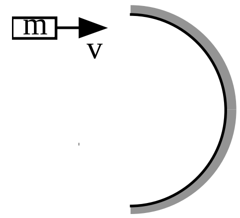
PHYSICS 211 **FINAL EXAM** **7 June 2004**

- Exam is closed book, closed notes. Use only your formula sheet.
- Write all work and answers in exam booklets.
- The backs of pages will not be graded unless you so request on the front of the page.
- **Show all your work** and explain your reasoning (except on #1).
- Partial credit will be given (not on #1). No credit will be given if no work is shown (not on #1).
- If you have a question, raise your hand or come to the front.

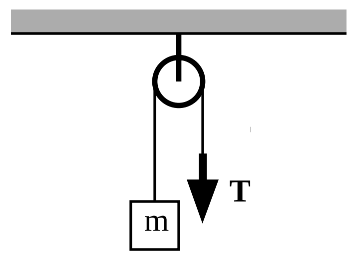
1. (35 points) For each of these multiple choice questions, indicate the correct response (A, B, C, or D (where needed)) on the page for problem 1 in your exam booklet.

i) A block of mass *m* is sliding on a horizontal, frictionless table and then encounters a fixed semi-circular wall as shown at right (looking down on the table). There is friction between the wall and the block, so that the kinetic energy of the block is reduced as it travels along the wall. If the initial velocity *v* is increased, does the loss of kinetic energy ΔK of the block increase, decrease, or stay the same?



A) Increase. B) Decrease. C) Stay the same.

ii) A tension T is applied to the rope while the mass *m* is moving upward with a decreasing speed. What is true about the magnitude of the tension?



A) It is larger than *mg*.
B) It is smaller than *mg*.
C) It is equal to *mg*.

iii) Sammy Sosa can swing his corked bat at 120 mph. Jamie Moyer can throw a fastball at 80 mph. When Jamie's fastball meets Sammy's bat in an elastic head-on collision, at what speed will the ball leave the bat? Assume that the mass of the ball is negligible compared to that of the bat.

A) 160 mph B) 240 mph C) 300 mph D) 320 mph

iv) A golf ball (very light) is fired at a bowling ball (very heavy), which is initially at rest. The golf ball bounces back elastically. After the collision, the golf ball, compared to the bowling ball, has

A) a larger magnitude of momentum but smaller kinetic energy.
B) a larger magnitude of momentum and larger kinetic energy.
C) a smaller magnitude of momentum and smaller kinetic energy.
D) a smaller magnitude of momentum but larger kinetic energy.

v) A spring-loaded gun shoots marble **A** horizontally off a table. Marble **A** lands on the floor a horizontal distance *d* from the launch point. A different spring-loaded gun shoots marble **B** vertically. Marble **B** travels up the same distance *d* vertically before falling back down. Both springs are then compressed twice their original compressions and the experiments are repeated. Which distance is now greater: the horizontal distance that marble **A** travels before hitting the floor, or the vertical distance marble **B** travels to its maximum height?

A) Marble **A** horizontal. B) Marble **B** vertical. C) They travel the same distance as each other.

vi) Two balls are dropped from a cliff. The second ball is dropped one second after the first ball is dropped. As both fall, does the distance between them increase, decrease, or stay the same?

A) Increase. B) Decrease. C) Remain the same.

vii) A block slides up a ramp, momentarily comes to rest, and slides back down. There is friction between the block and the surface of the ramp. Assume that the ramp is so steep that the block cannot remain at rest on the ramp. Which statement is true regarding the time that the block takes to go up the ramp compared to the time to come back down?

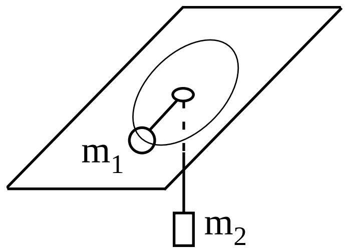
A) The time up is longer than the time down.
B) The time up is shorter than the time down.
C) The time up is equal to the time down.

2. (30 points) Two subway stops are separated by 400 *m*. A subway train accelerates at 4 m/s^2 from rest through the first half of the distance and decelerates at -4 m/s^2 through the second half.

a) What is the travel time between the two stops?

b) What is the maximum speed?

c) Graph *x*, *v*, and *a*, each as a function of time, for the complete trip.



3. (35 points) A mass m_1 on a horizontal table is attached to a hanging mass m_2 by a string through a hole in the table, as shown at right. Express all your answers in terms of the quantities given and *g*.

a) If there is friction between the table and the mass m_1, what coefficient of static friction is required to keep the system stationary? Is your value a maximum or minimum?

b) If the table is frictionless and mass m_1 moves in a circle of radius *R* (as shown in figure), what speed is required to keep m_2 at rest?

