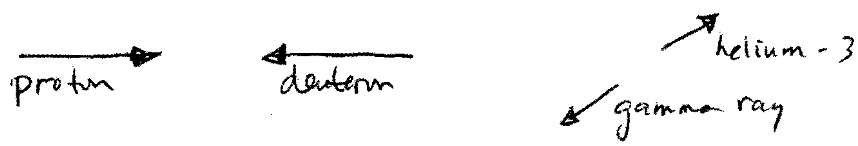
(23) (a) When a neutron collides with an electron, the neutron is almost unaffected, because $m_n \gg m_e$. On the other hand, when a neutron collides with a uranium nucleus, the neutron bounces back with little change in speed, because $m_n \ll m_u$.

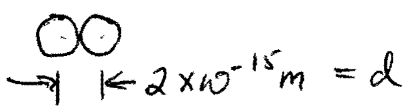
(b) Carbon (mass = 12u) is much lighter than uranium (m = 238u), so when a neutron collides with carbon the neutron loses some energy to the carbon.

(c) Water should be an even better moderator than carbon, because protons (H nuclei) have almost the same mass as neutrons.

(24) (a)



(b) In order for the proton and deuteron to get close enough to make contact, it is necessary to overcome the electrical potential energy due to their repulsion



$$U = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r} = \frac{(9\times10^9)(1.6\times10^{-19})^2}{2\times10^{-15}} = 1.15\times10^{-13}\,J$$

If the proton and deuteron have equal momentum, then $v_p \approx 2v_d$ because $m_p \approx \frac{1}{2}m_d$ and $K_p = 2K_d$ (we can use nonrelativistic KE).

$$K_p + K_d = 1.15\times10^{-13}\,J \qquad K_d = \frac{1}{3}(1.15\times10^{-13}\,J) = 3.8\times10^{-14}\,J$$
$$= 2.4\times10^{5}\,eV$$
$$K_p = 7.7\times10^{-14}\,J$$
$$= 4.8\times10^{5}\,eV$$

(c) Energy conservation:

$$K_p + m_pc^2 + K_d + m_dc^2 = K_{He} + m_{He}c^2 + E_\gamma$$

Momentum conservation: $p_{He} = p_\gamma$ $\qquad E_\gamma = cp_\gamma$ (because $m_\gamma = 0$)

$$K_{He} = \frac{p_{He}^2}{2m_{He}} = \frac{p_\gamma^2}{2m_{He}} = E_\gamma \frac{E_\gamma}{2m_{He}c^2} \ll E_\gamma$$

because $E_\gamma \ll m_{He}c^2$

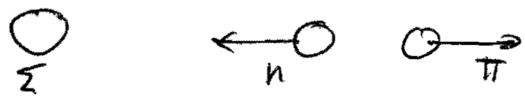
$$E_\gamma = K_p + K_d + m_pc^2 + m_dc^2 - m_{He}c^2$$

$$(m_p + m_d - m_{He})c^2 = (1.0073u + 2.0136u - 3.015u)(1.66\times10^{-27}\,kg/u)(3\times10^8\,m/s)^2$$
$$= 8.8\times10^{-13}\,J = 5.5\times10^6\,eV$$

$$E_\gamma = 0.24\times10^6\,eV + 0.48\times10^6\,eV + 5.5\times10^6\,eV = 6.2\times10^6\,eV$$

$$K_{He} = \frac{E_\gamma^2}{2m_{He}c^2} = \frac{(6.2\times10^6\,eV)^2}{2(3.015u)(1.66\times10^{-27}\,kg/u)(3\times10^8\,m/s)^2/1.6\times10^{-19}\,J/eV}$$
$$= 6.8\times10^3\,eV$$

(25)



Conservation of p ($p_i = p_f$) : $0 = p_\pi - p_n$ or $p_\pi = p_n$

Conservation of E : $E_\Sigma = E_n + E_\pi$

$$m_\Sigma c^2 = \sqrt{(p_n c)^2 + (m_n c^2)^2} + \sqrt{(p_\pi c)^2 + (m_\pi c^2)^2}$$

Use $p_\pi = p_n$:

$$m_\Sigma c^2 = \sqrt{(p_n c)^2 + (m_n c^2)^2} + \sqrt{(p_n c)^2 + (m_\pi c^2)^2}$$

Solve for p_n, then find $E_n = \sqrt{(p_n c)^2 + (m_n c^2)^2}$ and E_π.

