

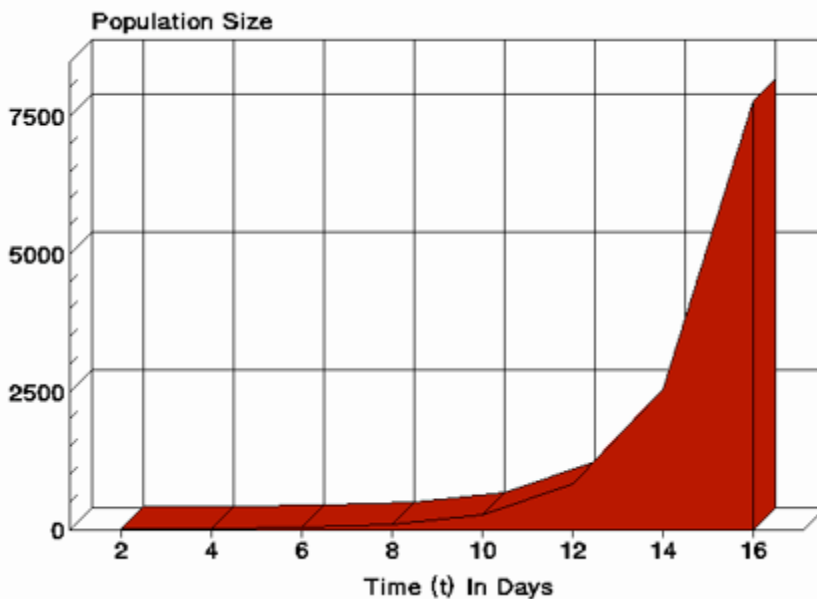
# Exponential Growth, Biological Case Study

## Project

Can we have a title of this project and some introductory sentences?

### Objectives

- To determine the growth rate  $\lambda$  from the doubling time  $\tau_2$  for *Wolffia microscopica*. Although it is experimentally hard to measure the change in the number of plants over time, especially for those small guys in the thimble, it is straight forward to measure the change in weight or in size of the biomass over time.
- To reproduce a previously-measured growth as a function of time using our exponential model. (Reproducing someone else's results is often done in the early stages of modeling.)
- To examine the population growth curve and determine which portions, if any, are exponential.



### Steps

1. The doubling time for *Wolffia microscopica* via budding is 30 hours = 1.25 days. What is its growth rate  $\lambda$ ?
2. Reproduce and plot sixteen days of *Wolffia* growth.
3. Compare to the experimental results in Fig.3 using a time interval of  $\Delta t = 1$  day.
  - a. Use excel to generate a table of *Wolffia* population size vs. time for an initial population  $N(0) = 1$ .
  - b. Verify that the same results are obtained using either the doubling time or the growth rate.

### 4. Create Vensim simulation of

#### Wolffia plant growth

- a. Save Vensim file of Radioactive Decay as **Wolffia growth** file
- b. Change the diagram to represent growth rather than decay
- c. Change the equations
- d. Use your Vensim model to reproduce the graphs you obtained in Excel.

Figure 1 Population growth in days

## World Population Growth

On the right of Figure 1 is presented a United Nation's plot of the past growth of the world's population and three predictions of the future population.

1. As best you can, convert this graph into a table of numbers.

2. Determine which portions of this graph, if any, correspond to exponential growth or decay,  $N \propto e^{\pm k t}$ , where  $k$  is a constant. Examine all three predictions of the future.
3. Determine which portions of this graph, if any, correspond to linear decay or growth,  $N \propto m t$ , where  $m$  is a constant. Examine all three predictions of the future.
4. Determine which portions of this graph, if any, correspond to a power-law decay or growth,  $N \propto k t^n$ , where  $k$  and  $n$  are constants. Examine all three predictions of the future.

*Hint:* The hard way to do this is to try to fit different functions to the graph. The easy way to do this is to 1) make a semilog plot of the data [ $\log N(t)$  versus  $t$ ] and 2) a log-log plot of the data [ $\log N(t)$  versus  $\log t$ ]. Exponential behavior would appear as a straight line on a semilog plot, while power-law behavior would appear as a straight line on a log-log plot.

*Proof:*  $N = A e^{\pm k t} \rightarrow \log N = \pm k t + \log A$   
 $N = k t^n \rightarrow \log N = \log t + \log k$