

This print-out should have 24 questions. Check that it is complete before leaving the printer. Also, multiple-choice questions may continue on the next column or page: find all choices before making your selection.

Note, only a few (usually 4) of the problems will have their scores kept for a grade. You may make multiple tries to get a problem right, although it's worth less each time. Worked solutions to a number of these problems (even some of the scored ones) may be found on the Ph211 home page.

AP M 1993 MC 16

09:01, calculus, multiple choice, < 1 min.

001

A balloon of mass M is floating motionless in the air. A person of mass less than M is on a rope ladder hanging from the balloon. The person begins to climb the ladder at a uniform speed v relative to the ground.

How does the balloon move relative to the ground?

1. Up with speed v
2. Up with a speed less than v
3. Down with a speed less than v **correct**
4. Down with speed v
5. The balloon does not move.

Explanation:

Let the mass of the person be m .

Total momentum is conserved (because the exterior forces on the system are balanced), especially the component in the vertical direction.

When the person begins to move, we have

$$\begin{aligned}
 mv + Mv_M &= 0, \\
 \implies v_M &= -\frac{m}{M} v < v \\
 \implies |v_M| &= \frac{m}{M} v < v,
 \end{aligned}$$

since $m < M \implies \frac{m}{M} < 1$.

Thus the balloon moves in the opposite direction.

Return to the Shuttle

09:01, trigonometry, numeric, > 1 min.

002

A(n) 63.1 kg astronaut becomes separated from the shuttle, while on a spacewalk. She finds herself 70.2 m away from the shuttle and moving with zero speed relative to the shuttle. She has a(n) 0.543 kg camera in her hand and decides to get back to the shuttle by throwing the camera at a speed of 12 m/s in the direction away from the shuttle.

How long will it take for her to reach the shuttle?

Correct answer: 11.3301 min.

Explanation:

Basic Concepts: Conservation of Linear Momentum Because of conservation of linear momentum, we have

$$0 = M V + m(-v)$$

$$m v = M V$$

where V is the velocity of the astronaut and it has a direction toward the shuttle.

So

$$\begin{aligned}
 V &= \frac{m v}{M} \\
 &= 0.103265 \text{ m/s}
 \end{aligned}$$

And the time it takes for her to reach the shuttle

$$\begin{aligned}
 t &= \frac{d}{V} \\
 &= \frac{70.2 \text{ m}}{0.103265 \text{ m/s}} \\
 &= 679.807 \text{ s} = 11.3301 \text{ min}
 \end{aligned}$$

Algorithm

$$\langle \frac{\text{min}}{\text{s}} \rangle = 0.0166667 \text{ min/s} \quad (1)$$

$$M = 63.1 \text{ kg} \left\{ \begin{matrix} 50 \\ 80 \end{matrix} \right\} \quad (2)$$

$$d = 70.2 \text{ m} \left\{ \begin{matrix} 25 \\ 75 \end{matrix} \right\} \quad (3)$$

$$m = 0.543 \text{ kg} \left\{ \begin{matrix} 0.5 \\ 0.95 \end{matrix} \right\} \quad (4)$$

$$v = 12.0 \text{ m/s} \quad (5)$$

$$V = \frac{m v}{M} \quad (6)$$

$$\begin{aligned}
 &= \frac{\langle 0.543 \rangle \langle 12 \rangle}{\langle 63.1 \rangle}
 \end{aligned}$$

$$\begin{aligned}
 &= 0.103265 \text{ m/s} \\
 \langle \text{m/s} \rangle &= \frac{\langle \text{kg} \rangle \langle \text{m/s} \rangle}{\langle \text{kg} \rangle} && \text{units} \\
 t &= \frac{d}{V} && (7) \\
 &= \frac{\langle 70.2 \rangle}{\langle 0.103265 \rangle} \\
 &= 679.807 \text{ s} \\
 \langle \text{s} \rangle &= \frac{\langle \text{m} \rangle}{\langle \text{m/s} \rangle} && \text{units} \\
 t_u &= t \langle \frac{\text{min}}{\text{s}} \rangle && (8) \\
 &= \langle 679.807 \rangle \langle 0.0166667 \rangle \\
 &= 11.3301 \text{ min} \\
 \langle \text{min} \rangle &= \langle \text{s} \rangle \langle \text{min/s} \rangle && \text{units}
 \end{aligned}$$

Force on a Golf Ball

09:02, calculus, numeric, > 1 min.

003

A golf ball ($m = 72.3 \text{ g}$) is struck a blow that makes an angle of 45.6° with the horizontal. The drive lands 187 m away on a flat fairway. If the golf club and ball are in contact for 3.59 ms, what is the average force of impact? (Neglect air resistance.)

Correct answer: 862.234 N.