(26)

Conservation of p (x dir) $p_{\pi i} = p_{\pi f}\cos\theta + p_X\cos\phi$

(y dir) $0 = p_{\pi f}\sin\theta - p_X\sin\phi$

Conserv of E: $E_{\pi i} + m_p c^2 = E_{\pi f} + E_X$

$$p_X\sin\phi = p_{\pi f}\sin\theta$$

$$p_X\cos\phi = p_{\pi i} - p_{\pi f}\cos\theta$$

Square and add:

$$p_x^2(\sin^2\phi + \cos^2\phi) = p_{\pi f}^2 \sin^2\theta + p_{\pi i}^2 - 2p_{\pi i}p_{\pi f}\cos\theta + p_{\pi f}^2\cos^2\theta$$

$$p_x^2 = p_{\pi f}^2 + p_{\pi i}^2 - 2p_{\pi i}p_{\pi f}\cos\theta \Rightarrow p_x = 2083 \text{ MeV}/c$$

$$E_{\pi i} = \sqrt{(p_{\pi i}c)^2 + (m_\pi c^2)^2} = \sqrt{(3000\text{ MeV})^2 + (140\text{ MeV})^2} = 3003.3 \text{ MeV}$$

$$E_{\pi f} = \sqrt{(p_{\pi f}c)^2 + (m_\pi c^2)^2} = \sqrt{(1510\text{ MeV})^2 + (140\text{ MeV})^2} = 1516.5 \text{ MeV}$$

$$E_x = E_{\pi i} + m_p c^2 - E_{\pi f} = 3003.3\text{ MeV} + 938\text{ MeV} - 1516.5\text{ MeV}$$
$$= 2425 \text{ MeV}$$

$$m_x c^2 = \sqrt{E_x^2 - (p_x c)^2} = \sqrt{(2425\text{ MeV})^2 - (2083\text{ MeV})^2}$$
$$= 1241 \text{ MeV}$$

9.P.23 Slowing down neutrons

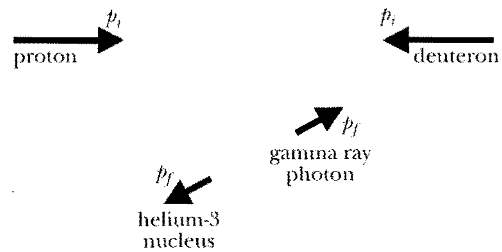
(a) The neutrons have almost no interaction with the electrons in a block of uranium, but they do interact through the strong interaction with the uranium nuclei. A collision of a neutron with a uranium nucleus is an example of a low-mass projectile striking a massive target that is at rest (the uranium nucleus has about 235 times the mass of the neutron). If the interaction is elastic (kinetic energy constant), the neutron will bounce off with almost its original momentum and kinetic energy. The uranium nucleus may acquire as much as twice the neutron's momentum (this happens in a head-on collision), but due to the huge mass of the uranium nucleus compared to the neutron, the uranium nucleus acquires little kinetic energy $p^2/(2M)$. So fast neutrons travel through uranium with little change in speed.

(b) Again, the neutrons have almost no interaction with the electrons in a block of carbon, but they do interact with the carbon nuclei, through the strong interaction. The mass of a carbon nucleus is only about 12 times the mass of the neutron, so much more kinetic energy can be transferred from the neutron to the carbon nucleus than is the case with uranium nuclei. Therefore carbon should be much more effective than uranium in slowing down fast neutrons.

