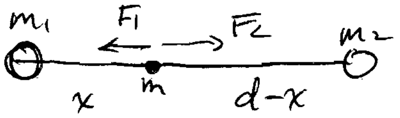
(1) (a)



$$|\vec{F}_1| = \frac{G m m_1}{x^2} \qquad |\vec{F}_2| = \frac{G m m_2}{(d-x)^2}$$

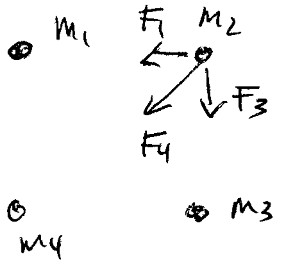
$$|\vec{F}_1| = |\vec{F}_2| \Rightarrow \frac{G m m_1}{x^2} = \frac{G m m_2}{(d-x)^2}$$

$$m_1 (d-x)^2 = m_2 x^2$$

$$\sqrt{\frac{m_2}{m_1}}\, x = d - x \quad \text{or} \quad x = \frac{d}{1+\sqrt{m_2/m_1}}$$

(b) Moves back to equilibrium position if displaced in y or z direction, moves away if displaced in x direction

(2)



$$\vec{F}_1 = -\frac{G m_1 m_2}{d^2}\,\hat{i}$$

$$F_3 = -\frac{G m_3 m_2}{d^2}\,\hat{j}$$

$$\vec{F}_4 = \frac{G m_2 m_4}{(\sqrt{2} d)^2}\left(-\frac{\sqrt{2}}{2}\hat{i} - \frac{\sqrt{2}}{2}\hat{j}\right)$$

$$\vec{F}_1 + \vec{F}_3 + \vec{F}_4 = \frac{G m_2}{d^2}\left[-\left(m_1 + \frac{\sqrt{2}}{4} m_4\right)\hat{i} - \left(m_3 + \frac{\sqrt{2}}{4} m_4\right)\hat{j}\right]$$

(3) (a) $F = mg = (1\,\text{kg})(9.8\,\text{N/kg}) = 9.8\,\text{N}$

(b) $$F = \frac{G m M_S}{(d_{SE} - R_E)^2} = \frac{(6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2)(1.0\,\text{kg})(1.99\times10^{30}\,\text{kg})}{(1.50\times10^{11}\,\text{m} - 6.37\times10^{6}\,\text{m})^2} = 0.0059\,\text{N}$$

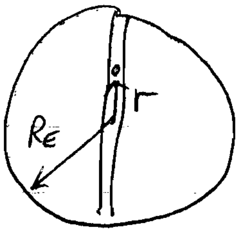
(c) $$F = \frac{G m M_m}{(d_{mE} - R_E)^2} = \frac{(6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2)(1.0\,\text{kg})(7.36\times10^{22}\,\text{kg})}{(3.82\times10^{8}\,\text{m} - 6.37\times10^{6}\,\text{m})^2} = 0.000035\,\text{N}$$

4. (a) $\Delta F = \dfrac{GmM_S}{(d_{SE}-R_E)^2} - \dfrac{GmM_S}{(d_{SE}+R_E)^2} = \dfrac{GmM_S}{d_{SE}^2}\left[\dfrac{1}{(1-R_E/d_{SE})^2} - \dfrac{1}{(1+R_E/d_{SE})^2}\right]$

$$\cong \frac{GmM_S}{d_{SE}^2}\left(4\frac{R_E}{d_{SE}}\right) = 1.0\times 10^{-6}\,N$$

(b) $\Delta F = \dfrac{GmM_M}{d_{ME}^2}\left(4\dfrac{R_E}{d_{ME}}\right) = 2.2\times 10^{-6}\,N$

Note that (from #3), the Sun's force is stronger but the force difference is large for the moon (which is why the moon has a greater effect on tides).

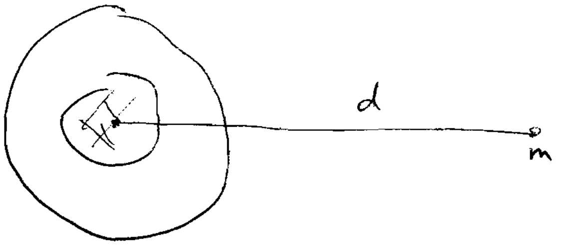
5. 

By the 2nd shell theorem, all the mass beyond radius r does not contribute to the gravitational force at r.
By the first shell theorem, all the mass inside r can be replaced by a point mass at the center.
Assuming the Earth to be of uniform density, the fraction of the mass inside radius r is

$$M_E\,\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R_E^3} = M_E\,\frac{r^3}{R_E^3}$$

$$F = \frac{Gm\left(M_E r^3/R_E^3\right)}{r^2} = \frac{GM_E m}{R_E^3}\,r$$

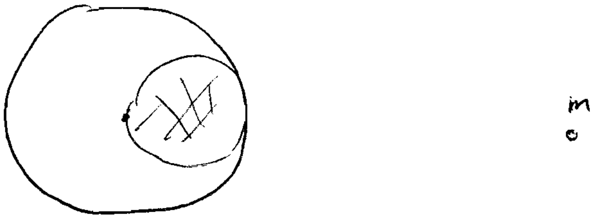
(6) (a)



The mass of the hollow sphere is $M - \frac{\frac{4}{3}\pi\left(\frac{R}{2}\right)^3}{\frac{4}{3}\pi R^3} M = \frac{7}{8} M$

$$F = \frac{G\, m \left(\frac{7}{8}M\right)}{d^2}$$

(b)

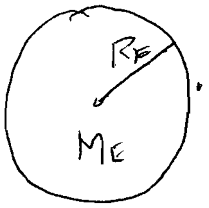


Let body A = original solid sphere
B = sphere with hole
C = sphere of radius $\frac{R}{2}$ removed to make hole.

$A = B + C$

$$F_B = F_A - F_C = \frac{GmM}{d^2} - \frac{Gm\left(\frac{1}{8}M\right)}{(d - R/2)^2}$$

(7)



$$\vec{F} = -G \frac{M_E m}{R_E^2}\hat{r} = -mg\,\hat{r} \Rightarrow g = \frac{GM_E}{R_E^2}$$

$$g' = \frac{GM_E}{(R_E/2)^2} = 4 \cdot \frac{GM_E}{R_E^2} = 4g = 4 \times 9.8\, m/s^2$$

$$g' = 39.2\ m/s^2$$