This print-out should have 14 questions. Check that it is complete before leaving the printer. Also, multiple-choice questions may continue on the next column or page: find all choices before making your selection.

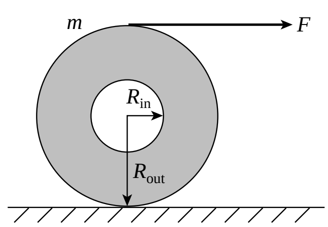
We are back in order (let's hope). Some solutions may be found on the class home page.

Rolling of a Cylinder

11:01, trigonometry, numeric, > 1 min.

004

An $m = 3.34$ kg hollow cylinder with $R_{in} = 0.3$ m and $R_{out} = 0.5$ m is pulled by a horizontal string with a force $F = 22.8$ N, as shown in the diagram.



What must the magnitude of the force of friction be if the cylinder is to roll without slipping?

Correct answer: 4.34286 N.

Explanation:

Suppose the cylinder has a length of l. Then the density of the cylinder is $\rho = \dfrac{m}{\pi(R_{out}^2 - R_{in}^2)l}$. The moment of inertia about the center is:

$$\begin{aligned} I_{cm} &= \int \rho r^2 \, dr \\ &= \frac{m}{\pi(R_{out}^2 - R_{in}^2)l} \int_{R_{in}}^{R_{out}} r^2 \, r \, dr \\ &\quad \times \int_0^{2\pi} d\theta \int_0^l dz \\ &= \frac{m}{\pi(R_{out}^2 - R_{in}^2)l} \frac{R_{out}^4 - R_{in}^4}{4} (2\pi)(l) \\ &= \frac{m}{2}(R_{out}^2 + R_{in}^2) \end{aligned}$$

By the parallel axis theorem, the moment of inertia about the ground is:

$$\begin{aligned} I &= I_{cm} + mR_{out}^2 \\ &= \frac{m}{2}(3R_{out}^2 + R_{in}^2) \end{aligned}$$

From the force equation, we have:

$$F + f = ma.$$

From the torque equation, we have:

$$F(2R_{out}) = I\alpha = I\frac{a}{R_{out}}.$$

Solving this pair of equations, we get:

$$\begin{aligned} f &= F\left(\frac{2mR_{out}^2}{I} - 1\right) \\ &= 22.8 \text{ N}\left(\frac{2(3.34 \text{ kg})(0.5 \text{ m})^2}{1.4028 \text{ kgm}^2} - 1\right) \\ &= 4.34286 \text{ N} \end{aligned}$$

005

In what direction is the frictional force?

1. To the left

2. To the right **correct**

3. Force is zero

Explanation:

The frictional force calculated has the same sign as the applied force, so it must be in the same direction as F, i.e. to the right.

006

What is the acceleration of the cylinder's center of mass?
Correct answer: 8.1266 $\mathrm{m/s^2}$.
Explanation:

From the pair of equations above, we can solve for a:

$$\begin{aligned} a &= \frac{2 F R_{out}^2}{I} \\ &= \frac{2(22.8\ \mathrm{N})(0.5\ \mathrm{m})^2}{1.4028\ \mathrm{kgm^2}} . \\ &= 8.1266\ \mathrm{m/s^2} \end{aligned}$$

