

Computational Problems for Physics Courses

Motivation & Examples

Rubin H Landau

<http://physics.oregonstate.edu/~landaur/>

CP Author, Founder CP Degree Program

Computational subatomic few-body systems (1966-2003)
CP Education (1988-)

Grad School: How creative? Plenty smarts; Exact Compute

DCOMP Boston, March 2019



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Contributors (Coauthors: Manuel Paez & Cristian Bordeianu-d)

In Addition: Suffering Students & Collaborators

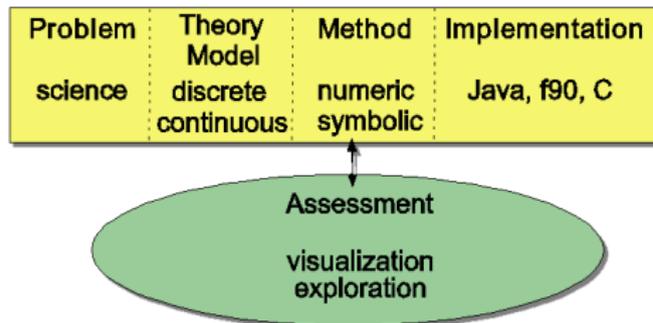
- C. E. Yaguna, J. Zuluaga, Oscar A. Restrepo, Guillermo Avendano-Franco
- Paul Fink, Robyn Wangberg, CoAuthors
- Justin Elser, Chris Sullivan (systems)
- Sally Haerer, Saturo S. Kano (consultants, producers)
- Melanie Johnson (Unix Tutorials)
- Hans Kowallik (CP text, sounds, codes, LAPACK, PVM)
- Matthew Ervin Des Voigne (tutorials)
- Bertrand Laubsch (Java sound, decay)
- Jon J Maestri (vizualizations, animations, quantum wave packets)
- Juan Vanegas (OpenDX)
- Al Stetz, David McIntyre (First Course)
- Connelly Barnes (OOP, PtPlot)
- Phil Carter, Donna Hertel (MPI)
- Zlatko Dimcovic (Wavelets, Java I/O)
- Joel Wetzel (figures)
- Pat Cannan, Don Corliss, Corvallis HS (N-D Newton Raphson)
- Brian Schlatter
- Daniel Moore, (REU, Sum 98; Whitman Coll)
- Justin Murray, (REU, Sum 98; Weber State)
- Brandon Smith, (REU, Sum 97; Chico State/SDSC)
- Paul D. Hillard, III (REU, Sum 96; Southern U)
- Kevin Wolver, (REU, Sum 96; St Ambrose)

How: Computation too important to leave to CS

- *“We are teaching the same things we taught 50 years ago”*

APS/AAPT Taskforce on Grad Ed., R Diehl, 2004

- OK that’s physics (APS/AAPT Taskforce)
- Do take math, then Math Mtds of Physics \Rightarrow
- Teach Computation within physics
- Use research-like examples