(c) A water molecule (H_2O) contains two very low-mass nuclei—the two hydrogen nuclei, each consisting of a single proton. The mass of a proton is nearly equal to the mass of a neutron, so our studies of elastic collisions between two equal masses, one initially at rest, apply here. As an example, a head-on collision will leave the neutron nearly at rest, and the proton acquires the neutron's momentum and kinetic energy. For this reason water should be a better moderator than carbon.

A comment: In part (c) we showed that water is a good moderator (that is, it does a good job of slowing down fast neutrons). There is however a problem with ordinary water. A neutron can fuse with a proton to form a deuteron (a proton plus a neutron), the nucleus of "heavy hydrogen" or deuterium. If the neutron is captured in this way, the neutron cannot contribute to triggering fission in a uranium nucleus. Because of this neutron capture effect, some reactors use as a moderator "heavy water," water consisting of D_2O molecules (an oxygen atom plus two deuterium atoms). A neutron can fuse with a deuteron to form a triton (proton plus two neutrons), but it happens that the probability for this neutron capture reaction is quite low compared to the probability of neutron capture in ordinary hydrogen. There is a trade-off between ordinary hydrogen doing a better job of slowing down the fast neutrons and heavy hydrogen removing fewer neutrons from circulation.

9.P.24 Fusion reaction revisited



(a) We're considering a situation where the total momentum of the system is initially zero. Let p_i be the magnitude of the momentum of the proton. Since the total momentum is zero, the deuteron has equal and opposite momentum as shown. After the collision, the helium-3 nucleus and the photon must have equal and opposite momenta as shown, so that the total momentum is still zero. The momentum of the helium-3 nucleus (or the photon) need not be the same as the original momentum of the proton (or the deuteron).

(b) Energy conservation—initial state is the proton (mass m_p) and deuteron (mass m_d) far away from each other with initial kinetic energy and negligible electric potential energy; we consider the final state to be at the instant when they are just touching, at a center-to-center distance $r \approx 2\times10^{-15}$ m, with zero kinetic energy. Assume that the speeds are small compared to c (and check this later).

$$\left(m_pc^2 + \frac{p_i^2}{2m_p}\right) + \left(m_dc^2 + \frac{p_i^2}{2m_d}\right) = (m_pc^2 + 0) + (m_dc^2 + 0) + \frac{1}{4\pi\varepsilon_0}\frac{(e)(e)}{r}$$

$$\frac{p_i^2}{2}\left(\frac{1}{m_p} + \frac{1}{m_d}\right) = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

$$p_i = \sqrt{\frac{2\left(\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}\right)}{\left(\frac{1}{m_p}+\frac{1}{m_d}\right)}} = \sqrt{\frac{2\left[\frac{(9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.6\times10^{-19}\ \text{C})^2}{(2\times10^{-15}\ \text{m})}\right]}{\left(\frac{1}{1.66\times10^{-27}\ \text{kg}}\right)\left(\frac{1}{1.0073}+\frac{1}{2.0136}\right)}} = 1.6\times10^{-20}\ \text{kg}\cdot\text{m/s}$$

$$K_p = \frac{p_i^2}{2m_p} = \frac{(1.6\times10^{-20}\ \text{kg}\cdot\text{m/s})^2}{2(1.0073)(1.66\times10^{-27}\ \text{kg})} = (7.7\times10^{-14}\ \text{J})\left(\frac{1\ \text{eV}}{1.6\times10^{-19}\ \text{J}}\right) = 4.8\times10^5\ \text{eV}$$

$$K_d = \frac{p_i^2}{2m_d} = \frac{(1.6\times10^{-20}\ \text{kg}\cdot\text{m/s})^2}{2(2.0136)(1.66\times10^{-27}\ \text{kg})} = (3.8\times10^{-14}\ \text{J})\left(\frac{1\ \text{eV}}{1.6\times10^{-19}\ \text{J}}\right) = 2.4\times10^5\ \text{eV}$$

$$\text{Total kinetic energy required: } K_p + K_d = 7.2\times10^5\ \text{eV} = 0.72\ \text{MeV}$$

If you approximate the closest approach as $r \approx 1\times10^{-15}$ m, the kinetic energies will be twice as large.