Algorithm

$$r{=}0.3\ \mathrm{m}\left\{{0.1 \atop 0.3}\right\} \quad (1)$$
$$R{=}0.5\ \mathrm{m}\left\{{0.4 \atop 0.6}\right\} \quad (2)$$
$$m{=}3.34\ \mathrm{kg}\left\{{2 \atop 5}\right\} \quad (3)$$
$$F{=}22.8\ \mathrm{N}\left\{{20 \atop 50}\right\} \quad (4)$$
$$\begin{aligned} I&{=}0.5\ m{\times}\left(3.0\ R^{2.0}{+}r^{2.0}\right) \quad (5) \\ &{=}0.5\ \langle 3.34\rangle{\times}\left(3.0\ \langle 0.5\rangle^{2.0}{+}\langle 0.3\rangle^{2.0}\right) \\ &{=}1.4028\ \mathrm{kgm^2} \end{aligned}$$
$$\langle \mathrm{kgm^2}\rangle{=}\langle\rangle\ \langle \mathrm{kg}\rangle{\times}\left(\langle\rangle\ \langle \mathrm{m}\rangle^{2.0}{+}\langle \mathrm{m}\rangle^{2.0}\right) \quad \text{units}$$
$$\begin{aligned} a&{=}\frac{2.0\ F\ R^{2.0}}{I} \quad (6) \\ &{=}\frac{2.0\ \langle 22.8\rangle\ \langle 0.5\rangle^{2.0}}{\langle 1.4028\rangle} \\ &{=}8.1266\ \mathrm{m/s^2} \end{aligned}$$
$$\langle \mathrm{m/s^2}\rangle{=}\frac{\langle\rangle\ \langle \mathrm{N}\rangle\ \langle \mathrm{m}\rangle^{2.0}}{\langle \mathrm{kgm^2}\rangle} \quad \text{units}$$
$$\begin{aligned} f&{=}F\left(\frac{2.0\ m\ R^{2.0}}{I}{-}1\right) \quad (7) \\ &{=}\langle 22.8\rangle\left(\frac{2.0\ \langle 3.34\rangle\ \langle 0.5\rangle^{2.0}}{\langle 1.4028\rangle}{-}1\right) \\ &{=}4.34286\ \mathrm{N} \end{aligned}$$
$$\langle \mathrm{N}\rangle{=}\langle \mathrm{N}\rangle\left(\frac{\langle\rangle\ \langle \mathrm{kg}\rangle\ \langle \mathrm{m}\rangle^{2.0}}{\langle \mathrm{kgm^2}\rangle}{-}\langle\rangle\right) \quad \text{units}$$

Contacting a Surface

11:01, advanced, numeric, > 1 min.

008

Determine the distance the disk travels before pure rolling occurs.
Correct answer: 1.11296 m.
Explanation:

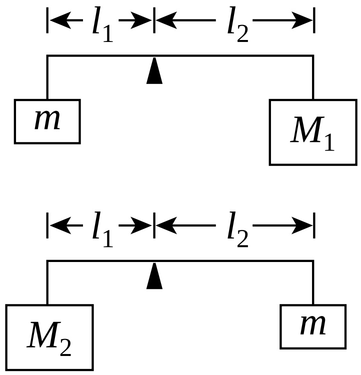
From the perspective of the ground and noting that the acceleration is constant, the distance traveled by the disk before the pure rolling occurs is,

$$\begin{aligned} s &= \frac{1}{2}\,a\,t^2 \\ &= \frac{1}{2}\,\mu\,g\left(\frac{R\,\omega_0}{3\ \mu\,g}\right)^2 \\ &= \frac{R^2\,\omega_0^2}{18\ \mu\,g} \\ &= \frac{(0.187\ \mathrm{m})^2\,(19\ \mathrm{rad/s})^2}{18\,(0.0643)\,(9.8\ \mathrm{m/s^2})} \\ &= 1.11296\ \mathrm{m}\,. \end{aligned}$$

Algorithm

$$\left\langle \tfrac{\mathrm{m}}{\mathrm{cm}}\right\rangle = 0.01\ \mathrm{m/cm} \quad (1)$$
$$g = 9.8\ \mathrm{m/s^2} \quad (2)$$
$$r = 18.7\ \mathrm{cm}\ \left\{{10 \atop 30}\right\} \quad (3)$$
$$\omega_0 = 19\ \mathrm{rad/s}\ \left\{{10 \atop 20}\right\} \quad (4)$$
$$\mu = 0.0643\ \ \left\{{0.05 \atop 0.2}\right\} \quad (5)$$
$$\begin{aligned} R &= r\ \left\langle \tfrac{\mathrm{m}}{\mathrm{cm}}\right\rangle \quad (6) \\ &= \langle 18.7\rangle\ \langle 0.01\rangle \\ &= 0.187\ \mathrm{m} \end{aligned}$$
$$\langle \mathrm{m}\rangle = \langle \mathrm{cm}\rangle\ \langle \mathrm{m/cm}\rangle \quad \text{units}$$
$$\begin{aligned} t &= \frac{R\,\omega_0}{3.0\,\mu\,g} \quad (7) \\ &= \frac{\langle 0.187\rangle\ \langle 19\rangle}{3.0\,\langle 0.0643\rangle\ \langle 9.8\rangle} \\ &= 1.87948\ \mathrm{s} \end{aligned}$$
$$\langle \mathrm{s}\rangle = \frac{\langle \mathrm{m}\rangle\ \langle \mathrm{rad/s}\rangle}{\langle\rangle\ \langle\rangle\ \langle \mathrm{m/s^2}\rangle} \quad \text{units}$$
$$\begin{aligned} s &= \frac{R^{2.0}\,\omega_0^{2.0}}{18.0\,\mu\,g} \quad (8) \\ &= \frac{\langle 0.187\rangle^{2.0}\ \langle 19\rangle^{2.0}}{18.0\,\langle 0.0643\rangle\ \langle 9.8\rangle} \\ &= 1.11296\ \mathrm{m} \end{aligned}$$
$$\langle \mathrm{m}\rangle = \frac{\langle \mathrm{m}\rangle^{2.0}\ \langle \mathrm{rad/s}\rangle^{2.0}}{\langle\rangle\ \langle\rangle\ \langle \mathrm{m/s^2}\rangle} \quad \text{units}$$