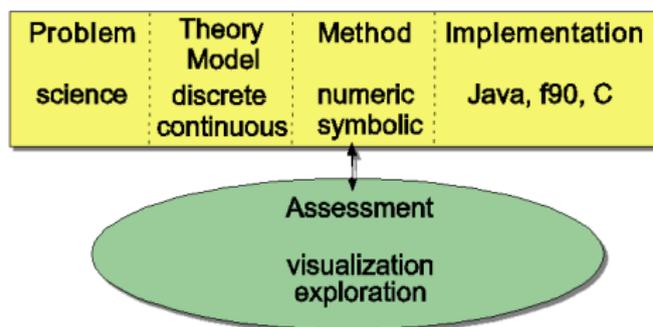
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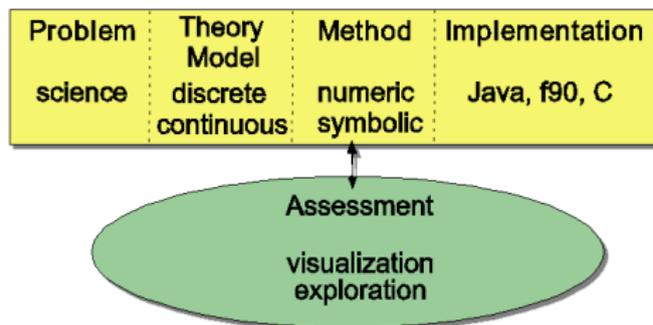
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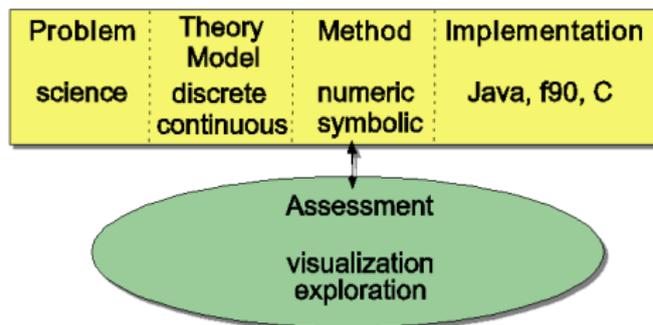
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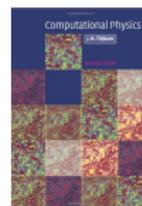
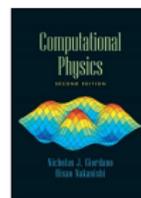
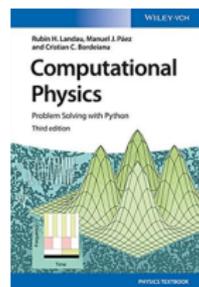
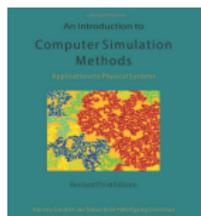
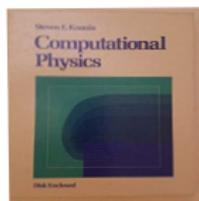
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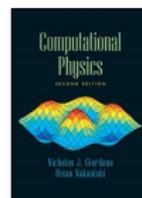
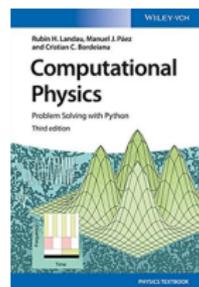
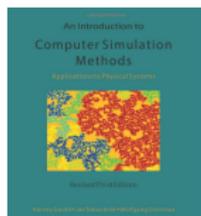
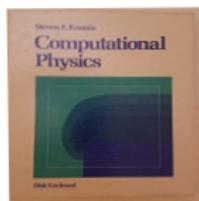
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Computational Physics Education has Progressed



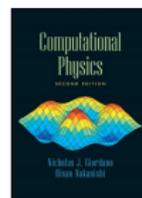
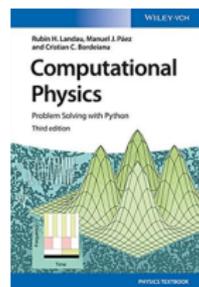
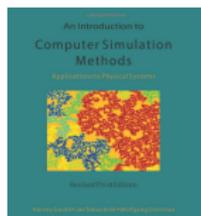
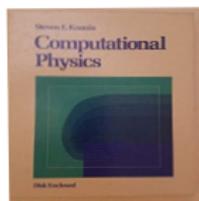
- 1980s: CP Texts, now plenty good ones
- ⇒ 1990s: CP Courses, Computational X programs, . . .
- No need repeat past developments (PICCUP)
- **Need integrate computation into physics courses**
- CRC Press *Series in CP*; S Gottlieb, R Landau, Eds

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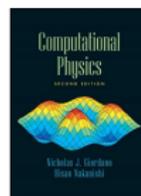
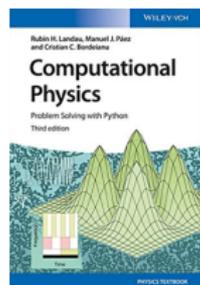
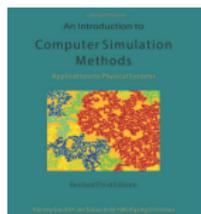
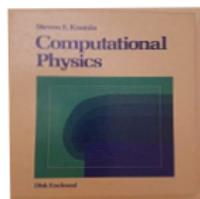
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Computational Competencies by Comp Physicists

≈ **AAPT Statement** (G, T, L, C, 2011)