4. (35 points) Lisa (mass m_L = 60 *kg*) and Kate (mass m_K = 40 *kg*) are standing at the center of a symmetric, stationary railroad car (mass m_{RR} = 100 *kg*, length *L* = 20 *m*) that can roll without friction along a straight track. Lisa runs toward one end of the railroad car with a speed of 6 *m/s* with respect to the railroad car and Kate runs toward the other end with a speed of 2 *m/s* with respect to the railroad car. Lisa runs all the way to her end of the car, but Kate stops after running a distance of 4 *m* with respect to the railroad car.

b) What is the velocity of the railroad car (with respect to the ground) while Lisa and Kate are running?

c) What is the velocity of the railroad car (with respect to the ground) after both Lisa and Kate have stopped running?

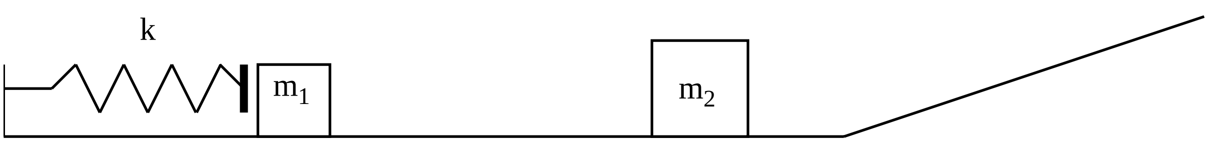
d) What is the displacement of the railroad car (with respect to the ground) after both Lisa and Kate have stopped running?

5. (35 points) A rocket is launched vertically from the surface of planet Claire, which has a mass $M = 5.0 \times 10^{21}$ *kg* and a radius $R = 1.2 \times 10^5$ *m* and does not rotate. The rocket reaches a speed of 1000 *m/s* in a negligible distance and then the rocket engines are turned off. Assume $G = 6.0 \times 10^{-11}\ Nm^2/s^2$.

a) What is the maximum altitude (above the surface of the planet) reached by the rocket?

b) When the rocket reaches its maximum altitude, the rocket engines are briefly turned on again to give the rocket a velocity in the horizontal direction (i.e., perpendicular to the vertical path taken from Claire). What speed is required to put the rocket into a circular orbit around Claire?

6. (30 points) A block of mass m_1 = 2 *kg* is pushed up against a spring (*k* = 8 *N/m*) as shown below. The spring is compressed a distance *d* = 3 *m* before the block is released. The block slides on a frictionless surface and, after leaving the spring, collides elastically with another block of mass m_2 = 4 *kg* that is initially at rest. The second block then slides up the frictionless ramp. What is the height *h* (measured vertically from the bottom of the ramp) of the second block when it momentarily comes to rest on the ramp?



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1\. i) A The loss of kinetic energy is given by $-f_k d$, where $f_k = \mu_k N$ and d is the arc length of the semicircle. Since the normal force N is the only horizontal force, it must produce the centripetal acceleration v^2/r. As v increases, the normal force, $N = mv^2/r$, will increase, so the frictional force will increase. Thus, for larger v, the kinetic energy loss will increase.

ii) B The mass is moving up, but with a decreasing speed, which means that the acceleration is downward. Since the acceleration is downward, the net force must be down. Hence, the weight (down) must be larger than the tension (up).

iii) D In elastic collisions the speed of approach of the two objects is equal to the speed of recession. The relative speed of approach here is 200 mph (120 + 80), so the speed of recession will be 200 mph with respect to the moving bat, which is not slowed down in the collision because of its dominant mass. Hence the speed of the ball is 320 mph (200 +120).

iv) D Before the collision, the golf ball (and the total system) has momentum mv. After the elastic collision, the golf ball will have momentum approximately $-mv$ and the bowling ball will have momentum approximately $2mv$ (in order to have total system momentum mv). The kinetic energy of the golf ball will be approximately $\frac{1}{2}mv^2$ and that of the bowling ball will be approximately

$\frac{1}{2}MV^2 = \frac{1}{2}M\left(\frac{2mv}{M}\right)^2 = \frac{4m}{M}\left(\frac{1}{2}mv^2\right)$. Since $m << M$, the bowling ball has less kinetic energy.

v) B Twice the spring compression implies four times the kinetic energy and hence twice the initial velocity. Conservation of energy ($KE_i = mgh$) implies that marble **B** goes four times as high as originally ($4d$). Marble **A** still takes the same time to hit the floor, and since its horizontal velocity is twice the original, it will go twice as far ($2d$).

vi) A As the balls fall, the speed of the first ball is always greater than the speed of the second ball at any given time. This speed differential causes the first ball to keep increasing its lead over the second ball.

vii) B Since kinetic friction dissipates energy, the speed of the returning block at the bottom of the ramp will be less than the initial speed up the ramp. This means that the average speed on the way up is larger than on the way down. Since the distances are the same, the block takes longer to get down.