009



A rod of negligible mass is pivoted at a point that is off-center, so that length ℓ_1 is diferent from length ℓ_2. the figures above show two cases in which masses are suspended from the ends of the rod. In each case the unknown mass m is balanced by a known mass, M_1 or M_2, so that the rod remains horizontal. What is the value of m in terms of the known masses?

1. $M_1 + M_2$

2. $\dfrac{M_1 + M_2}{2}$

3. $M_1 M_2$

4. $\sqrt{M_1 M_2}$ **correct**

5. $\dfrac{M_1 M_2}{2}$

Explanation:

The balance in the first case requires $m\ell_1 = M_1\ell_2$. And the balance in the second case requires $M_2\ell_1 = m\ell_2$. Cancel ℓ_1 and ℓ_2 from the above equations. So $m^2 = M_1 M_2$, i.e. $m = \sqrt{M_1 M_2}$.

010

A 2 kg object moves in a circle of radius 4 m at a constant speed of 3 m/s. A net force of 4.5 N acts on the object. What is the angular momentum of the object with respect to an axis perpendicular to the circle and through its center?

1. $24 \dfrac{\text{kg}\cdot\text{m}^2}{\text{s}}$ **correct**

2. $12 \dfrac{\text{m}^2}{\text{s}}$

3. $13.5 \dfrac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$

4. $18 \dfrac{\text{N}\cdot\text{m}}{\text{kg}}$

5. $9 \dfrac{\text{N}\cdot\text{m}}{\text{kg}}$

Explanation:

The angular momentum is

$$\begin{aligned} L &= m v r \\ &= (2\,\text{kg})(3\,\text{m/s})(4\,\text{m}) \\ &= 24 \frac{\text{kg}\cdot\text{m}^2}{\text{s}}. \end{aligned}$$

Mass on Solid Cylinder

11:01, calculus, numeric, > 1 min.

011

A 7.9 kg mass is attached to a light cord, which is wound around a pulley. The pulley is a uniform solid cylinder of radius 11.6 cm and mass 1.89 kg. What is the resultant net torque **on the system** about the center of the wheel?

Correct answer: 8.98072 $\text{kg m}^2/\text{s}^2$.

Explanation:

The net torque *on the system* is the torque by the external force, which is the weight of the mass. So it is given by:

$$\begin{aligned} \tau &= r\,F\,\sin\phi = r\,m\,g\,\sin 90^\circ = r\,m\,g \\ &= 0.116\text{ m}\cdot 7.9\text{ kg}\cdot 9.8\text{ m/s}^2 \\ &= 8.98072\text{ kg m}^2/\text{s}^2. \end{aligned}$$

012

When the falling mass has a speed of 5.47 m/s, the pulley has an angular velocity of v/r. Determine the total angular momentum of the system about the center of the wheel.

Correct answer: 5.61233 $\text{kg m}^2/\text{s}$.

Explanation:

The total angular momentum has two parts, one of the pulley and one of the mass.

So it is

$$
\begin{aligned}
|\vec{\mathbf{L}}| &= |\vec{\mathbf{r}} \times m\,\vec{\mathbf{v}} + I\,\vec{\omega}| \\
&= r\,m\,v + \frac{1}{2}\,M\,r^2\,\left(\frac{v}{r}\right) = r\,\left(m + \frac{M}{2}\right)\,v \\
&= 11.6\text{ cm}\,\left(7.9\text{ kg} + \frac{1.89\text{ kg}}{2}\right)\,v = 1.02602\text{ kg m}\,v \\
&= 1.02602\text{ kg m} \cdot 5.47\text{ m/s} = 5.61233\text{ kg m}^2\text{/s}\,.
\end{aligned}
$$

Mass on Solid Cylinder

11:02, calculus, multiple choice, < 1 min.