Let's check that the speeds are small compared to c:

$$v_p = \frac{p_i}{m_p} = \frac{1.6\times10^{-20}\ \text{kg}\cdot\text{m/s}}{(1.0073)(1.66\times10^{-27}\ \text{kg})} = 9.6\times10^6\ \text{m/s}$$

This is only 3% of the speed of light, and the speed of the deuteron is even smaller. So assuming $v << c$ is okay.

(c) Let p_f be the magnitude of the momentum of the outgoing helium-3 nucleus, and of the gamma ray photon. They have to have equal and opposite momenta so that the total momentum is zero, since the initial total momentum of proton and deuteron was zero. Therefore the magnitude of the photon's momentum is p_f.

The energy of the photon is obtained from the general relation $E^2 = (pc)^2 + (mc^2)^2$, with $m = 0$ for a photon, so that the photon energy E is equal to p_fc.

Energy conservation—initial state is the proton and deuteron far away from each other; consider the final state to be the helium-3 nucleus and the photon moving directly away from each other. Assume the helium-3 is slow compared to the speed of light:

$$\left(m_p c^2 + \frac{p_i^2}{2m_p}\right) + \left(m_d c^2 + \frac{p_i^2}{2m_d}\right) = \left(m_{\text{He}} c^2 + \frac{p_f^2}{2m_{\text{He}}}\right) + (p_f c)$$

Note that $\frac{p_f^2}{2m_{\text{He}}} = p_f\left(\frac{m_{\text{He}} v_f}{2m_{\text{He}}}\right) = p_f\left(\frac{v_f}{2}\right) << p_f c$ (assuming the helium-3 nucleus moves slowly), so we can conveniently neglect the kinetic energy of the helium-3 nucleus compared to the energy of the photon in solving for p_f:

$$p_f c \approx \left(m_p c^2 + \frac{p_i^2}{2m_p}\right) + \left(m_d c^2 + \frac{p_i^2}{2m_d}\right) - m_{\text{He}} c^2$$

Dividing by c we have $p_f \approx m_p c + m_d c - m_{\text{He}} c + \frac{K_p + K_d}{c}$

Use the initial kinetic energies calculated in part (a):

$$p_f = (3\times10^8 \text{ m/s})[1.0073 + 2.0136 - 3.015](1.66\times10^{-27} \text{ kg}) + \frac{(7.7\times10^{-14} \text{ J}) + (3.8\times10^{-14} \text{ J})}{(3\times10^8 \text{ m/s})}$$

$$p_f = 3.3\times10^{-21} \text{ kg·m/s}$$

This is only one-fifth the momentum of the incoming proton or deuteron. The energy of the photon is this:

$$p_f c = (3.3\times10^{-21} \text{ kg·m/s})(3\times10^8 \text{ m/s}) = (1\times10^{-12} \text{ J})\left(\frac{1 \text{ eV}}{1.6\times10^{-19} \text{ J}}\right) = 6.3\times10^6 \text{ eV} = 6.3 \text{ MeV}$$

Photons with such high energies are called gamma rays. The kinetic energy of the helium-3 nucleus is this:

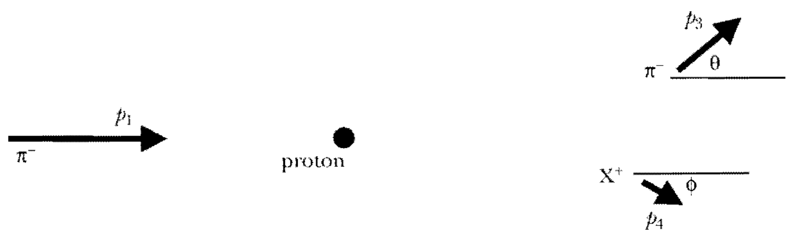
$$\frac{p_f^2}{2m_{\text{He}}} = \frac{(3.3\times10^{-21} \text{ kg·m/s})^2}{2(3.015)(1.66\times10^{-27} \text{ kg})} = (1.1\times10^{-15} \text{ J})\left(\frac{1 \text{ eV}}{1.6\times10^{-19} \text{ J}}\right) = 7\times10^3 \text{ eV} = 7 \text{ keV}$$

