

Activities II (Exploration) Bug Dynamics: The Logistic Map's Bifurcation Diagram

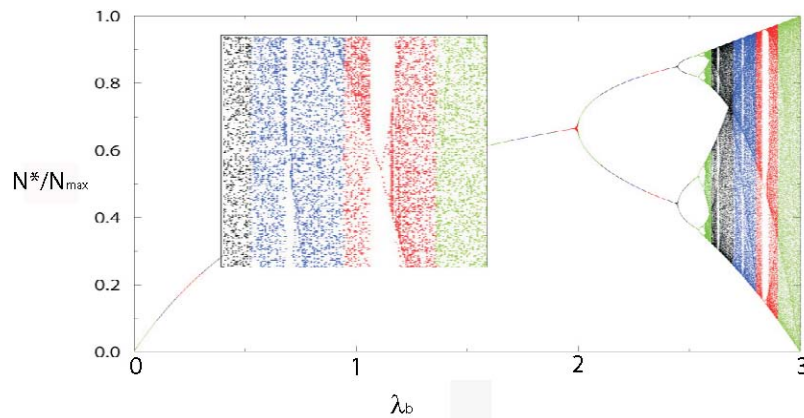


Figure 3: A bifurcation plot of attractor populations versus growth rate λ_b for the logistic map. The attractor populations have been divided by the maximum population in order to make the figure more universal.

6.5 A Bifurcation Diagram*

Computing and watching bug populations change with year number does give a good idea of the basic dynamics at play, at least until the oscillations get too complicated for you to discern patterns. In particular, as the number of bifurcations keeps increasing and the system approaches chaos, it is hard for us to see a simple underlying structure within the complicated behavior. One way to visualize what is going on is to concentrate on the attractors (also called long-term iterates), that is, those populations N^* that appear to attract the solutions and to which the solutions continuously return. A plot of these attractors as a function of the growth rate λ , such as shown in Figure 3, is called a bifurcation diagram and is an elegant way to summarize an extensive collection of computer calculations.

To create this type of diagram:

1. Break up the range $0 \leq \lambda \leq 3$ into 50 steps and loop through them, either by running your program 50 times, or by extending the program so that this is done automatically (a loop).
2. Wait 200 years of model time (or whatever number you found) for the transients to die out. Then print out the next 50 values of (x, y) points $(\lambda, N_j/N_{max})$, and plot them. Alternatively, you can plot directly from your program and skip the print out. We divide the ordinate (y value) by N_{max} to present the results in a more universal form and to save work. Hint: save the values of N_j/N_{max} to only two decimal places to avoid excessive plotting; you will not be able to see any fine details on your plots anyway.
3. Optional: Enlarge a section of your plot where where the density of points is high. Notice that similar bifurcations are contained within the magnified portion of the plot. This is called self similarity and is characteristic of fractal objects (of which sections of this plot are).

7. Going Beyond the Logistic Map

The logistic map includes the effect of a maximum population limit, but does not include separate rates for birth and death. In the *Predator Prey* module we extend the model by introducing a second species that feeds upon the bugs, and then observe how these populations appear to be related. You can attempt to make your own predator – prey model or look at the one in our module.