

Activities I: Bug Dynamics: The Logistic Map

Your problem is to develop and test the simplest mathematical model you can think of for a population that contains growth, yet also contains a mechanism to limit growth so that it remains below a maximum value. Once you have this model, you should then explore whether it can produce the types of behaviors described in the paragraph above.

3.5 Mathematical Model

Recall the mathematical model we have derived in Section 3.5:

N_j : the “number of bugs” in the current generation.

N_{j+1} : the “number of bugs” in the next generation.

λ : the “growth rate” constant.

N_{\max} = the maximum population

$$(11) \quad N_{j+1} = N_j (1 + \lambda) - \lambda N_j^2 / N_{\max}$$

6 Assessment of Model

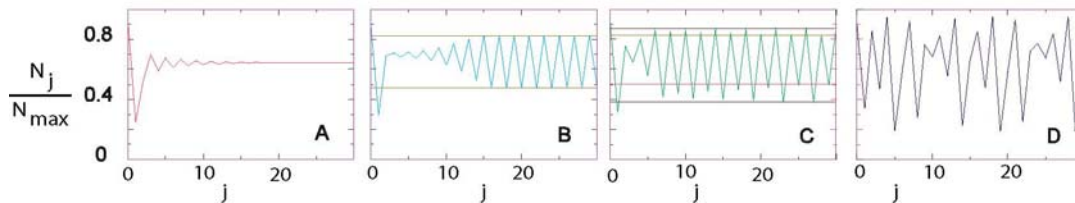


Figure 2: The insect population N_j versus the year number j for four different growth rates: (A) $\lambda = 1.8$, a single, stable population; (B) $\lambda = 2.3$, a two cycle; (C) $\lambda = 2.5$, a four cycle; (D) $\lambda = 2.8$, a chaotic regime.

6.1 Looking for Stable Populations (See Figure 2)

As the first exploration with our model, we want to see if it can produce a *stable* population. By “stable” we mean a population that may pass through a transient stage but eventually reaches a value at which it remains essentially the same from generation to generation. By “transient” we mean something that lasts only a short time and so is not a permanent feature.

1. You need a computer program written that computes and outputs the next-year population N_{j+1} as a function of year number j based on Equation (9). (This is so simple that a calculator is sufficient for the task.)

2. To set the scale for the calculations¹, assume a value for the maximum population, say $N_{\max} = 1000$.
3. Since the model generates a series of populations for a given initial population, we need to specify a value for the initial population N_0 . This is also called the seed population. (While this may seem like another parameter, it really is just the initial value for one of the variables in the model.) Start with a small value, say $N_0 = 100$, but feel free to experiment with other values.
4. The value assumed for the growth rate λ is very important. Too large a growth rate may lead to instabilities that “blow up”, and too small a value may lead to nothing of much interest. So let’s be careful. To make sure that the model is behaving reasonably before going exploring, let try some cases for which we can be fairly sure of what the model should predict. One clever way to do this is to try out some negative and zero values for λ . This should lead to a decaying population, or a static population. (A *static* population is one that does not change in time. A *stable* population is one that may experience deviations from some average value, but which always returns to this average value.) We suggest $\lambda = -1, -0.75, -0.5, -0.25, 0$.
5. Make plots of N_j versus j for each of these values of λ . Seeing pure decay, growth or no change serves in part as a validation of the model, that is, as a check that the model works the way it should in situations where we now the correct behavior.
6. Now that you have some confidence in the model, let’s see if we can increase the population to some stable values. With the same initial population as before, try $\lambda = 0, 0.5, 1.0, 1.5, 2$. Make plots of N_j versus j for each of these values of λ .
7. Take note of the *transient* behavior in these plots. These transients are the various values that the population passes through before a steady state or more regular behavior sets in. (They are called transients because they the population just passes through them without settling there.)

6.2 Checking Validity of Model

You should have found by now that with positive growth rates λ , this model does yield a variety of stable populations, with the bugs approaching the maximum population more rapidly for larger values of λ . This also serves in part as a validation of the model, that is, a test that this simple model predicts stable populations. Next, check the validity of this model, that is, that it is general and is not highly dependent upon the specific values of the parameters:

1. For fixed values of the growth rate λ and initial population N_0 , try different values for the maximum population N_{\max} . Verify that increasing values of N_{\max} lead to proportionally larger maximum populations, but do not affect that rate at which the maximum is attained. The transients may be affected.
2. For a fixed value of maximum population N_{\max} and growth rate λ , try different values for the seed population N_0 . Verify that differing values of N_0 do not affect the values of the maximum populations, although it may take longer to reach that maximum. Again, transients may be affected, but not the values for the stable populations.

¹ This sets the scale of the calculation since it is the ratio N_j/N_{\max} that enters into the formula, and so, for example, we would say that $N_j < 1000$ is small, whereas $N_j = 900$ is large.

3. For several combinations of N_{\max} , λ and N_0 , record the number of cycles that the system passes through before a stable population is obtained. (Variations of $\sim 3\%$ can be considered stable for this investigation.) This number will be useful when later when we create a bifurcation plot.

6.3 Modeling Empirical Data

In Table 1 we have presented some empirical data on the tent caterpillar. See if you can set the parameters of our model to make it consistent with those data.