Week 6 - In-class problem

① (a) AB $+$ $a>0, v>0 \Rightarrow F$ is in same direction as motion $\Rightarrow \vec{F}\cdot\vec{d}>0$

BC 0 $a=0 \Rightarrow F=0 \Rightarrow W=0$

CD $-$ $v>0$ but $a<0 \Rightarrow \vec{F}\cdot\vec{d}<0$

DE $+$ $v<0, a<0 \Rightarrow \vec{F}$ in same direction as $\vec{d}$ so $\vec{F}\cdot\vec{d}>0$

(b) AB $\Delta K>0$

BC $\Delta K=0$

CD $\Delta K<0$

DE $\Delta K>0$

Discuss: Would any answers be different if the graph showed only the points A, B, C, D, E and not the connecting lines?

② See attached

③ Use $W=\Delta K$, because potential energy may not yet have been introduced.

(a) $W_{grav}=mgh$ $\Delta K = K_f - K_i = \frac{1}{2}mv^2 - 0$

$\frac{1}{2}mv^2 = mgh \Rightarrow v=\sqrt{2gh}$

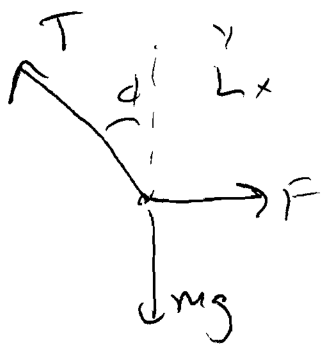
(b) $\Delta K = K_f - K_i = 0-0 = 0$

$W_{grav} = mg(h+x)$ $W_{spring} = -\frac{1}{2}kx^2$

$W=\Delta K \Rightarrow mg(h+x)-\frac{1}{2}kx^2=0$

$x = \dfrac{mg \pm \sqrt{m^2g^2+2mghk}}{k}$ 2 roots, one positive + one negative

Discuss: Did we set up the problem so that our desired sol. is $x>0$ or $x<0$?

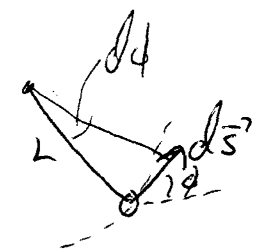


x: $F - T\sin\phi = ma_x = 0$

y: $T\cos\phi - mg = ma_y = 0$

$T = mg/\cos\phi$, $F = T\sin\phi$

$\Rightarrow F = mg\tan\phi$



$ds = L\,d\phi$

① T: $\vec{T}\cdot d\vec{s} = 0 \Rightarrow W_T = 0$

② F: $W_F = \int_i^f \vec{F}\cdot d\vec{s} = \int_i^f F\cos\phi\, ds$

$= \int_0^{\phi_m} mg\tan\phi\cos\phi\, L\, d\phi$

$= mgL\int_0^{\phi_m} \sin\phi\, d\phi$

$= mgL\left[1-\cos\phi_m\right]$

$= mgh$

③ mg: $W_g = \int_i^f m\vec{g}\cdot d\vec{s} = \int_i^f mg\cos(90°+\phi)\, ds$

$= \int_i^f mg(-\sin\phi)\, ds$

$= -mgL\int_{\phi=0}^{\phi_m} \sin\phi\, d\phi$

$= -mgh$

$\Rightarrow W_T + W_F + W_g = 0 = \Delta K$ as expected.

4. (a) Discuss $W = \Delta K$ comes from $F = ma$ which applies only to particles — all parts of the system must move in the same way. Clearly not true in this case.

Note that point of application of the force on the car by the wall does not move, so $W = 0$ as we have defined W. Then $W \neq \Delta K$

(b) Same situation — she pushes on railing, railing pushes back (Newton's 3rd law). But point of application of force does not move, so $W = 0$. But $\Delta K \neq 0$.

Note that not all parts of her body move in the same way, so we can't treat the skater as a particle.

Discuss: Where does her KE come from? (It is not provided by an external agent.)