013

Using the fact that $\tau = d\mathbf{L}/dt$ and your result from the previous part, calculate the acceleration of the falling mass.

Correct answer: $8.75297\ \text{m/s}^2$.

Explanation:

Use the torque-angular momentum relation, we have

$$\tau = \frac{dL}{dt} = \frac{d}{dt}(1.02602\text{ kg m}\,v) = 1.02602\text{ kg m}\,a\,,$$

Solving for acceleration:

$$
\begin{aligned}
a &= \frac{\tau}{1.02602\text{ kg m}} \\
&= \frac{8.98072\text{ kg m}^2/\text{s}^2}{1.02602\text{ kg m}} \\
&= 8.75297\text{ m/s}^2.
\end{aligned}
$$

Algorithm

$$
\begin{aligned}
\langle{\scriptstyle \frac{\text{m}}{\text{cm}}}\rangle &= 0.01\text{ m/cm} && (1)\\
m &= 7.9\text{ kg }\left\{{4 \atop 8}\right\} && (2)\\
r &= 11.6\text{ cm }\left\{{8 \atop 15}\right\} && (3)\\
r_u &= r\langle{\scriptstyle \frac{\text{m}}{\text{cm}}}\rangle && (4)\\
&= \langle 11.6\rangle\langle 0.01\rangle \\
&= 0.116\text{ m} \\
\langle\text{m}\rangle &= \langle\text{cm}\rangle\langle\text{m/cm}\rangle && \text{units}\\
M &= 1.89\text{ kg }\left\{{1 \atop 3}\right\} && (5)\\
g &= 9.8\text{ m/s}^2 && (6)\\
\tau &= r_u m g && (7)\\
&= \langle 0.116\rangle\langle 7.9\rangle\langle 9.8\rangle \\
&= 8.98072\text{ kg m}^2/\text{s}^2 \\
\langle\text{kg m}^2/\text{s}^2\rangle &= \langle\text{m}\rangle\langle\text{kg}\rangle\langle\text{m/s}^2\rangle && \text{units}\\
v &= 5.47\text{ m/s }\left\{{5 \atop 10}\right\} && (8)\\
I &= \frac{1.0 M r_u^{2.0}}{2.0} && (9)\\
&= \frac{1.0\langle 1.89\rangle\langle 0.116\rangle^{2.0}}{2.0} \\
&= 0.0127159\text{ kg m}^2 \\
\langle\text{kg m}^2\rangle &= \frac{\langle\rangle\langle\text{kg}\rangle\langle\text{m}\rangle^{2.0}}{\langle\rangle} && \text{units}\\
\omega &= \frac{v}{r_u} && (10)\\
&= \frac{\langle 5.47\rangle}{\langle 0.116\rangle} \\
&= 47.1552\text{ s}^{-1} \\
\langle\text{s}^{-1}\rangle &= \frac{\langle\text{m/s}\rangle}{\langle\text{m}\rangle} && \text{units}\\
L_1 &= r_u m v && (11)\\
&= \langle 0.116\rangle\langle 7.9\rangle\langle 5.47\rangle \\
&= 5.01271\text{ kg m}^2/\text{s} \\
\langle\text{kg m}^2/\text{s}\rangle &= \langle\text{m}\rangle\langle\text{kg}\rangle\langle\text{m/s}\rangle && \text{units}\\
L_2 &= I\omega && (12)\\
&= \langle 0.0127159\rangle\langle 47.1552\rangle \\
&= 0.599621\text{ kg m}^2/\text{s} \\
\langle\text{kg m}^2/\text{s}\rangle &= \langle\text{kg m}^2\rangle\langle\text{s}^{-1}\rangle && \text{units}\\
L &= L_1 + L_2 && (13)\\
&= \langle 5.01271\rangle + \langle 0.599621\rangle \\
&= 5.61233\text{ kg m}^2/\text{s} \\
\langle\text{kg m}^2/\text{s}\rangle &= \langle\text{kg m}^2/\text{s}\rangle + \langle\text{kg m}^2/\text{s}\rangle && \text{units}\\
b &= \frac{L}{v} && (14)\\
&= \frac{\langle 5.61233\rangle}{\langle 5.47\rangle} \\
&= 1.02602\text{ kg m} \\
\langle\text{kg m}\rangle &= \frac{\langle\text{kg m}^2/\text{s}\rangle}{\langle\text{m/s}\rangle} && \text{units}\\
a &= \frac{\tau v}{L} && (15)\\
&= \frac{\langle 8.98072\rangle\langle 5.47\rangle}{\langle 5.61233\rangle} \\
&= 8.75297\text{ m/s}^2 \\
\langle\text{m/s}^2\rangle &= \frac{\langle\text{kg m}^2/\text{s}^2\rangle\langle\text{m/s}\rangle}{\langle\text{kg m}^2/\text{s}\rangle} && \text{units}
\end{aligned}
$$

Child on a MerryGoRound

11:03, trigonometry, multiple choice, < 1 min.

