1 v_{1c} = -m_2 V$$

$$v_{1c} = -\frac{m_2}{m_1} V = -\frac{10 \text{ kg}}{40 \text{ kg}} 8 \text{ m/s}$$

$$\boxed{v_{1c} = -2 \text{ m/s}}$$

Now plug this into the 2nd equation to get:

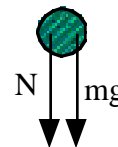
$$v_{3c} = -\frac{m_1}{m_1 + m_2} v_{1c}$$

$$v_{3c} = -\frac{m_1}{m_1 + m_2} \left(-\frac{m_2}{m_1} V \right) = \frac{m_2}{m_1 + m_2} V$$

$$v_{3c} = \frac{10 \text{ kg}}{40 \text{ kg} + 10 \text{ kg}} 8 \text{ m/s}$$

$$\boxed{v_{3c} = 1.6 \text{ m/s}}$$

4. a) Energy is conserved throughout the motion. If the block is to leave the track it will do so at the top of the loop, so we must look there first. The free body diagram and the equations of motion are:



$$N + mg = m \frac{v^2}{R}$$

$$N = m \frac{v^2}{R} - mg$$

To keep the block on the track requires $N \geq 0$, which leads to $v^2 \geq gR$. The velocity at the top of the loop can be found from conservation of energy. Choose $y = 0$ at the bottom of the loop for convenience in calculating the gravitational potential energy.

$$\frac{1}{2} kx^2 + mgR = mg2R + \frac{1}{2} mv^2$$

The minimum compression will correspond to the minimum velocity $v_{\min} = \sqrt{gR}$:

$$\frac{1}{2} kx_{\min}^2 + mgR = mg2R + \frac{1}{2} mgR$$

$$\frac{1}{2} kx_{\min}^2 = \frac{3}{2} mgR$$

$$x_{\min}^2 = \frac{3mgR}{k}$$

$$x_{\min} = \sqrt{\frac{3mgR}{k}}$$

- b) To find the distance the block takes to stop, use the energy budget equation, noting that $f_k = \mu_k N$ and $N = mg$ in this case:

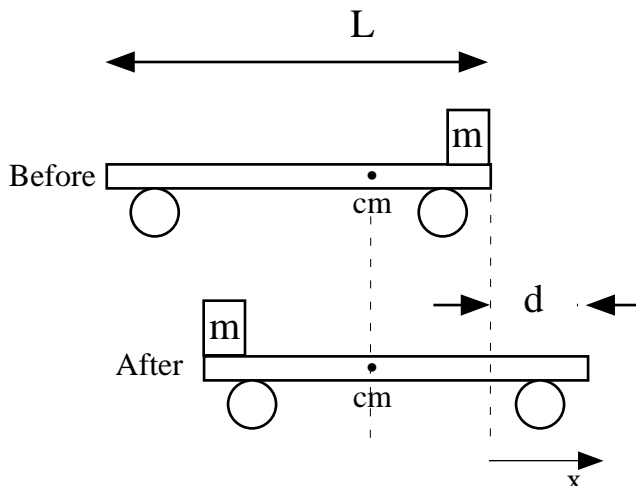
$$\frac{1}{2} kx_{\min}^2 + mgR - f_k d = 0$$

$$\frac{1}{2} kx_{\min}^2 + mgR - \mu_k mgd = 0$$

$$d = \frac{\frac{1}{2} kx_{\min}^2 + mgR}{\mu_k mg} = \frac{\frac{3}{2} mgR + mgR}{\mu_k mg}$$

$$d = \frac{5R}{2\mu_k}$$

5. a) Since there are no horizontal forces acting, the center of mass will not move, as shown at right. Let $M = 30 \text{ kg}$ be the mass of the cart and Junior and all groceries combined. Let m be the mass of the can (spinach in part (a) and soup in part(b)). Put the origin of the x -axis at the front of the cart, where the can of spinach is initially. Let $L = 1.5 \text{ m}$ be the length of the cart and $d = 0.050 \text{ m}$ be the distance that the cart travels when the can is tossed to the back.



Let x_2 be the position of the can of spinach (0 to start) and let x_1 be the position of the center of mass of all the rest of the system (cart, Junior, other groceries). Then the center of mass of the whole system before Junior throws the spinach is

$$x_{\text{cm}} = \frac{(M - m)x_1 + mx_2}{M} = \frac{(M - m)x_1}{M}$$

The center of mass after Junior throws the spinach is

$$x_{\text{cm}} = \frac{(M - m)(x_1 + d) + m(-L + d)}{M}$$

These two expressions must be equal since the center of mass does not move:

$$(M - m)x_1 = (M - m)(x_1 + d) - m(L - d)$$

$$0 = Md - md - mL + md$$

$$Md = mL$$

$$m = \frac{d}{L}M = \frac{0.050\text{m}}{1.5\text{m}}30\text{kg}$$

$$\boxed{m_{\text{spinach}} = 1.0\text{kg}}$$

- b) Since there are no horizontal forces, horizontal momentum will be conserved. Let $v_1 = 1.0 \text{ m/s}$ be Junior's velocity and $v_2 = -19 \text{ m/s}$ be the soup can's velocity.

