Class problems – Nov 6.

① ~~Ex~~: At initial point $K_1 = 0 \quad U_1 = mgL$

at bottom $K_2 = \frac{1}{2}mv^2 \quad U_2 = 0$

define system

Cons of E $\quad U_1 + K_1 = U_2 + K_2 \Rightarrow mgL = \frac{1}{2}mv^2$

$v = \sqrt{2gL} = 4.8 \text{ m/s}$

(b) at top of arc around peg

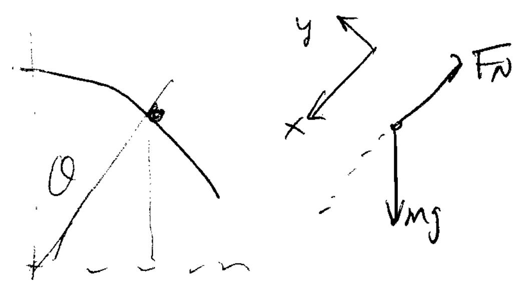
$U_3 = mg(2r) \qquad r = L - d = 1.2 - .75 = .45$

$K_3 = \frac{1}{2}mv_3^2$

$U_1 = U_3 + K_3 \qquad mgL = mg(2r) + \frac{1}{2}mv_3^2$

$\sqrt{2g(L-2r)} = 2.42 \text{ m/s}$

②



define system

$F_{net,r} = mg\cos\theta - F_N = \frac{mv^2}{r}$

$F_N = 0 \qquad mg\cos\theta = \frac{mv^2}{r} \qquad v^2 = gr\cos\theta$

at Top. $\quad K = 0 \quad U = mg(2r)$

where he leaves: $K = \frac{1}{2}mv^2 \quad U = mg(R + R\cos\theta)$

If friction, $\theta > 48.2°$

$mg(2r) = \frac{1}{2}mv^2 + mg(r + r\cos\theta)$

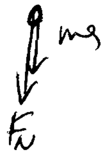
$\cancel{mg}(2\cancel{r}) = \frac{1}{2}\cancel{m}\cancel{g}\cancel{r}\cos\theta + \cancel{mg}(\cancel{r} + \cancel{r}\cos\theta)$

$2 = \frac{1}{2}\cos\theta + 1 + \cos\theta$

$1 = \frac{3}{2}\cos\theta \qquad \cos\theta = \frac{2}{3} \qquad \theta = 48.2°$

(3) at top of loop, at minimum speed

defies simple



$$mg + F_N = \frac{mv^2}{r}$$

$$F_N = 0 \qquad \cancel{m}g = \frac{\cancel{m}v^2}{r}$$

$$v^2 = gr$$

at h: $K = 0$ $\quad U = mgh$

at top of loop $\quad K = \frac{1}{2}mv^2 \quad U = mg(2r)$

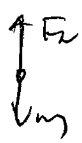
$$\frac{1}{2}\cancel{m}v^2 + \cancel{m}g(2r) = \cancel{m}gh$$

$$\frac{1}{2}\cancel{g}r + 2\cancel{g}r = \cancel{mg}h \qquad \underline{\underline{h = 2.5r}}$$

(b) at new release point $\quad U = mgh' = mg(5r)$

at bottom $\quad U = 0 \quad K = \frac{1}{2}mv^2 \qquad \cancel{m}g(5r) = \frac{1}{2}\cancel{m}v^2$

$$v^2 = 10gr$$



$$F_N - mg = \frac{mv^2}{r}$$

$$F_N = mg + \frac{mv^2}{r} = mg + \frac{m}{\cancel{r}}10g\cancel{r} = \underline{\underline{11mg}}$$

at top $\quad \cancel{U = mg(2r) \quad K = \frac{1}{2}mv^2}$

$$\cancel{mg(2r) + \frac{1}{2}mv^2}$$

$$U = mg(2r) \quad K = \frac{1}{2}mv^2$$

$$mg(2r) + \frac{1}{2}mv^2 = mg(5r)$$

$$\frac{1}{2}\cancel{m}v^2 = \cancel{m}g(3r) \qquad v^2 = 6gr$$

$$F_N + mg = \frac{mv^2}{r} \qquad F_N = \frac{mv^2}{r} - mg = \frac{m}{r}6gr - mg = \underline{\underline{5mg}}$$

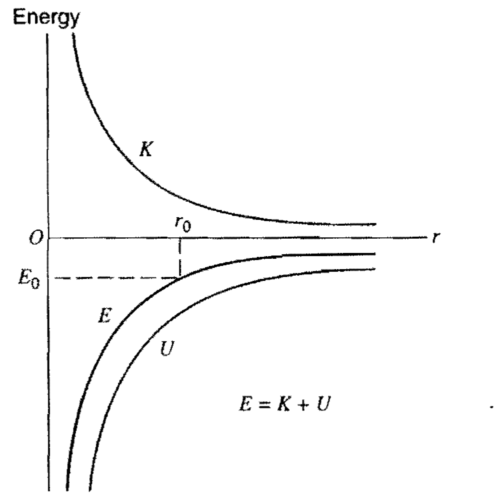


FIGURE 14-17. Kinetic energy K, potential energy U, and total energy $E = K + U$ of a body in circular planetary motion. A planet with total energy $E_0 < 0$ will remain in an orbit with radius r_0. The greater the distance from the Sun, the greater (that is, less negative) its total energy E. To escape from the center of force and still have kinetic energy at infinity, the planet would need positive total energy.