≈ **HPC University** (Ohio State) Computational Competencies

Tools All Physicists Should Know

● Basic Numeric Tools

- Integration ⇒ Guassian
- Differentiation
- Floating Point Math [Errors]
- Search Techniques
- Linear Algebra [Libes]

● Languages, Environments

- Program compiled (2) (≠ CS)
- Symbolic
- Operating sys (2)

● ODEs Solutions ⇒ rk4

- Planets, 3-B Orbits
- CM & QM Chaotic Scattering
- Tools for Analysis
- Data fitting
- Visualize functions & data
- Document prep (\LaTeX)
- Fourier, DFT, FFT
- Wavelets*, Principal Components*

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Computational Competencies by Phys Educators

AAPT Undergraduate Curriculum Task Force, 2016

- Computation ubiquitous in physics
- Deeper understanding of physics via fundamental laws
- Spreadsheets: “see exactly what’s happening”
- Mathematical computing packages: get computing over with quickly, don’t emphasize computing
- Programming language: worthwhile “in the long term”
- Special-purpose software best choice for classrooms, not empowering
- Process data
- Represent data visually
- Prepare documents and presentations

AAPT 2016 ⇒ Don’t take computation too seriously!

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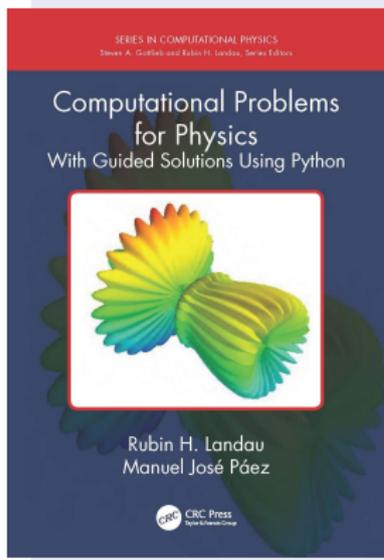
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Computational Problems for Physics Courses

2018 Book, collected problems, Projects & Demos [shameless commerce]

Chapters; Some borrowed, some new, all long overdue



- Computational Basics
- Data Analytics
- Classical, Nonlinear Dynamics
- Waves, Fluids
- E & M
- Quantum Mechanics
- Thermo, Stat Phys
- Bio Models: Population Dynamics, Plant Growth
- More Entry-Level Problems
- Python Codes

Problems to Include in Physics Courses (will demo)

Applications

- Explore Nonlinear Dynamics
- Bifurcations, phase space (CM)
- Double & Chaotic Pendula (CM)
- Fractals, Stat Growth (StatMech)
- Integral Equations (QM, CM)
- Monte Carlo, Stochastic
- Spontaneous Decay (QM)
- Random Walk (Thermo)
- Thermal Simulations (StatMech)
- Molecular Dynamics (\neq MC)
- Feynman Path Integrals (QM)
- PDEs (relax, t step, split t)
- Laplace/Possion (EM)
- Heat [x-t diffusion] (Thermo)
- Realistic Strings [waves] (CM)
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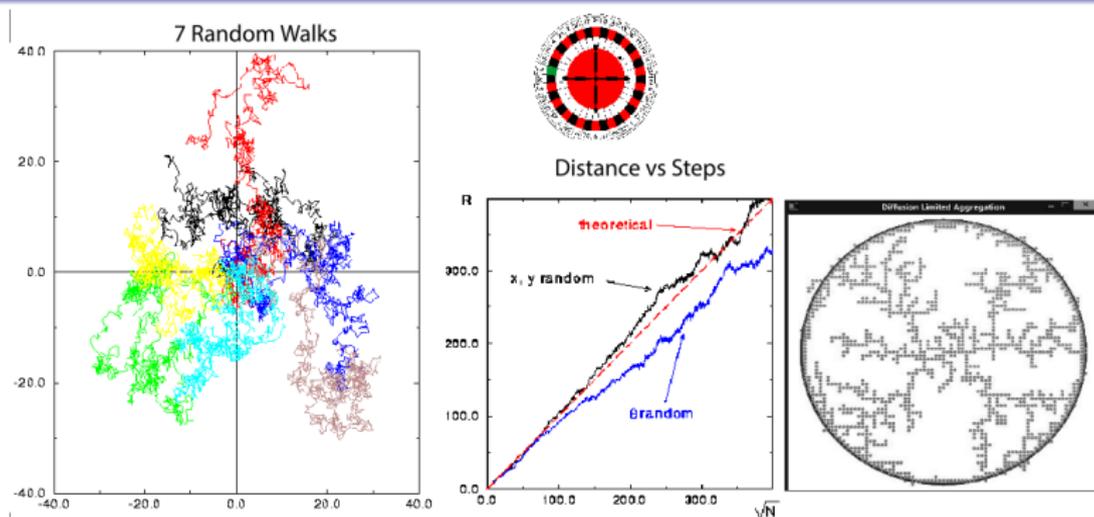
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Monte-Carlo Random Walks *Walk3DVis.py*