014

A playground merry-go-round of radius 2.5 m has a moment of inertia 200 $\mathrm{kg\,m^2}$ and is rotating at 9.5 rev/min. A child with mass 21 kg jumps on the edge of the merry-go-round.

What is the new moment of inertia of the merry-go-round and child, together?

Correct answer: 331.25 $\mathrm{kg\,m^2}$.

Explanation:

The moment of inertia will be the combination of the individual moments of inertia of each component.

$$I_{merry-go-round} + I_{child} = I_{total}$$

Child on a MerryGoRound

11:03, calculus, numeric, > 1 min.

015

Assuming that the boy's initial speed is negligible, what is the new angular speed of the merry-go-round?

Correct answer: 5.73585 rev/min.

Explanation:

Basic Concepts:

$$\sum \vec{L} = const$$

The net angular momentum of the system remains constant, therefore, from conservation of the angular momentum we have:

$$I_1\omega_1 = (I_1 + m\,R^2)\,\omega_2$$

And

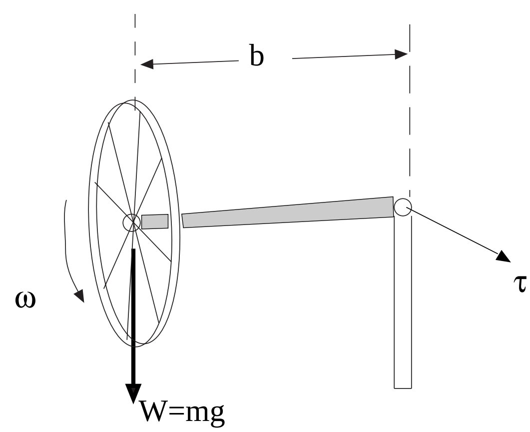
$$\begin{aligned}
\omega_2 &= \omega_1 \frac{I_1}{I_1 + m\,R^2} \\
&= \frac{(9.5 \text{ rev/min}) \times (200 \text{ kg m}^2)}{(200 \text{ kg m}^2) + (21 \text{ kg})\,(2.5 \text{ m})^2} \\
&= 5.73585 \text{ rev/min}
\end{aligned}$$

Algorithm

$$R = 2.5 \text{ m} \left\{{1.5 \atop 2.5}\right\} \quad (1)$$
$$I_1 = 200 \text{ kg m}^2 \left\{{100 \atop 300}\right\} \quad (2)$$
$$\omega_1 = 9.5 \text{ rev/min} \left\{{5 \atop 15}\right\} \quad (3)$$
$$m = 21 \text{ kg} \left\{{20 \atop 35}\right\} \quad (4)$$
$$I_2 = I_1 + m\,R^{2.0} \quad (5)$$
$$= \langle 200\rangle + \langle 21\rangle\,\langle 2.5\rangle^{2.0}$$
$$= 331.25 \text{ kg m}^2$$
$$\langle \text{kg m}^2\rangle = \langle \text{kg m}^2\rangle + \langle \text{kg}\rangle\,\langle \text{m}\rangle^{2.0} \text{ units}$$
$$\omega_2 = \frac{\omega_1\,I_1}{I_2} \quad (6)$$
$$= \frac{\langle 9.5\rangle\,\langle 200\rangle}{\langle 331.25\rangle}$$
$$= 5.73585 \text{ rev/min}$$
$$\langle \text{rev/min}\rangle = \frac{\langle \text{rev/min}\rangle\,\langle \text{kg m}^2\rangle}{\langle \text{kg m}^2\rangle} \quad \text{units}$$

018

A bicycle wheel of mass m rotating at an angular velocity ω has its shaft supported on one side, as shown in the figure. When viewing from the left, one sees that the wheel is rotating in a counterclockwise manner. The distance from the center of the wheel to the pivot point is b. We assume the wheel is a hoop of radius R, and the shaft is horizontal.