2. Define a coordinate system with the *x*-axis along the subway line. Let the first stop be at position $x_0 = 0$, where the train starts at time $t_0 = 0$. At time t_1 the train reaches the halfway point $x_1 = d/2$, and at time t_2 it reaches the second stop at the position $x_2 = d$. During the first half of the journey, the acceleration is $a_1 = +4\ m/s^2$, and during the second half of the journey, the acceleration is $a_2 = -4\ m/s^2 = -a_1$. Since the train starts at rest and finishes at rest, we also know that $v_0 = 0$ and $v_2 = 0$.

a) The first thing to notice is that since the acceleration has the same magnitude on the two halves of the journey, we expect that the two halves take the same time. To show this simply, use the expression for the acceleration:

$$a_1 = \frac{v_1 - v_0}{t_1 - t_0} = \frac{v_1}{t_1}$$

$$a_2 = \frac{v_2 - v_1}{t_2 - t_1} = -\frac{v_1}{t_2 - t_1}$$

$$t_2 - t_1 = -\frac{v_1}{a_2} = \frac{v_1}{a_1} = t_1$$

The equation of motion for the first half of the journey can then be used to find t_1.

$$x = x_0 + v_0 t + \frac{1}{2} a_1 t^2$$

$$x_1 = \frac{1}{2} a_1 t_1^2 = \frac{d}{2}$$

$$t_1 = \sqrt{\frac{d}{a_1}}$$

which gives for the total time t_2:

$$t_2 = 2t_1 = 2\sqrt{\frac{d}{a_1}} = 2\sqrt{\frac{400m}{4m/s^2}}$$

$$\boxed{t_2 = 20s}$$

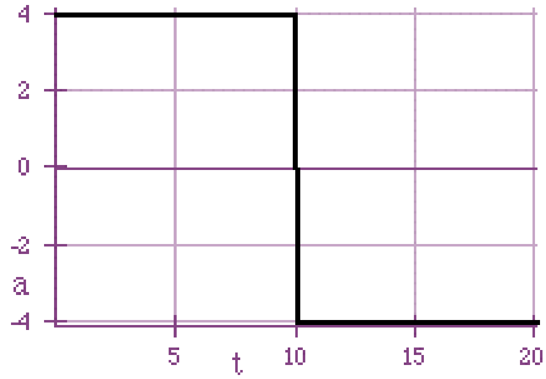
b) The maximum speed is just the speed at the midway point v_1, which can be obtained using:

$$v_1 = v_0 + a_1 t_1$$

$$v_1 = a_1 t_1 = a_1 \sqrt{\frac{d}{a_1}} = \sqrt{a_1 d} = \sqrt{4m/s^2 (400m)}$$

$$\boxed{v_1 = 40m/s}$$

c) The graphs for the trip are: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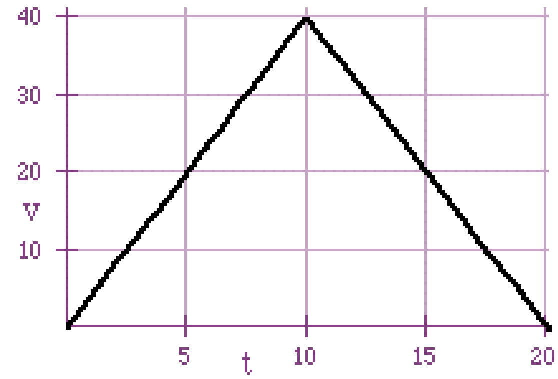
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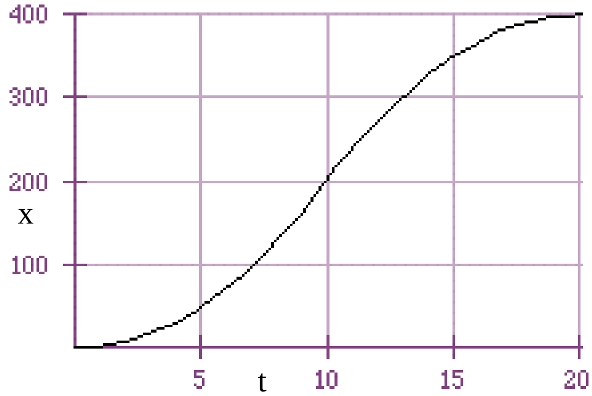
400
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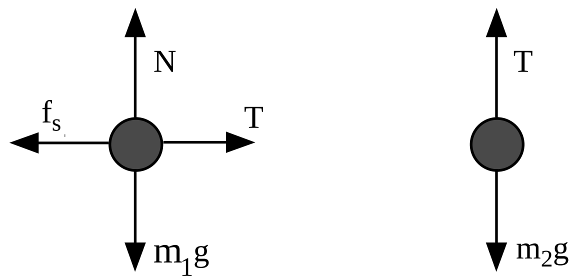
3. a) The free body diagrams for the static case are shown below:



Since there is no motion of the system, all accelerations are zero and the equations of motion are:

$$T - f_s = 0 \quad \Rightarrow \quad T = f_s$$
$$N - m_1 g = 0 \quad \Rightarrow \quad N = m_1 g$$
$$T - m_2 g = 0 \quad \Rightarrow \quad T = m_2 g \quad \Rightarrow \quad f_s = m_2 g$$

The static friction must obey the equation $f_s \le \mu_s N$, which gives

$$m_2 g \le \mu_s m_1 g$$
$$\boxed{\mu_s \ge \frac{m_2}{m_1}}$$

So the ratio of masses is the minimum value of μ_s needed to keep the blocks stationary.

b) When there is no friction and mass m_1 moves in a circle while mass m_2 is stationary, the free body diagrams are as above except there is no frictional force.

The equations of motion in this case are:

$$T = m_1 \frac{v^2}{R}$$
$$N - m_1 g = 0 \quad \Rightarrow \quad N = m_1 g$$
$$T - m_2 g = 0 \quad \Rightarrow \quad T = m_2 g$$

Equating the two expressions for the tension, we get:

$$m_1 \frac{v^2}{R} = m_2 g$$
$$v^2 = \frac{m_2}{m_1} gR$$
$$\boxed{v = \sqrt{\frac{m_2}{m_1} gR}}$$

4. a) The system (Lisa, Kate, & car) is at rest and no horizontal forces act, so the center of mass of the system will not move and horizontal momentum will be conserved. The initial momentum is zero, so the final momentum must also be zero. The equations we need are conservation of momentum and the equations relating the motion of Lisa (L) and Kate (K) relative to the car (RR) to their motion relative to the ground (G). Let Lisa move in the positive direction, which means that Kate moves in the negative direction.

$$m_L v_{L/G} + m_K v_{K/G} + m_{RR} v_{RR/G} = 0$$

$$v_{L/G} = v_{L/RR} + v_{RR/G}$$

$$v_{K/G} = v_{K/RR} + v_{RR/G}$$

$$m_L\left(v_{L/RR} + v_{RR/G}\right) + m_K\left(v_{K/RR} + v_{RR/G}\right) + m_{RR} v_{RR/G} = 0$$

$$\left(m_L + m_K + m_{RR}\right) v_{RR/G} + m_L v_{L/RR} + m_K v_{K/RR} = 0$$

$$v_{RR/G} = -\frac{m_L v_{L/RR} + m_K v_{K/RR}}{m_L + m_K + m_{RR}} = -\frac{60kg(6\,m/s) + 40kg(-2\,m/s)}{60kg + 40kg + 100kg}$$

$$\boxed{v_{RR} = -1.4\,m/s}$$

The minus sign means that the car moves opposite to the motion of Lisa.

b) After Lisa and Kate have stopped running, the car must also stop so that there is no net momentum in the system. This is also clear if we apply the result from above with $v_{L/RR} = v_{K/RR} = 0$.

$$\boxed{v_{RR} = 0\,m/s}$$

c) Put the origin at the center of the car where Lisa and Kate start, so all three things start at $x = 0$. The equations we need are the position of the system center of mass (which must be zero) and the equation relating the positions of Lisa (L) and Kate (K) relative to the car (RR) to their positions relative to the ground (G).

$$x_{cm} = \frac{m_L x_{L/G} + m_K x_{K/G} + m_{RR} x_{RR/G}}{m_L + m_K + m_{RR}} = 0$$

$$\Rightarrow \quad m_L x_{L/G} + m_K x_{K/G} + m_{RR} x_{RR/G} = 0$$

$$x_{L/G} = x_{L/RR} + x_{RR/G} \qquad\qquad x_{K/G} = x_{K/RR} + x_{RR/G}$$

$$m_L(x_{L/RR} + x_{RR/G}) + m_K(x_{K/RR} + x_{RR/G}) + m_{RR} x_{RR/G} = 0$$

$$x_{RR/G} = -\frac{m_L x_{L/RR} + m_K x_{K/RR}}{m_L + m_K + m_{RR}} = -\frac{60kg(10m) + 40kg(-4m)}{60kg + 40kg + 100kg}$$

$$\boxed{\Delta x_{RR/G} = -2.2m}$$

The minus sign means that the car moves opposite to the motion of Lisa.

