

PH202H/212H
W09

Problems Involving Relativistic Collisions and Decays
(with solutions)

1. A π meson of rest energy 139.6 MeV moving at a speed of $0.906c$ collides with and sticks to a proton of rest energy 938.3 MeV that is at rest. (a) Find the total relativistic energy of the resulting composite particle. (b) Find the total linear momentum of the composite particle. (c) Using the results of (a) and (b), find the rest energy of the composite particle.
2. An electron and a positron (an antielectron) make a head-on collision, each moving at $v = 0.99999c$. In the collision the electrons disappear and are replaced by two muons ($mc^2 = 105.7$ MeV) which move off in opposite directions. What is the kinetic energy of each of the muons?
3. It is desired to create a particle of mass $9700 \text{ MeV}/c^2$ in a collision between a proton and an antiproton traveling at the same speed in opposite directions. What speed is necessary for this to occur?
4. A particle of rest energy mc^2 is moving with speed v in the positive x direction. The particle decays into two particles, each of rest energy 140 MeV. One particle, with kinetic energy 282 MeV, moves in the positive x direction, and the other particle, with kinetic energy 25 MeV, moves in the negative x direction. Find the rest energy of the original particle and its speed.

Solutions

1. (a)

$$E = E_\pi + E_p = \frac{m_\pi c^2}{\sqrt{1-v^2/c^2}} + m_p c^2 = \frac{139.6 \text{ MeV}}{\sqrt{1-(0.906)^2}} + 938.3 \text{ MeV} = 1268.1 \text{ MeV}$$

(b)

$$p = p_\pi + p_p = \frac{m_\pi v}{\sqrt{1-v^2/c^2}} = \frac{1}{c} \frac{m_\pi c^2 (v/c)}{\sqrt{1-v^2/c^2}} = \frac{1}{c} \frac{(139.6 \text{ MeV})(0.906)}{\sqrt{1-(0.906)^2}} = 298.8 \text{ MeV}/c$$

(c)

$$mc^2 = \sqrt{E^2 - (pc)^2} = \sqrt{(1268.1 \text{ MeV})^2 - (298.8 \text{ MeV})^2} = 1232 \text{ MeV}$$

2. Before the collision, the total relativistic energy of each electron is

$$E_e = \frac{m_e c^2}{\sqrt{1-v^2/c^2}} = \frac{0.511 \text{ MeV}}{\sqrt{1-(0.99999)^2}} = 114.3 \text{ MeV}$$

The total energy in the collision is therefore $2 \times 114.3 \text{ MeV} = 228.6 \text{ MeV}$. The total momentum is zero before the collision, because the two particles move with equal and opposite velocities and have equal masses. After the collision, the total momentum is still zero, so we know that the two muons must move with equal speeds and thus have equal energies. The total energy of each muon is then 114.3 MeV and its kinetic energy is

$$K_\mu = E_\mu - m_\mu c^2 = 114.3 \text{ MeV} - 105.7 \text{ MeV} = 8.6 \text{ MeV}$$

3. The two protons have equal (and opposite) momenta and thus equal energies E_1 and E_2 . The new particle is created with zero momentum (at rest), so its total energy is equal to its rest energy $Mc^2 = 9700 \text{ MeV}$. Conservation of energy then gives $E_1 + E_2 = Mc^2$, so $E_1 = E_2 = Mc^2/2$.

$$E_1 = \frac{m_p c^2}{\sqrt{1-v^2/c^2}} = \frac{Mc^2}{2}$$

$$1 - \frac{v^2}{c^2} = \left[\frac{2(m_p c^2)}{Mc^2} \right]^2 = \left[\frac{2(938.3 \text{ MeV})}{9700 \text{ MeV}} \right]^2 = 0.0374 \quad \text{so} \quad v = 0.981c$$

4. For particle 1, moving in the positive x direction,

$$E_1 = K_1 + mc^2 = 282 \text{ MeV} + 140 \text{ MeV} = 422 \text{ MeV}$$

$$cp_1 = \sqrt{E_1^2 - (mc^2)^2} = \sqrt{(422 \text{ MeV})^2 - (140 \text{ MeV})^2} = +398 \text{ MeV}$$

For particle 2, moving in the negative x direction,

$$E_2 = K_2 + mc^2 = 25 \text{ MeV} + 140 \text{ MeV} = 165 \text{ MeV}$$

$$cp_2 = -\sqrt{E_2^2 - (mc^2)^2} = -\sqrt{(165 \text{ MeV})^2 - (140 \text{ MeV})^2} = -87 \text{ MeV}$$

The net final momentum is $p_f = p_1 + p_2 = 398 \text{ MeV}/c - 87 \text{ MeV}/c = 311 \text{ MeV}/c$, and the net final energy is $E_f = E_1 + E_2 = 422 \text{ MeV} + 165 \text{ MeV} = 587 \text{ MeV}$. Because of the conservation laws, these must be equal to the momentum and the energy of the initial particle, so that its rest energy is then

$$m_i c^2 = \sqrt{E_f^2 - (cp_f)^2} = \sqrt{(587 \text{ MeV})^2 - (311 \text{ MeV})^2} = 498 \text{ MeV}$$

Solving $E = mc^2 / \sqrt{1 - v^2 / c^2}$ for v , we obtain

$$v = c \sqrt{1 - \left(\frac{mc^2}{E} \right)^2} = c \sqrt{1 - \left(\frac{498 \text{ MeV}}{587 \text{ MeV}} \right)^2} = 0.529c$$