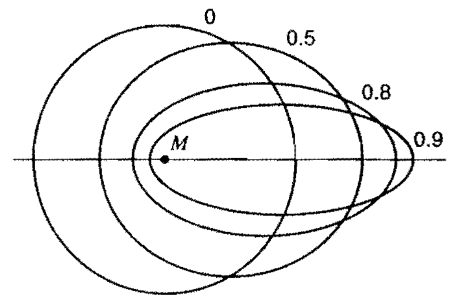


FIGURE 14-18. All four orbits have the same semimajor axis a and thus correspond to the same total energy E. Their eccentricities are marked.

must go to zero as the separation goes to infinity. The potential energy is always negative except for its zero value at infinite separation. The meaning of the total negative energy then is that the system is a closed one, the planet m always being bound to the attracting solar center M and never escaping from it (Fig. 14-17).

It can be shown* that Eq. 14-25 is also valid for elliptical orbits, if we replace r by the semimajor axis a. The total energy is still negative, and it is also constant, because gravitational forces are conservative. Hence both the total energy and the total angular momentum are constant in planetary motion. These quantities are often called *constants of the motion.*

Because the total energy does not depend on the eccentricity of the orbit, all orbits with the same semimajor axis a have the same total energy. Figure 14-18 shows several different orbits that have the same energy.

If we supply the proper amount of kinetic energy, we can arrange for the total energy to be zero or positive, in which case the orbits are no longer elliptical. The orbits are parabolic for $E = 0$ and hyperbolic for $E > 0$. This case often occurs in the scattering of particles from a nucleus, where the electrostatic force also varies as $1/r^2$. The spacecraft *Pioneer 10* was given enough initial kinetic energy to allow it to escape from the solar system; launched on March 3, 1972, it passed the orbit of Pluto, the outermost planet, on June 14, 1983, outward bound on a hyperbolic path.

Equation 14-25 shows that we cannot change the speed of an orbiting satellite without also changing the radius of its orbit. For example, suppose two satellites are following one another in the same circular orbit. If the trailing satellite tries to catch the leading one by accelerating forward, thereby increasing the kinetic energy, the total energy becomes less negative and the radius increases. Docking two spacecraft is not just a simple exercise in edging one craft forward! In fact, as the following sample problem shows, the proper procedure to use in overtaking an orbiting spacecraft often involves slowing down rather than speeding up.

SAMPLE PROBLEM 14-10. Two identical spacecraft, each with a mass of 3250 kg, are in the same circular orbit at a height of 270 km above the Earth's surface. Spacecraft A leads spacecraft B by 105 s; that is, A arrives at any fixed point 105 s before B. At a particular point P (Fig. 14-19), the pilot of B fires a short rocket burst in the *forward* direction, reducing the speed of B by 0.95%. Find the orbital parameters (energy, period, semimajor axis) of B before and after the "burn," and find the order of the two ships when they next return to point P.

Solution For $h = 270$ km, $r = R_E + h = 6370\text{ km} + 270\text{ km} = 6640$ km. Thus, before firing the rockets, $a = 6640$ km and, from Eq. 14-25,

$$E = -\frac{GmM_E}{2a}$$

$$= -\frac{(6.67 \times 10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2)(3250\text{ kg})(5.98 \times 10^{24}\text{ kg})}{2(6.64 \times 10^6\text{ m})}$$

$$= -9.76 \times 10^{10}\text{ J}.$$

The period follows from Eq. 14-23:

$$T = \left(\frac{4\pi^2 a^3}{GM_E}\right)^{1/2}$$

$$= \left(\frac{4\pi^2(6.64 \times 10^6\text{ m})^3}{(6.67 \times 10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\text{ kg})}\right)^{1/2}$$

$$= 5380\text{ s}.$$

Equations 14-24 and 14-25 show that (for a circular orbit only!) the kinetic energy is numerically equal to the negative of the total energy, so $K = +9.76 \times 10^{10}$ J and

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(9.76 \times 10^{10}\text{ J})}{3250\text{ kg}}} = 7.75 \times 10^3\text{ m/s}.$$

*See reference on p. 312.