- $\Delta x_i = r_i, \Delta y_i = r_j$
- Good to “see” a walk
- Stochastic processes & math: very interesting, $R_{rms} \propto \sqrt{N}$
- Random, probability, experimental statistics \leq taught

Monte-Carlo Decay Simulation *DecaySoundVisMod.py*

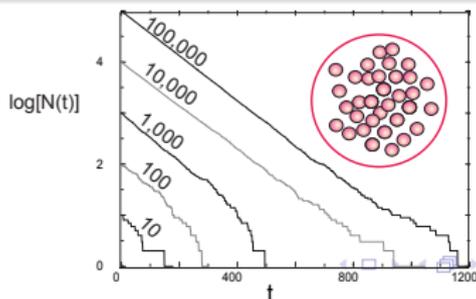
Analytic = $e^{-t/\tau} \simeq$ Simulation (discrete): Closer Nature

$$\mathcal{P} = \frac{\Delta N(t)/N(t)}{\Delta t} = -\lambda \quad (\text{Law of Nature}) \quad (1)$$

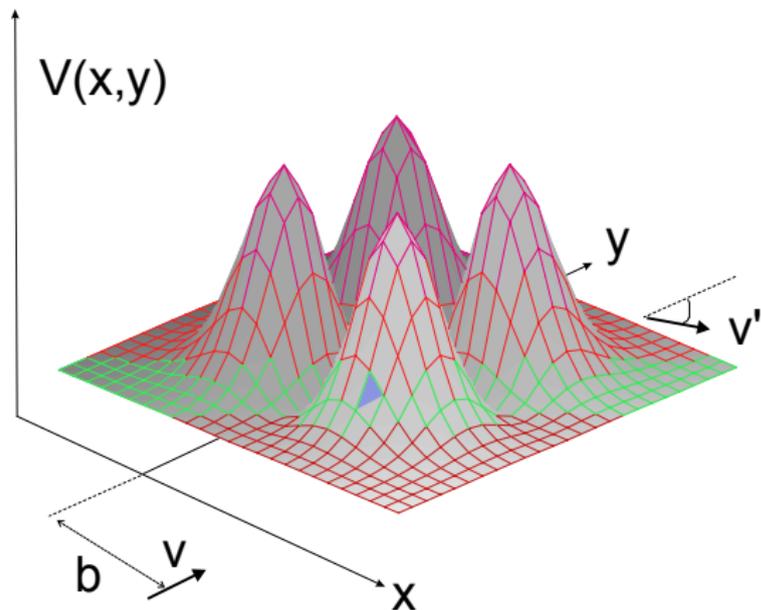
$$\Rightarrow \frac{\Delta N(t)}{\Delta t} = -\lambda N(t) \quad (\text{Real Physics}) \quad (2)$$

$$\Rightarrow \frac{dN(t)}{dt} = \frac{dN}{dt}(0)e^{-\lambda t} \quad (\text{Approximate Physics}) \quad (3)$$