5. a) Conserve mechanical energy as the rocket travels from the surface (R) to the height h above the surface. Let the rocket have mass m (which will not be important).

$$-\frac{GMm}{R}+\frac{1}{2}mv^2=-\frac{GMm}{R+h}$$

$$\frac{GM}{R+h}=\frac{GM}{R}-\frac{1}{2}v^2$$

$$R+h=\frac{GM}{\frac{GM}{R}-\frac{1}{2}v^2}=\frac{R}{1-\frac{v^2R}{2GM}}$$

$$h=R\left(\frac{1}{1-\frac{v^2R}{2GM}}-1\right)=R\left(\frac{1}{1-\frac{(1000m/s)^2(1.2\times10^5m)}{2(6.0\times10^{-11}Nm^2/kg^2)(5\times10^{21}kg)}}-1\right)$$

$$h=R\left(\frac{1}{1-\frac{1}{5}}-1\right)=\frac{R}{4}=\frac{1.2\times10^5m}{4}$$

$$\boxed{h=3.0\times10^4m}$$

b) A circular orbit requires that the gravitational force provide the centripetal force needed to move in a circle. Let the velocity needed for the circular orbit be V.

$$F=ma$$

$$\frac{GMm}{(R+h)^2}=m\frac{V^2}{(R+h)}$$

$$V^2=\frac{GM}{(R+h)}$$

$$V=\sqrt{\frac{GM}{(R+h)}}=\sqrt{\frac{(6.0\times10^{-11}Nm^2/kg^2)(5\times10^{21}kg)}{1.2\times10^5m+3.0\times10^4m}}$$

$$\boxed{V=1000\sqrt{2}m/s}$$

6. This problem can be broken into three pieces: (1) the spring expands, converting elastic potential energy into kinetic energy of block 1, (2) the two blocks collide, conserving both momentum and kinetic energy of the two block system, and (3) block 2 goes up the ramp and stops, converting its kinetic energy into gravitational potential energy. For step (1) we get:

$$\frac{1}{2}kd^2 = \frac{1}{2}m_1 v_{1i}^2$$

$$v_{1i} = d\sqrt{\frac{k}{m_1}} \qquad (\Rightarrow \quad v_{1i} = 6\,m/s)$$

For step (2) we conserve energy and momentum. We are only interested in the final velocity of block 2.

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \;\Rightarrow\; m_1\left(v_{1i} - v_{1f}\right) = m_2 v_{2f}$$

$$\tfrac{1}{2}m_1 v_{1i}^2 = \tfrac{1}{2}m_1 v_{1f}^2 + \tfrac{1}{2}m_2 v_{2f}^2 \;\Rightarrow\; m_1\left(v_{1i}^2 - v_{1f}^2\right) = m_2 v_{2f}^2$$

$$m_1\left(v_{1i} - v_{1f}\right)\left(v_{1i} + v_{1f}\right) = m_2 v_{2f}^2$$

$$\left(v_{1i} + v_{1f}\right) = v_{2f}$$

$$v_{1f} = v_{1i} - \frac{m_2}{m_1}v_{2f}$$

$$v_{1i} + v_{1i} - \frac{m_2}{m_1}v_{2f} = v_{2f} \;\Rightarrow\; v_{2f} = \frac{2m_1}{m_1 + m_2}v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2}v_{1i} = \frac{2m_1}{m_1 + m_2}d\sqrt{\frac{k}{m_1}} \qquad (\Rightarrow \quad v_{2f} = 4\,m/s)$$

Now apply conservation of energy up the ramp.

$$\frac{1}{2}m_2 v_{2f}^2 = m_2 g h$$

$$h = \frac{v_{2f}^2}{2g} = \frac{1}{2g}\left(\frac{2m_1}{m_1 + m_2}d\right)^2 \frac{k}{m_1} = \frac{2m_1 k d^2}{g(m_1 + m_2)^2} = \frac{2(2kg)(8\,N/m)(3m)^2}{10\,m/s^2\,(2kg + 4kg)^2}$$

$$\boxed{h = 0.8m}$$