Class problems - Oct 4, 2007

① (a) Each bullet carries momentum mv
In one second, r bullets hit the block so $\frac{\Delta p}{\Delta t} = mvr = F_{avg}$.

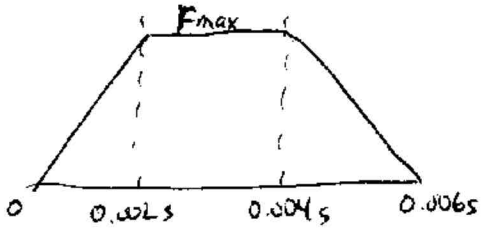
(b) $\Delta p = F_{avg}\,\Delta t = (mvr)(T)$ change in momentum in time T

$= M(v_f - v_i) \Rightarrow v_f = \frac{mvrT}{M}$

② Initial momentum of the two skaters is zero
$\Rightarrow$ Final momentum $= 0 = m_1 v_1 + m_2 v_2$

$$v_2 = -v_1 \frac{m_1}{m_2} = -(3.0\,m/s)\frac{75\,kg}{50\,kg} = -4.5\,\frac{m}{s}$$

③



$\Delta p = m(v_f - v_i) = (0.058\,kg)(44\,m/s - 12\,m/s)$
$= 1.856\,kg\cdot m/s$

$$F_{avg} = \frac{\Delta p}{\Delta t} = \frac{1.856\,kg\cdot m/s}{0.006\,s} = 309.3\,N$$

From 0 to 0.002 s $F_{avg} = \frac{1}{2} F_{max}$ (also from 0.004s to 0.006s)
From 0.002s to 0.004 s, $F_{avg} = F_{max}$
Overall average $= \frac{1}{3}\left(\frac{1}{2}F_{max} + F_{max} + \frac{1}{2}F_{max}\right) = \frac{2}{3}F_{max}$

$$F_{max} = \frac{3}{2} F_{avg} = 464\,N$$

④ (a) $\vec{\Delta p} = \vec{F}_{avg}\,\Delta t = \left(\langle -7\times10^3, -9.2\times10^2, 0\rangle N\right)(0.2\,s)$

$= \langle -1.4\times10^2, -1.84\times10^2, 0\rangle\ kg\cdot m/s$

$\vec{p}_f = \vec{p}_i + \vec{\Delta p} = \langle 4.4\times10^4, -7.6\times10^3, 0\rangle + \langle -1.4\times10^2, -1.84\times10^2, 0\rangle$
$= \langle 4.26\times10^4, -7.78\times10^3, 0\rangle\ kg\cdot m/s$

(b) $\vec{v}_{avg} = \frac{1}{2}(v_i + v_f) = \frac{1}{2}\,\frac{\langle 4.4\times10^4, -7.6\times10^3, 0\rangle + \langle 4.26\times10^4, -7.78\times10^3, 0\rangle}{240}$

$= \langle 180, -32, 0\rangle\ m/s$

$\vec{r}_f = \vec{r}_i + \vec{v}_{avg}\,\Delta t = \langle 4.3\times10^3, 8.7\times10^3, 0\rangle + \langle 180, -32, 0\rangle(20)$
$= \langle 4336, 864, 0\rangle\ m$

(5) (a) $v_y = 0$ at max. height, so $\vec{v} = \langle 5, 0, 0 \rangle$ m/s

(b) $v_f = v_i - g\Delta t \qquad \Delta t = \frac{v_i}{g} = \frac{8 \text{ m/s}}{9.8 \text{ m/s}^2} = 0.816 \text{ s}$

(c) $\vec{v}_{avg} = \frac{1}{2}(\vec{v}_i + \vec{v}_f) = \frac{1}{2}[\langle 5, 8, 0 \rangle + \langle 5, 0, 0 \rangle] = \langle 5, 4, 0 \rangle$ m/s

(d) $\Delta \vec{r} = \vec{v}_{avg}\,\Delta t = \left(\langle 5, 4, 0 \rangle \frac{m}{s}\right)(0.816 \text{ s}) = \langle 4.1, 3.3, 0 \rangle$ m

(e) $\Delta t = 2(0.816 \text{ s}) = 1.632 \text{ s}$

(f) $\Delta x = v_x \Delta t = (5 \text{ m/s})(1.632 \text{ s}) = 8.16 \text{ m}$

(g) v_y is reversed from initial v_y or -8 m/s

(h) $\tan\theta = v_y / v_x = 8 \text{ m/s} / 5 \text{ m/s} = 1.6 \quad \Rightarrow \quad \theta = 58°$

(i) $-58°$