This print-out should have 8 questions. Check that it is complete before leaving the printer. Also, multiple-choice questions may continue on the next column or page: find all choices before making your selection.

This HW 7 should be done *Before* assignment HW 6. Yes, this means they are out of order. However, since HW 7 is short and HW 6 is long, you are advised to start HW 6 early.

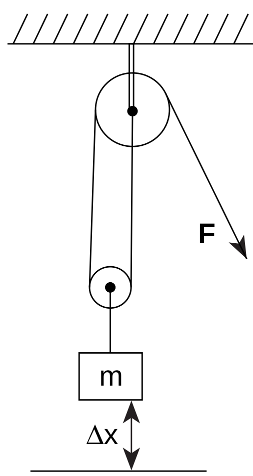
Block and Tackle

08:02, trigonometry, numeric, > 1 min.

004

A weight of 888 N is raised by a two-pulley arrangement as shown in the figure. Assume that the pulleys are weightless, the rope does not stretch, and the system moves at a constant speed which is slow enough that the kinetic energy is negligible.

How much work is done by the agent (force **F**) to raise the weight by a vertical distance of 22.3 m?



Correct answer: 19802.4 J.

Explanation:

Optional questions (not graded, but deal with concepts that can be tested in quiz problems)

(a) What is the magnitude of force F? (b) What is the distance that the force F move?

Basic Concepts:

$W = \mathbf{F} \cdot \mathbf{s}$

$U_{\text{grav.}} = mgh$

$\mathbf{F}_{net} = \sum_i \mathbf{F}_i$

Solution :

The work done by the force F results in the change of potential energy of the system. The final energy is the potential energy $PE_{final} = mg\Delta x$; the initial energy is zero. Thus,

$$\begin{aligned} \Delta E &= mg\Delta x \\ &= (888 \text{ N})(22.3 \text{ m}) \\ &= 19802.4 \text{ J} \quad . \end{aligned}$$

005

Which of the following is the SI unit of the force?

1. kg m/s^2 **correct**

2. W

3. kg

4. m/s^2

5. Nm

6. kg m/s

7. m/kg s

8. J

9. J s

10. N/s

Explanation:

The SI unit of force is kg m/s^2. This combination of units is called a Newton (N).

Block and Tackle

08:02, trigonometry, numeric, > 1 min.

006

The change in potential energy of the weight is ____________ .

1. $2mg\Delta x$

2. $mg\Delta x$ **correct**

3. $F\Delta x$

4. $\frac{1}{2}(2F - mg)\Delta x$

5. $\frac{1}{2}(F - mg)\Delta x$

6. $(F - 2mg)\Delta x$

7. $\frac{3}{2}F\Delta x$

8. $(F - mg)\Delta x$

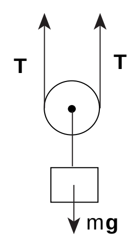
9. $2(F - mg)\Delta x$

10. $2(F + mg)\Delta x$

Explanation:

See Part 1. **Optional problems**

(a) If we look at the forces on the pulley we have



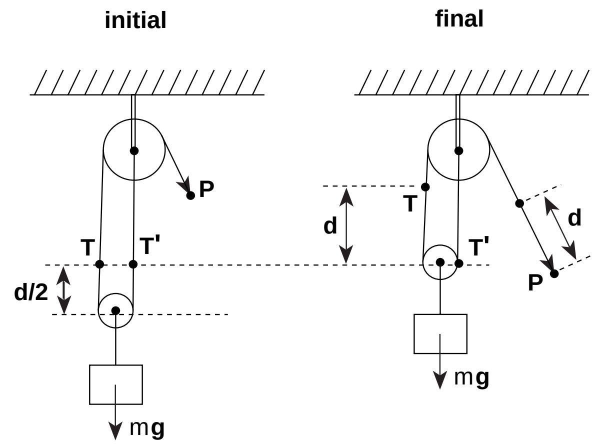
As the weight is lifted with no acceleration we have, by force balance, $2T = mg$ or

$$T = F = \frac{mg}{2}$$

(b) If d is the distance the force F acts, then the work done is $W = Fd = \frac{mg}{2}d$. Using the result of Part 1, $W = mg\Delta x = \frac{mg}{2}d$ and so

$$d = 2\Delta x$$

Let us examine the geometry of the pulley in order to convince ourselves that this results is the correct answer, namely that if the mass is lifted a distance $d/2$ then the force F acts over a distance d.



Note that the length of the cord between points T and T′ is d, and initially both points are a distance $d/2$ above the moving assembly of the pulley, which is of negligible size. When the cord moves a distance d, the point T moves a distance d above the its initial position and the point T′ stays where it is (it is of course still a distance d from T); but now the assembly has reached the point T′, which means the assembly has moved up a distance $d/2$.

