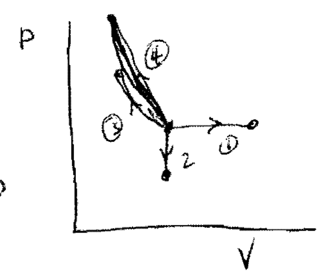
Ph202H/212H
W08 Class worksheet – January 31, 2008

1. One mole of a monatomic ideal gas undergoes 4 different processes: isobaric (constant pressure), isochoric (constant volume), isothermal (constant temperature), and adiabatic (no heat transfer). Fill in the missing final-to-initial ratios of volume, pressure, and temperature for each process. For each process, state whether the work done on the gas, heat transfer, and internal energy change is positive, negative, or zero. Sketch each process on a PV diagram. Imagine the gas to be in a cylinder with a moveable piston on the top, and describe how each process would be carried out.

	Process	V_f/V_i	P_f/P_i	T_f/T_i	W	Q	ΔE_{int}
1	Isobaric	2	1	2	−	+	+
2	Isochoric	1	1/2	1/2	0	−	−
3	Isothermal	1/2	2	1	+	−	0
4	Adiabatic	1/2	$2^{5/3}$	$2^{2/3}$	+	0	+



① Isobaric $\Rightarrow P_f/P_i = 1$

$$\frac{T_f}{T_i} = \frac{P_f V_f}{P_i V_i} = 2$$

$V_f > V_i \Rightarrow W < 0$; $T_f > T_i \Rightarrow \Delta E_{int} > 0$, $Q > 0$

② Isochoric $\Rightarrow V_f = V_i$

$$\frac{T_f}{T_i} = \frac{P_f V_f}{P_i V_i} = \frac{1}{2}$$

$V_f = V_i \Rightarrow W = 0$; $T_f < T_i \Rightarrow Q < 0$, $\Delta E_{int} < 0$

③ Isothermal $\Rightarrow T_f = T_i$

$$\frac{P_f}{P_i} = \frac{T_f}{T_i}\frac{V_i}{V_f} = 2$$

$V_f < V_i \Rightarrow W > 0$; $T_f = T_i \Rightarrow \Delta E_{int} = 0$; $\Delta E_{int} = Q + W \Rightarrow Q < 0$

④ $\gamma = 5/3$ $P_i V_i^{\gamma} = P_f V_f^{\gamma}$ $\frac{P_f}{P_i} = \left(\frac{V_i}{V_f}\right)^{\gamma} = 2^{5/3}$

$$\frac{T_f}{T_i} = \frac{P_f V_f}{P_i V_i} = (2^{5/3})\left(\frac{1}{2}\right) = 2^{2/3}$$

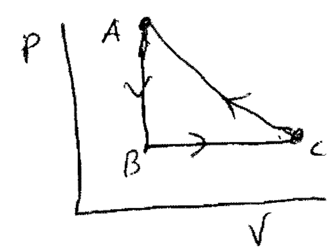
$Q = 0$; $T_f > T_i \Rightarrow \Delta E_{int} > 0$; $\Delta E_{int} = Q + W \Rightarrow W > 0$

2. A quantity of 0.75 mole of an ideal diatomic gas ($C_V = 5R/2$, $C_P = 7R/2$) is carried through the following cycle, starting at point A where $P_A = 3.2 \times 10^3$ Pa and $V_A = 0.21$ m^3: (1) a reduction in pressure at constant volume to point B where $P_B = 1.2 \times 10^3$ Pa; (2) an increase in volume at constant pressure to point C; (3) an isothermal compression back to point A. Sketch the processes on a PV diagram and fill in the missing information.

	P (Pa)	V (m^3)	T (K)
A	3.2×10^3	0.21	108
B	1.2×10^3	0.21	40
C	1.2×10^3	0.56	108

	W	Q	ΔE_{int}
$A \to B$	0	−1060 J	−1060 J
$B \to C$	−420 J	1480 J	1060 J
$C \to A$	660 J	−660 J	0
$A \to B \to C \to A$	240 J	−240 J	0

A: $T_A = \frac{P_A V_A}{nR} = \frac{(3.2\times10^3\,\text{Pa})(0.21\,\text{m}^3)}{(0.75\,\text{mole})(8.31\,\text{J/mole}\cdot\text{K})} = 108\,\text{K}$



B: $V_B = V_A$ $\quad T_B = \frac{P_B V_B}{nR} = \frac{(1.2\times10^3)(0.21)}{(0.75)(8.31)} = 40\,\text{K}$

C: $T_A = T_C$ $\quad V_C = \frac{nRT_C}{P_C} = \frac{(0.75)(8.31)(108)}{1.2\times10^3} = 0.56\,\text{m}^3$

$P_C = P_B$

AB: $Q = nC_V(T_B - T_A) = (0.75\,\text{mole})(\frac{5}{2})(8.31\,\text{J/mole}\cdot\text{K})(40\text{K} - 108\text{K}) = -1060\text{J}$

$W = 0$ ($V = \text{const.}$) $\quad \Delta E_{int} = Q + W = -1060\text{J}$

BC: $W = -P_B(V_C - V_B) = -(1.2\times10^3\,\text{Pa})(0.56\text{m}^3 - 0.21\text{m}^3) = -420\text{J}$

$Q = nC_P(T_C - T_B) = (0.75)(\frac{7}{2})(8.31)(108 - 40) = 1480\text{J}$

$\Delta E_{int} = Q + W = 1060\text{J}$

CA: $\Delta E_{int} = 0$ (isothermal)

$W = -nRT_C \ln\frac{V_A}{V_C} = -(0.75\,\text{mole})(8.31\,\frac{\text{J}}{\text{mole}\cdot\text{K}})(108\text{K}) \ln\frac{0.21}{0.56} = 660\text{J}$

$Q = -W = -660\text{J}$