The magnitude of the angular momentum of the wheel is given by

1. $\frac{1}{4} m\,R^2\,\omega$

2. $m\,R^2\,\omega^2$

3. $\frac{1}{2} m\,R^2\,\omega^2$

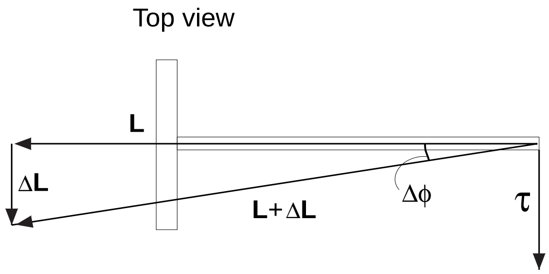
4. $\frac{1}{4} m\,R^2\,\omega^2$

5. $\frac{1}{2} m\, R^2\, \omega$

6. $m\, R^2\, \omega$ **correct**

Explanation:
Solution: Basic Concepts:

$$\vec{\tau} = \frac{d\vec{\mathbf{L}}}{dt}$$



The magnitude of the angular momentum of the wheel, L, is

$$L = I\,\omega = m\,R^2\,\omega,$$

since the moment of inertia of the wheel, I, is $m\,R^2$.

019

Given: the mass 3 kg, the angular velocity 15 rad/s, the axil length $b = 0.5$ m, and the radius of the wheel $R = 0.49$ m. Find the precession angle in the time interval $t = 1.1$ s.
Correct answer: 85.7488 °.

Explanation:

From the figure below, we get $\Delta\phi = \frac{\Delta L}{L}$. Using the relation, $\Delta L = \tau\,\Delta t$, where τ is the magnitude of the torque, $mg \cdot b$, we get

$$\begin{aligned}
\Delta\phi &= \frac{\Delta L}{L} \\
&= \frac{\tau\,\Delta t}{L} \\
&= \frac{m\,g\,b\,\Delta t}{m\,R^2\,\omega} \\
&= \frac{g\,b\,\Delta t}{R^2\,\omega} \\
&= \frac{(9.8\ \text{m/s}^2)(0.5\ \text{m})(1.1\ \text{s})}{(0.49\ \text{m})^2(15\ \text{rad/s})} \\
&= 1.4966\ \text{rad} \\
&= 85.7488^\circ.
\end{aligned}$$

Precession

11:05, calculus, numeric, > 1 min.

020

The direction of precession as viewed from the top is:

1. along the direction of rotation of the wheel

2. counterclockwise **correct**

3. clockwise

4. opposite to the direction of rotation of the wheel

Explanation:

From the figure, we can see the direction of precession is counterclockwise.

Algorithm

$$\begin{aligned}
\langle\tfrac{\text{deg}}{\text{rad}}\rangle &= 57.2958\ \text{deg/rad} && (1) \\
g &= 9.8\ \text{m/s}^2 && (2) \\
m &= 3.0\ \text{kg} && (3) \\
\omega &= 15\ \text{rad/s}\ \left\{{10 \atop 15}\right\} && (4) \\
b &= 0.5\ \text{m}\ \left\{{0.4 \atop 0.6}\right\} && (5) \\
R &= 0.49\ \text{m}\ \left\{{0.4 \atop 0.6}\right\} && (6) \\
t &= 1.1\ \text{s}\ \left\{{1 \atop 2}\right\} && (7) \\
\Phi &= \frac{gbt}{R^{2.0}\omega} && (8) \\
&= \frac{\langle 9.8\rangle\langle 0.5\rangle\langle 1.1\rangle}{\langle 0.49\rangle^{2.0}\langle 15\rangle} \\
&= 1.4966\ \text{rad} \\
\langle\text{rad}\rangle &= \frac{\langle\text{m/s}^2\rangle\langle\text{m}\rangle\langle\text{s}\rangle}{\langle\text{m}\rangle^{2.0}\langle\text{rad/s}\rangle} && \text{units} \\
\Phi_{deg} &= \Phi\langle\tfrac{\text{deg}}{\text{rad}}\rangle && (9) \\
&= \langle 1.4966\rangle\langle 57.2958\rangle \\
&= 85.7488^\circ \\
\langle^\circ\rangle &= \langle\text{rad}\rangle\langle\text{deg/rad}\rangle && \text{units}
\end{aligned}$$