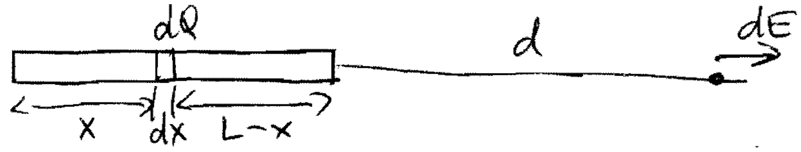
① (a)



$$dQ = \frac{Q}{L}\,dx$$

$$dE_x = \frac{1}{4\pi\epsilon_0}\frac{dQ}{(d+L-x)^2} = \frac{1}{4\pi\epsilon_0}\frac{Q}{L}\frac{dx}{(d+L-x)^2}$$

For the portion of the rod from $x=0$ to $x=L/2$, E points in the negative x direction.

$$E_x = \frac{1}{4\pi\epsilon_0}\frac{Q}{L}\left[-\int_0^{L/2}\frac{dx}{(d+L-x)^2} + \int_{L/2}^{L}\frac{dx}{(d+L-x)^2}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\frac{Q}{L}\left[-\frac{1}{d+L-x}\bigg|_0^{L/2} + \frac{1}{d+L-x}\bigg|_{L/2}^{0}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\frac{Q}{L}\left[-\frac{2}{d+L/2} + \frac{1}{d} + \frac{1}{d+L}\right]$$

(b) At large d the rod should look like a dipole with $E_x \propto 1/d^3$.

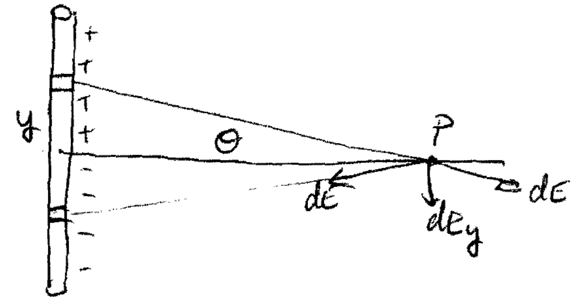
$$E_x \simeq \frac{1}{4\pi\epsilon_0}\frac{Q}{L}\frac{1}{d}\left[1 + \frac{1}{1+L/d} - \frac{2}{1+L/2d}\right]$$

$$\approx \frac{1}{4\pi\epsilon_0}\frac{Q}{L}\frac{1}{d}\left[1 + \left(1 - \frac{L}{d} + \left(\frac{L}{d}\right)^2\right) - 2\left(1 - \frac{L}{2d} + \frac{L^2}{4d^2}\right)\right]$$

$$= \frac{1}{4\pi\epsilon_0}\frac{Q}{L}\frac{1}{d}\left[\frac{L^2}{d^2} - \frac{L^2}{2d^2}\right] = \frac{1}{4\pi\epsilon_0}\frac{Q}{L}\frac{L^2}{2d^3}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{Q\cdot L/2}{d^3}$$

(2)



(a) For each element dQ in the top half with + charge, there is a corresponding element in the bottom half with − charge, such that the x components of the fields cancel and only the y component survives.

$$dE_y = dE \sin\theta = \frac{1}{4\pi\epsilon_0}\frac{Q}{L}\frac{dy}{d^2+y^2}\frac{y}{\sqrt{d^2+y^2}}$$

Integrate from 0 to $L/2$ and include factor of 2 to account for lower half:

$$E_y = \frac{2}{4\pi\epsilon_0}\frac{Q}{L}\int_0^{L/2}\frac{y\,dy}{(d^2+y^2)^{3/2}} = \frac{1}{4\pi\epsilon_0}\frac{Q}{L}\int_0^{L/2}\frac{2y\,dy}{(d^2+y^2)^{3/2}}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{Q}{L}\left[-2(d^2+y^2)^{-1/2}\right]_0^{L/2}$$

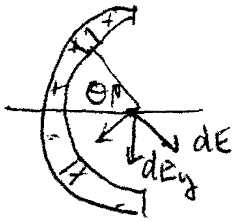
$$= \frac{1}{2\pi\epsilon_0}\frac{Q}{L}\left(-\frac{1}{d} + \frac{1}{\sqrt{d^2+L^2/4}}\right)$$

(b) For $d \gg L$

$$E_y \approx \frac{1}{2\pi\epsilon_0}\frac{Q}{L}\left(-\frac{1}{d}+\frac{1}{d}\left(1-\frac{1}{2}\frac{L^2}{4d^2}\right)\right) = \frac{-1}{4\pi\epsilon_0}\frac{Q/4L}{d^3}$$

The field depends on $1/d^3$, like a dipole.