One line algorithm: if $r_i < \lambda$, $\Delta N = \Delta N + 1$



Classical Chaotic Scattering, 3-Body Problems



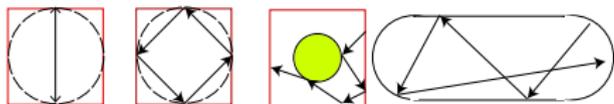
- Just Coupled ODEs

$$\mathbf{F} = m\mathbf{a}$$

$$\mathbf{F} = -\nabla \left(x^2 y^2 e^{-x^2 - y^2} \right)$$

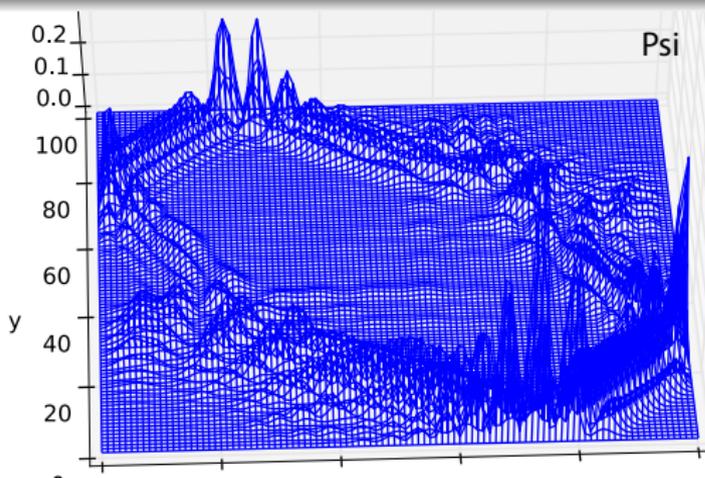
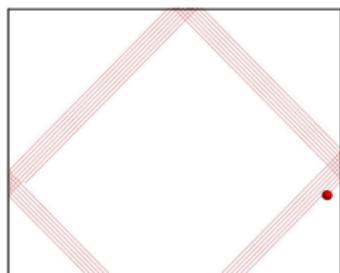
- **Chaotic Scattering**
- **3 Body Applet**

Classical & Quantum Chaos



Solve Same Problem Classically & Quantum Mechanically

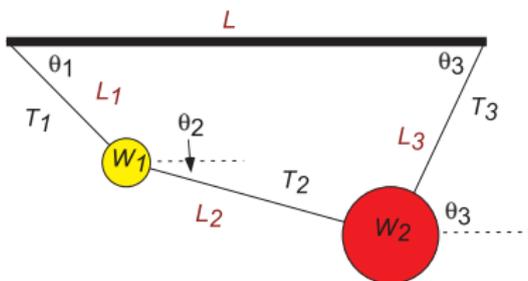
- Billiards in enclosed figure can be chaotic (BC)
- Quantum Chaos hard to “see”
- Look for signature of classical chaos in QM



Linear Algebra Packages, HS Problem \Rightarrow HPC

NewtonNAnimate.py 

<Ex: 3 masses>



- $T_i, \theta_i = ?$
- 9 nonlinear equations

$$L_1 \cos \theta_1 + L_2 \cos \theta_2 + L_3 \cos \theta_3 = L,$$

$$L_1 \sin \theta_1 + L_2 \sin \theta_2 - L_3 \sin \theta_3 = 0,$$

$$\sin^2 \theta_1 + \cos^2 \theta_1 = 1$$

$$\sin^2 \theta_2 + \cos^2 \theta_2 = 1$$

$$\sin^2 \theta_3 + \cos^2 \theta_3 = 1$$

- Newton-Raphson search
- Use matrix libes

$$T_1 \sin \theta_1 - T_2 \sin \theta_2 - W_1 = 0$$

$$T_1 \cos \theta_1 - T_2 \cos \theta_2 = 0$$

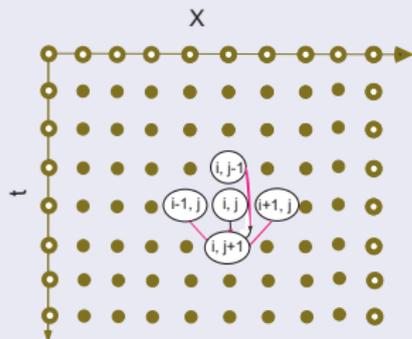
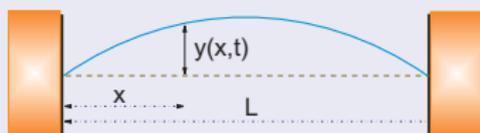
$$T_2 \sin \theta_2 + T_3 \sin \theta_3 - W_2 = 0$$

$$T_2 \cos \theta_2 - T_3 \cos \theta_3 = 0$$

Realistic Waves: Catenary + Friction

EqStringAnimate.py 

PDE with Time Stepping



$$c^2 \frac{\partial^2 y}{\partial x^2} - \frac{2\kappa}{\rho} \frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial t^2} \quad (\text{with Friction}) \quad (1)$$

$$\frac{\partial T(x)}{\partial x} \frac{\partial y(x,t)}{\partial x} + T(x) \frac{\partial^2 y(x,t)}{\partial x^2} = \rho(x) \frac{\partial^2 y(x,t)}{\partial t^2} \quad (\text{Variable } \rho \text{ \& } T) \quad (2)$$