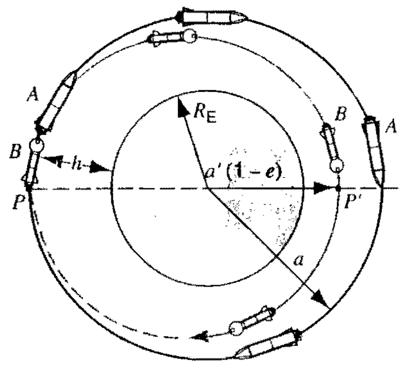


FIGURE 14-19. Sample Problem 14-10. The orbits of spacecraft A and B are shown. Note that B catches A by moving to a noncircular orbit at lower height above the Earth. The relative size of the Earth and the orbital heights is not to scale.

After the burn, the speed decreases by the given amount of 0.95% to $v' = (1 - 0.0095)v = 7.68 \times 10^3$ m/s, and the new kinetic energy of B is

$$K' = \tfrac{1}{2}(3250 \text{ kg})(7.68 \times 10^3 \text{ m/s})^2 = 9.58 \times 10^{10} \text{ J}.$$

The potential energy of B at point P immediately after the short burn is unchanged, equal to the initial value $E - K$ or $2E$, according to Eq. 14-25. The total energy E' of B after the burn must then be

$$E' = K' + U' = 9.58 \times 10^{10} \text{ J} + 2(-9.76 \times 10^{10} \text{ J})$$
$$= -9.94 \times 10^{10} \text{ J},$$

and the new semimajor axis is, from Eq. 14-25,

$$a' = -\frac{GmM_E}{2E'}$$
$$= -\frac{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(3250 \text{ kg})(5.98 \times 10^{24} \text{ kg})}{2(-9.94 \times 10^{10} \text{ J})}$$
$$= 6.52 \times 10^6 \text{ m} = 6520 \text{ km},$$

a reduction of 1.8% from the value in the original orbit. The corresponding period is

$$T' = \left(\frac{4\pi^2 a'^3}{GM_E}\right)^{1/2}$$
$$= \left(\frac{4\pi^2(6.52 \times 10^6 \text{ m})^3}{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}\right)^{1/2}$$
$$= 5240 \text{ s}.$$

The difference in the periods is 140 s. That is, if A originally passes through point P at $t = 0$ and B passes through (and fires its rockets) at $t = 105$ s, then A returns to P at $t = 5380$ s (determined by the period T), and B returns to P at 5240 s after its initial passage, or at $t = 105 \text{ s} + 5240 \text{ s} = 5345$ s. Thus B is now 35 s *ahead of A* at point P. Now B can fire a second rocket burst identical in strength and duration to the first but in the reverse direction. This returns B to the original circular orbit, now 35 s ahead of A. Figure 14-19 shows the relationship between A and B during the first orbit after the burn. Note that after the burn, B moves in an elliptical orbit and so can pass A without colliding because A remains in the original circular orbit.

See Exercise 38 to help understand how B can reduce its speed at P and still get ahead of A.

14-8 THE GRAVITATIONAL FIELD (Optional)

A basic fact of gravitation is that two particles exert forces on one another. We can think of this as a direct interaction between the two particles, if we wish. This point of view is called *action-at-a-distance,* the particles interacting even though they are not in contact. Another point of view is the *field* concept, which regards a particle as modifying the space around it in some way and setting up a *gravitational field.* This field, the strength of which depends on the mass of the particle, then acts on any other particle, exerting the force of gravitational attraction on it. The field therefore plays an intermediate role in our thinking about the force that one particle exerts on another.

According to this view we have two separate parts to our problem. First, we must determine the gravitational field established by a given distribution of particles. Second, we must calculate the gravitational force that this field exerts on another particle placed in it.

We use this same approach later in the text when we study electromagnetism, in which case particles with electric charge set up an *electric* field, and the force on another charged particle is determined by the strength of the electric field at the location of the particle.

Let us consider the Earth as an isolated particle and ignore all rotational and other nongravitational effects (so that g and g_0 are equivalent). We use a small test body of mass m_0 as a probe of the gravitational field. If this body is placed in the vicinity of the Earth, it will experience a force having a definite direction and magnitude at each point in space. The direction is radially in toward the center of the Earth, and the magnitude is $m_0 g$. We can associate with each point near the Earth a vector $\vec{\mathbf{g}}$, which is the acceleration that a body would experience if it were released at this point. We define the *gravitational field strength* at a point as the *gravitational force per unit mass* at that point or, in terms of our test mass,

$$\vec{\mathbf{g}} = \frac{\vec{\mathbf{F}}}{m_0}. \tag{14-26}$$

By moving the test mass to various positions, we can make a map showing the gravitational field at any point in space. We can then find the force on a particle at any point in that field by multiplying the mass m of the particle by the value of the gravitational field $\vec{\mathbf{g}}$ at that point: $\vec{\mathbf{F}} = m\vec{\mathbf{g}}$. Figure 14-20 shows examples of gravitational fields.

The gravitational field is an example of a *vector field,* each point in this field having a vector associated with it. There are also *scalar fields,* such as the temperature field in a heat-conducting solid. The gravitational field arising from