Algorithm

$$b = 888 \text{ N} \left\{ {200 \atop 900} \right\} \quad (1)$$

$$a = 22.3 \text{ m} \left\{ {5 \atop 25} \right\} \quad (2)$$

$$\begin{aligned} d &= 2.0\ a \quad (3) \\ &= 2.0\ \langle 22.3 \rangle \\ &= 44.6 \text{ m} \\ \langle \text{m} \rangle &= \langle\rangle\ \langle \text{m} \rangle \quad \text{units} \end{aligned}$$

$$\begin{aligned} F &= \frac{b}{2.0} \quad (4) \\ &= \frac{\langle 888 \rangle}{2.0} \\ &= 444 \text{ N} \\ \langle \text{N} \rangle &= \frac{\langle \text{N} \rangle}{\langle\rangle} \quad \text{units} \end{aligned}$$

$$\begin{aligned} W &= F\ d \quad (5) \\ &= \langle 444 \rangle\ \langle 44.6 \rangle \\ &= 19802.4 \text{ J} \\ \langle \text{J} \rangle &= \langle \text{N} \rangle\ \langle \text{m} \rangle \quad \text{units} \end{aligned}$$

AP M 1993 MC 18

08:02, calculus, numeric, > 1 min.

007

When an object is moved from rest at point A to rest at point B in a gravitational field, the net work done by the field depends on the mass of the object and

1. the positions of A and B only. **correct**

2. the path taken between A and B only.

3. both the positions of A and B and the path taken between them.

4. the velocity of the object as it moves between A and$B$.

5. the nature of the external force moving the object from A to B.

Explanation:

Gravitational force is a conservative force, so in addition to the mass, only the positions are needed.

008

A block initially at rest is allowed to slide down a frictionless ramp and attains a speed v at the bottom. To achieve a speed $2v$ at the bottom, how many times as high must a new ramp be?

1. 4 **correct**

2. 2

3. 3

4. 1

5. 5

6. 6

Explanation:

The gain in kinetic energy, proportional to the square of the block's speed at the bottom of the ramp, is equal to the loss in potential energy. This, in turn, is proportional to the height of the ramp.

009

Consider a compact car that is being driven at 99.6 km/h. From what height would the car have to be dropped to have the same kinetic energy?

Correct answer: 39.0533 m.

Explanation:

Assume the car is dropped from the height h. By conservation of energy,

$$KE_o + PE_o = KE_f + PE_f$$

$$PE_o = KE_f$$

Thus to attain the same kinetic energy as a car of the same mass driven at a speed of v,

$$mgh = \frac{1}{2} m v^2$$

$$h = \frac{v^2}{2g}$$

The velocity must be in m/s.

Algorithm

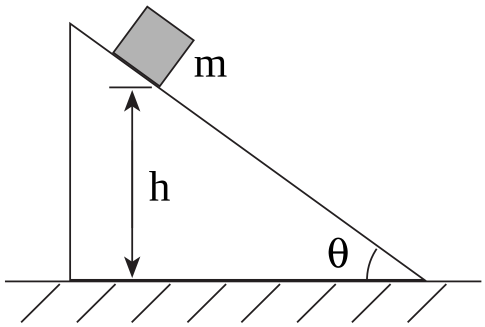
$$\begin{aligned}
\langle \tfrac{\mathrm{m}}{\mathrm{km}} \rangle &= 1000 \text{ m/km} && (1)\\
\langle \tfrac{\mathrm{h}}{\mathrm{s}} \rangle &= 0.000277778 \text{ h/s} && (2)\\
g &= 9.8 \text{ m/s}^2 && (3)\\
v &= 99.6 \text{ km/h} \left\{ {80 \atop 120} \right\} && (4)\\
v_{mps} &= v \langle \tfrac{\mathrm{m}}{\mathrm{km}} \rangle \langle \tfrac{\mathrm{h}}{\mathrm{s}} \rangle && (5)\\
&= \langle 99.6 \rangle \langle 1000 \rangle \langle 0.000277778 \rangle \\
&= 27.6667 \text{ m/s} \\
\langle \text{m/s} \rangle &= \langle \text{km/h} \rangle \langle \text{m/km} \rangle \langle \text{h/s} \rangle && \text{units}\\
h &= \frac{v_{mps}^{2.0}}{2.0 g} && (6)\\
&= \frac{\langle 27.6667 \rangle^{2.0}}{2.0 \langle 9.8 \rangle} \\
&= 39.0533 \text{ m} \\
\langle \text{m} \rangle &= \frac{\langle \text{m/s} \rangle^{2.0}}{\langle \rangle \langle \text{m/s}^2 \rangle} && \text{units}
\end{aligned}$$

Sliding Down a Plane

08:02, calculus, multiple choice, < 1 min.