(3)



$$dQ = \frac{Q}{\pi R} R\, d\theta = \frac{Q}{\pi}\, d\theta$$

For every + dQ on the upper half, there will be a − dQ on the lower half such that the x components of the field will cancel.

$$dE_y = dE \sin\theta = \frac{1}{4\pi\epsilon_0}\frac{dQ}{R^2}\sin\theta = \frac{1}{4\pi\epsilon_0}\frac{Q}{\pi R^2}\sin\theta\, d\theta$$

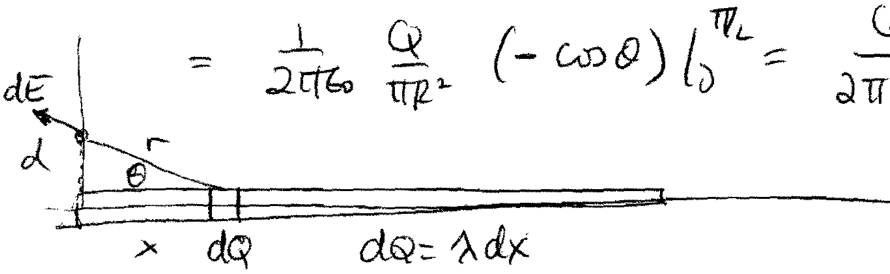
Integrate from $\theta = 0$ to $\pi/2$ and double the result to account for the lower half:

$$E_y = 2\int_0^{\pi/2} \frac{1}{4\pi\epsilon_0}\frac{Q}{\pi R^2}\sin\theta\, d\theta$$

$$= \frac{1}{2\pi\epsilon_0}\frac{Q}{\pi R^2}\int_0^{\pi/2}\sin\theta\, d\theta$$

$$= \frac{1}{2\pi\epsilon_0}\frac{Q}{\pi R^2}\left(-\cos\theta\right)\Big|_0^{\pi/2} = \frac{Q}{2\pi^2\epsilon_0 R^2}$$

(4)



$dQ = \lambda\, dx$

$$dE = \frac{dQ}{4\pi\epsilon_0 r^2} = \frac{\lambda}{4\pi\epsilon_0}\frac{dx}{x^2+d^2}$$

$$dE_x = dE\cos\theta = \frac{\lambda}{4\pi\epsilon_0}\frac{dx}{x^2+d^2}\frac{x}{\sqrt{x^2+d^2}}$$

$$E_x = \frac{\lambda}{4\pi\epsilon_0}\int_0^\infty \frac{x\,dx}{(x^2+d^2)^{3/2}} = \frac{\lambda}{4\pi\epsilon_0}\frac{1}{2}\int_0^\infty \frac{2x\,dx}{(x^2+d^2)^{3/2}}$$

$$= \left(\frac{\lambda}{4\pi\epsilon_0}\frac{1}{2}\right)\left(-2(x^2+d^2)^{-1/2}\Big|_0^\infty\right) = \frac{\lambda}{4\pi\epsilon_0 d} \text{ in } -x \text{ direction}$$

$$dE_y = dE \sin\theta = \frac{\lambda}{4\pi\epsilon_0} \frac{dx}{x^2+d^2} \frac{d}{\sqrt{x^2+d^2}}$$

$$E_y = \frac{\lambda d}{4\pi\epsilon_0} \int_0^\infty \frac{dx}{(x^2+d^2)^{3/2}}$$

$$= \frac{\lambda d}{4\pi\epsilon_0} \left. \frac{x}{d^2\sqrt{x^2+d^2}} \right|_0^\infty$$

$$= \frac{\lambda}{4\pi\epsilon_0 d}$$ in $+y$ direction