$$P_i = P_f$$

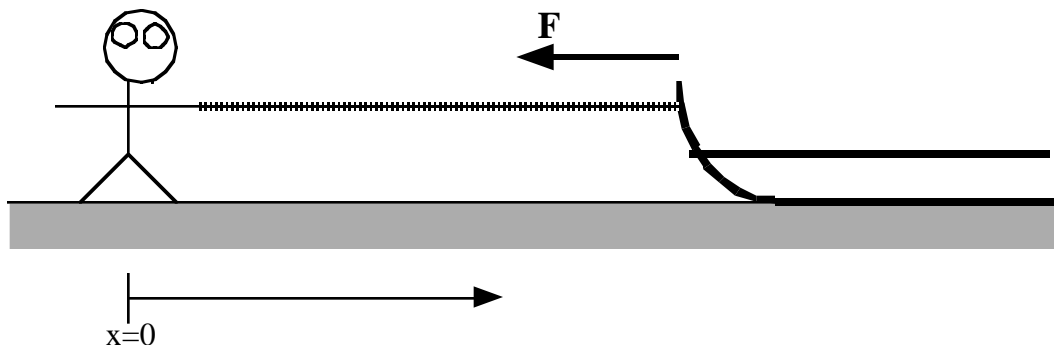
$$0 = (M - m)v_1 + mv_2$$

$$Mv_1 + m(v_2 - v_1) = 0$$

$$m = \frac{v_1}{v_1 - v_2}M = \frac{1.0\text{ m/s}}{1.0\text{ m/s} - (-19\text{ m/s})}30\text{kg}$$

$$\boxed{m_{\text{soup}} = 1.5\text{kg}}$$

6. The situation is as shown below. Let the 1-dimensional coordinate system have its origin at the girl so that $x_{g0} = 0$, $x_{s0} = d = 16\text{ m}$.



- a) The force on the sled is to the left and so is negative. Thus we get

$$-F = m_s a_s$$

$$a_s = \frac{-F}{m_s} = \frac{-150\text{ N}}{30\text{ kg}}$$

$$\boxed{a_s = -5.0\text{ m/s}^2}$$

- b) Since the girl exerts a force on the sled, the sled must exert an equal and opposite force on the girl, by Newton's third law. This force is just the tension in the rope. Thus we get:

$$F = m_g a_g$$

$$a_g = \frac{F}{m_g} = \frac{150\text{ N}}{50\text{ kg}}$$

$$\boxed{a_g = +3.0\text{ m/s}^2}$$

- c) The girl and the sled both start from rest, with the girl at the origin and the sled at $x_{s0} = d = 16\text{ m}$. The positions of the two are thus given by

$$x_g = \frac{1}{2} a_g t^2 \qquad x_s = d + \frac{1}{2} a_s t^2$$

If we now equate the two positions at the common time t when they meet, then we get

$$\frac{1}{2} a_g t^2 = d + \frac{1}{2} a_s t^2$$

$$t^2 = \frac{d}{\frac{1}{2}(a_g - a_s)}$$

$$t = \sqrt{\frac{2d}{a_g - a_s}} = \sqrt{\frac{2(16\text{ m})}{3.0\text{ m/s}^2 - (-5.0\text{ m/s}^2)}} = \sqrt{\frac{32\text{ m}}{8.0\text{ m/s}^2}}$$

$$\boxed{t = 2.0\text{ s}}$$

7. a) Energy is conserved in this process, and if we consider the initial state as the jumper on the bridge and the final state as the jumper just touching the water, then we only have to consider potential energy. Since the bungee only stretches, its potential energy only contributes at the bottom of the jump. Let the height of the bridge be $h = 60\text{ m}$ and the relaxed length of the bungee be $L = 20\text{ m}$. Then the stretch in the bungee is $h - L$.

$$mgh = \frac{1}{2}k(h - L)^2$$

$$k = \frac{2mgh}{(h - L)^2}$$

$$k = \frac{2 \cdot 40\text{ kg} \cdot 10\text{ m/s}^2 \cdot 60\text{ m}}{(60\text{ m} - 20\text{ m})^2}$$

$$\boxed{k = 30\text{ N/m}}$$

- b) At the bottom of the fall the forces acting on the jumper are gravity (down) and the spring force (up).

$$F = kx - mg$$

$$F = k(h - L) - mg$$

$$F = 30\text{ N/m} \cdot (60\text{ m} - 20\text{ m}) - 40\text{ kg} \cdot 10\text{ m/s}^2$$

$$F = 1200\text{ N} - 400\text{ N}$$

$$\boxed{F = 800\text{ N} \quad \text{upwards}}$$