The kinetic energy of the helium-3 nucleus is indeed very small (with small speed), and this kinetic energy is negligible compared to the energy of the photon, though the magnitudes of their momenta are equal. (If you approximate the closest approach of the proton and deuteron as $r \approx 1\times10^{-15}$ m, the photon momentum is 3.7×10^{-21} kg·m/s, the photon energy is 6.9 MeV, and the helium-3 kinetic energy is 8.5 keV.)

Note: The net energy release in this reaction is about 5.6 MeV—the final energy of about 6.3 MeV minus the 0.72 MeV of kinetic energy that had to be supplied to overcome the "Coulomb" (electric) barrier, to get the proton and deuteron close enough to react through the strong interaction. The presence of the Coulomb barrier is what makes it so difficult to build a fusion power reactor. The *net* energy release is high, but it is difficult to give the protons and deuterons enough kinetic energy to overcome their mutual electric repulsion.

9.P.26 Pion production of a particle

Take the system to be the incoming pion of mass m and the target proton of mass M. The system is free of external influences, so the total momentum and total energy do not change. Let the angles θ and ϕ of the outgoing particles to the horizontal axis be considered to be positive numbers.



$$p_x\text{: } p_1 + 0 = p_3\cos\theta + p_4\cos\phi$$

$$p_y\text{: } 0 + 0 = p_3\sin\theta + -p_4\sin\phi$$

$$E\text{: } E_1 + Mc^2 = E_3 + E_4$$

There are various algebraic approaches to solving for the mass of the X^+. We present one approach. Start with the mass in terms of the energy and momentum of the X^+:

$$(M_Xc^2)^2 = (E_4)^2 - (p_4c)^2$$

Note that

$$p_4^2 = p_x^2 + p_y^2 = (p_4\cos\phi)^2 + (p_4\sin\phi)^2 = (p_1 - p_3\cos\theta)^2 + (p_3\sin\theta)^2$$

$$E_4 = E_1 + Mc^2 - E_3$$

Therefore we have

$$(M_Xc^2)^2 = (E_1 + Mc^2 - E_3)^2 - [(p_1 - p_3\cos\theta)^2 + (p_3\sin\theta)^2]c^2$$

Evaluate numerically, noting that pc has units of energy and can be simply expressed in MeV:

$$E_1 = \sqrt{(p_1c)^2 + (mc^2)^2} = \sqrt{(3000\text{ MeV})^2 + (140\text{ MeV})^2} = 3003\text{ MeV}$$

$$E_3 = \sqrt{(p_3c)^2 + (mc^2)^2} = \sqrt{(1510\text{ MeV})^2 + (140\text{ MeV})^2} = 1516\text{ MeV}$$

$$(M_Xc^2)^2 = (3003 + 938 - 1516)^2(\text{MeV})^2 - [(3000 - 1510\cos(40°))^2 + (1510\sin(40°))^2](\text{MeV})^2$$

$$(M_Xc^2)^2 = 5.88\times10^6(\text{MeV})^2 - [3.40\times10^6 + 9.4\times10^5](\text{MeV})^2 = 1.54\times10^6(\text{MeV})^2$$

$$M_Xc^2 = 1240\text{ MeV}$$

1240 MeV/c^2 is the mass of the particle called the Δ^+. It was experiments of this kind in the 1950's that led to the discovery of this particle. It can be considered to be an excited state of three quarks, where the proton represents the ground state. The Δ^+ has a very short lifetime and almost immediately decays into a proton and a neutral pion (or into a neutron and a positive pion).