012

A 3.51 kg block starts at a height of 51.3 cm on a plane that has an inclination angle of 51.1° as in figure.



Upon reaching the bottom, the block slides along a horizontal surface. The coefficient of friction on both surfaces is $\mu = 0.14$

How far does the block slide on the horizontal surface before coming to rest?

Correct answer: 3.25035 m.

Explanation:

From the conservation of energy for the part of the motion on the inclined plane

$$\frac{1}{2}\, m\, v_{end}^2 = m\, g\, h - W$$

where the work done is

$$\begin{aligned} W &= \int_0^{h/\sin\theta} (-f)\, d\,x \\ &= \mu\, m\, g \cos\theta (\frac{h}{\sin\theta}) \end{aligned}$$

From the conservation of energy on the horizontal plane:

$$m\, g\, h - \mu\, m\, g\, h \cot\theta = \mu\, m\, g\, x$$

Note that the mass cancels out, therefore

$$\begin{aligned} x &= h(1 - \mu \cot(\theta))/\mu \\ &= \frac{(0.01 \text{ m/cm})(51.3 \text{ cm})(1 - (0.14)\cot(51.1^\circ))}{0.14} \\ &= 3.25035 \text{ m} \end{aligned}$$

Algorithm

$$\langle \tfrac{\text{m}}{\text{cm}} \rangle = 0.01 \text{ m/cm} \quad (1)$$

$$h = 51.3 \text{ cm } \left\{ {24 \atop 52} \right\} \quad (2)$$

$$m = 3.51 \text{ kg } \left\{ {3.1 \atop 3.9} \right\} \quad (3)$$

$$\theta = 51.1^\circ \left\{ {48 \atop 55} \right\} \quad (4)$$

$$\mu = 0.14 \left\{ {0.12 \atop 0.32} \right\} \quad (5)$$

$$x = \frac{\left(1.0 - \frac{\mu}{\tan\left(\frac{\theta\,\pi}{180.0}\right)}\right) \langle \tfrac{\text{m}}{\text{cm}} \rangle\, h}{\mu} \quad (6)$$

$$= \frac{\left(1.0 - \frac{\langle 0.14 \rangle}{\tan\left(\frac{\langle 51.1 \rangle \langle 3.1415926 \rangle}{180.0}\right)}\right) \langle 0.01 \rangle \langle 51.3 \rangle}{\langle 0.14 \rangle}$$

$$= 3.25035 \text{ m}$$

$$\langle \text{m} \rangle = \frac{\left(\langle\rangle - \frac{\langle\rangle}{\tan\left(\frac{\langle^\circ\rangle \langle\rangle}{\langle\rangle}\right)}\right) \langle \text{m/cm} \rangle \langle \text{cm} \rangle}{\langle\rangle} \quad \text{units}$$

Rising Elevator

08:04, arithmetic, multiple choice, < 1 min.

013

An elevator is rising at constant speed. Consider the following statements: I. The upward cable force is constant. II. The kinetic energy of the elevator is constant. III. The gravitational potential energy of the earth-elevator system is constant. IV. The acceleration of the elevator is zero. V. The mechanical energy of the earth-elevator system is constant. Which of the statements are true?

1. all five are true

2. only II and V are true

3. only I, II, and IV are true **correct**

4. only I, II, and III are true

5. only IV and V are true

Explanation:

Basic Concepts: Potential Energy, Kinetic Energy

[redacted]e will consider these statements one at a [redacted]ime. **I.** Since the elevator is moving at con[redacted]tant speed, the net force on the elevator [redacted]ust be zero. The tension in the cable must be equal and opposite to the weight of the elevator, and since the weight is constant, the tension in the cable must also be constant. **II.** The kinetic energy depends only on the mass of the elevator and its velocity ($KE = \frac{1}{2}mv^2$). Since the mass and velocity are both constant, the kinetic energy is also constant. **III.** The gravitational potential energy of the earth-elevator system is increasing, because the distance between the elevator and the earth is increasing. (The potential energy depends only on the mass of the earth, the mass of the elevator, and the distance between them.) **IV.** Since the elevator is rising at constant speed, its acceleration is zero. **V.** The mechanical energy of the earth-elevator system is not constant, because the potential energy is increasing (see explanation of III) while the kinetic energy is constant. Since the mechanical energy is the sum of the kinetic and potential energies, the mechanical energy is increasing.

Thus only I, II, and IV are true.