Time-Dependent Schrödinger Eqn

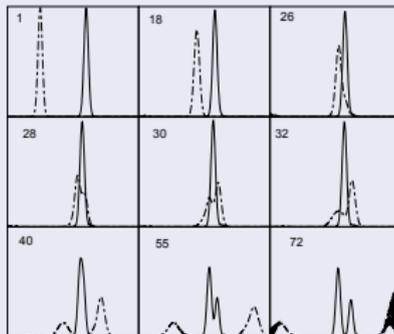
Wavepacket – Wavepacket Interactions

$$i \frac{\partial \psi(x, t)}{\partial t} = -\frac{1}{2m} \frac{\partial^2 \psi(x, t)}{\partial x^2} + V(x) \psi(x, t) \quad (1 \text{ particle})$$

$$i \frac{\partial \psi(x_1, x_2, t)}{\partial t} = -\frac{1}{2m_1} \frac{\partial^2 \psi(x_1, x_2, t)}{\partial x_1^2} - \frac{1}{2m_2} \frac{\partial^2 \psi(x_2, x_2, t)}{\partial x_2^2} + V(x_1, x_2) \psi(x_1, x_2, t) \quad (2 \text{ particles})$$

- Often ignored in QM
- 1 packet, 2 Slits
- Packet(x_1)-Packet(x_2)

m=10m, Attractive Vsquare
KE = -V/2



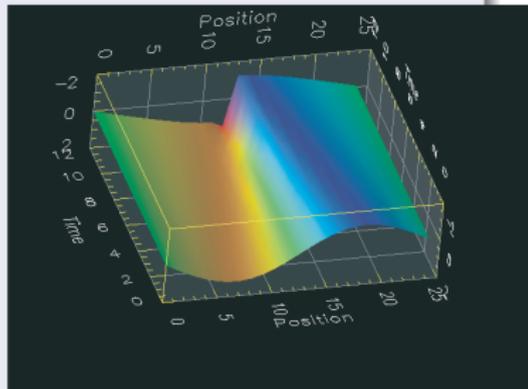
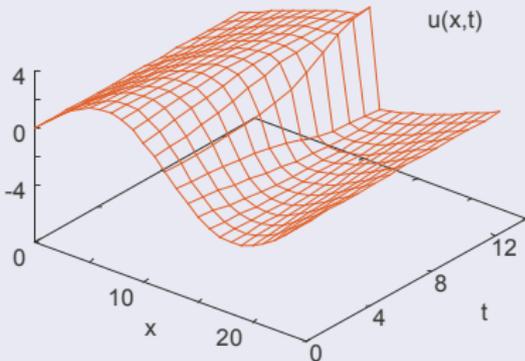
Shock Wave Physics

Singularity ⇒ Better Algorithm (Lax)

Singular Nature?

$$\frac{\partial \rho(x, t)}{\partial t} + c \frac{\partial \rho(x, t)}{\partial x} = 0 \quad (\text{Advection/Continuity}) \quad (1)$$

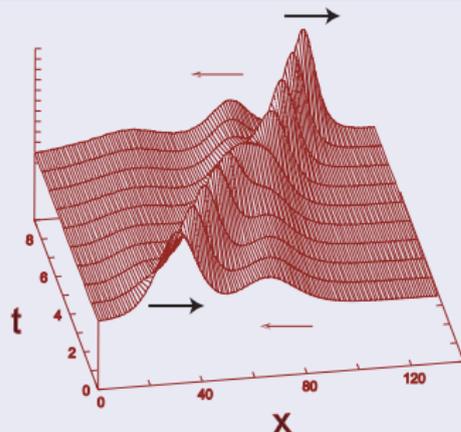
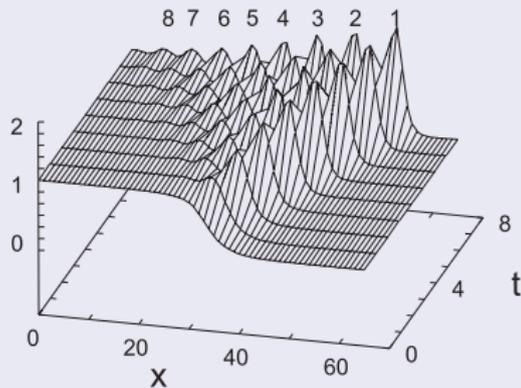
$$\frac{\partial \rho(x, t)}{\partial t} + \epsilon \rho(x, t) \frac{\partial \rho(x, t)}{\partial x} = 0 \quad (\text{Burgers' Eqn} \Rightarrow \text{Shock}) \quad (2)$$



(Shock Waves) Dispersion, Solitons

SolitonAnimateVis.py 

KdeV Equation: Dispersion Balances Shock

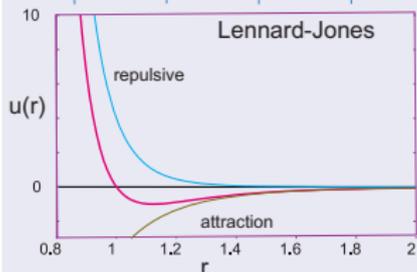
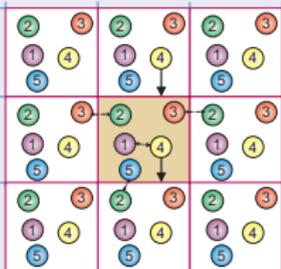


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$$\frac{\partial \rho(x, t)}{\partial t} + \epsilon \rho(x, t) \frac{\partial \rho}{\partial x} + \mu \frac{\partial^3 \rho(x, t)}{\partial x^3} = 0 \quad \text{Shock + Dispersion} \quad (2)$$

Molecular Dynamics

Straightforward, Obvious, Ridiculously Effective



- > Chem 101: walls $\Rightarrow PV = nRT$
- Just $F_{QM} = ma_i$; 10^8 times
- Deterministic \neq statistics (kT)

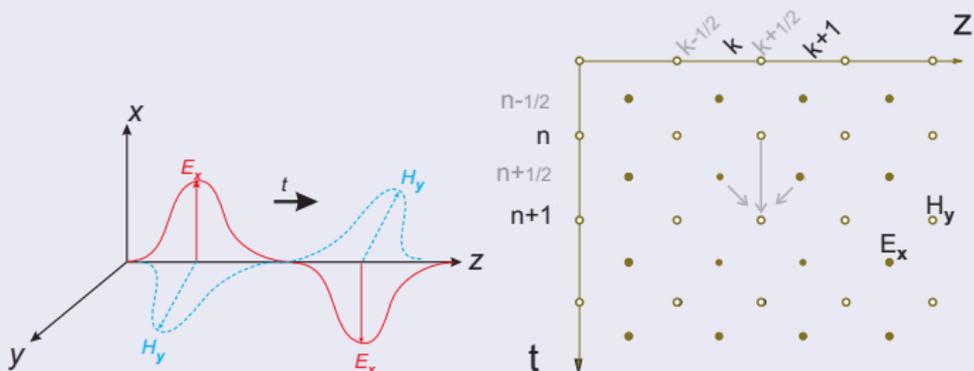
Maxwell: Finite Difference Time Domain

Split Space-Time Steps

Problem: waveguide

$$\tilde{E}_x^{k,n+1/2} = \tilde{E}_x^{k,n-1/2} + \beta \left(H_y^{k-1/2,n} - H_y^{k+1/2,n} \right) \quad (1)$$

$$H_y^{k+1/2,n+1} = H_y^{k+1/2,n} + \beta \left(\tilde{E}_x^{k,n+1/2} - \tilde{E}_x^{k+1,n+1/2} \right) \quad (2)$$



- Coupled E_x, H_y drive other
- Easy: even Excell

Take Home Lessons: Rejuvenate Physics Ed

Computations Part of Real-World Physics

- Include Research-like & Computation problems
- Within physics course
- Computation Problems Book may help
- Agree: faulty math \Rightarrow bad science?
- So uncertain computation \Rightarrow bad physics?
- Computation too important to leave to CS