Explanation:

Note that the range of the golf ball is given by:

$$L = \frac{v^2 \sin 2\theta}{g}$$

Then the initial velocity of the ball is:

$$v_0 = \sqrt{Lg / \sin 2\theta} = 42.8136 \text{ m/s}$$

The average force exerted is the change in its momentum over the time of contact:

$$\begin{aligned}
 F &= \frac{mv_0}{t} \\
 &= \frac{(72.3 \text{ g})(0.001 \text{ kg/g})(42.8136 \text{ m/s})}{(3.59 \text{ ms})(0.001 \text{ s/ms})} \\
 &= 862.234 \text{ N}
 \end{aligned}$$

Algorithm

$$\begin{aligned}
 \langle \frac{\text{kg}}{\text{g}} \rangle &= 0.001 \text{ kg/g} && (1) \\
 \langle \frac{\text{s}}{\text{ms}} \rangle &= 0.001 \text{ s/ms} && (2) \\
 g &= 9.8 \text{ m/s}^2 && (3)
 \end{aligned}$$

$$m = 72.3 \text{ g} \left\{ \begin{matrix} 18.4 \\ 73.6 \end{matrix} \right\} \quad (4)$$

$$\theta = 45.6^\circ \left\{ \begin{matrix} 18 \\ 72 \end{matrix} \right\} \quad (5)$$

$$L = 187 \text{ m} \left\{ \begin{matrix} 80 \\ 320 \end{matrix} \right\} \quad (6)$$

$$t = 3.59 \text{ ms} \left\{ \begin{matrix} 2.8 \\ 11.2 \end{matrix} \right\} \quad (7)$$

$$v_0 = \sqrt{\frac{Lg}{\sin \left(\frac{2.0\theta\pi}{180.0} \right)}} \quad (8)$$

$$= \sqrt{\frac{\langle 187 \rangle \langle 9.8 \rangle}{\sin \left(\frac{2.0 \langle 45.6 \rangle \langle 3.1415926 \rangle}{180.0} \right)}}$$

$$= 42.8136 \text{ m/s}$$

$$\langle \text{m/s} \rangle = \sqrt{\frac{\langle \text{m} \rangle \langle \text{m/s}^2 \rangle}{\sin \left(\frac{\langle \rangle \langle \rangle}{\langle \rangle} \right)}} \quad \text{units}$$

$$F = \frac{m \langle \frac{\text{kg}}{\text{g}} \rangle v_0}{t \langle \frac{\text{s}}{\text{ms}} \rangle} \quad (9)$$

$$= \frac{\langle 72.3 \rangle \langle 0.001 \rangle \langle 42.8136 \rangle}{\langle 3.59 \rangle \langle 0.001 \rangle}$$

$$= 862.234 \text{ N}$$

$$\langle \text{N} \rangle = \frac{\langle \text{g} \rangle \langle \text{kg/g} \rangle \langle \text{m/s} \rangle}{\langle \text{ms} \rangle \langle \text{s/ms} \rangle} \quad \text{units}$$

Bullet Moves a Block

09:03, trigonometry, numeric, > 1 min.

004

A(n) 18.7 g bullet is shot into a(n) 6310 g wooden block standing on a frictionless surface. The block, with the bullet in it, acquires a velocity of 0.869 m/s.

Calculate the velocity of the bullet before striking the block.

Correct answer: 294.098 m/s.

Explanation:

Basic concepts:

Momentum of any object is

$$p = mv$$

The collision is inelastic, and by conservation of momentum,

$$p_{\text{before}} = p_{\text{after}}$$

$$m_b v_b + 0 = (m_b + m_w) v_f$$

$$v_b = \frac{(m_b + m_w) v_f}{m_b}$$

Algorithm

$$m_b = 18.7 \text{ g} \left\{ \begin{matrix} 10 \\ 30 \end{matrix} \right\} \quad (1)$$

$$m_w = 6310 \text{ g} \left\{ \begin{matrix} 4500 \\ 6500 \end{matrix} \right\} \quad (2)$$

$$v_f = 0.869 \text{ m/s} \left\{ \begin{matrix} 0.75 \\ 2 \end{matrix} \right\} \quad (3)$$

$$v = \frac{(m_b + m_w) v_f}{m_b} \quad (4)$$

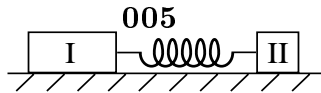
$$= \frac{(\langle 18.7 \rangle + \langle 6310 \rangle) \langle 0.869 \rangle}{\langle 18.7 \rangle}$$

$$= 294.098 \text{ m/s}$$

$$\langle \text{m/s} \rangle = \frac{(\langle \text{g} \rangle + \langle \text{g} \rangle) \langle \text{m/s} \rangle}{\langle \text{g} \rangle} \quad \text{units}$$

AP B 1993 MC 7

09:04, trigonometry, multiple choice, > 1 min.



Two pucks are attached by a stretched spring and are initially held at rest on a frictionless surface, as shown above. The pucks are then released simultaneously. If puck I has three times the mass of puck II, which of the following quantities is the same for both pucks as the spring pulls the two pucks toward each other?

1. Speed
2. Velocity
3. Acceleration
4. Magnitude of momentum **correct**
5. Kinetic energy

Explanation:

The total momentum is conserved here (no friction).

$$\begin{aligned} 0 &= \mathbf{P}_i = \mathbf{P}_f = \mathbf{P}_I + \mathbf{P}_{II} \\ \implies \mathbf{P}_I &= -\mathbf{P}_{II} \\ |\mathbf{P}_I| &= |\mathbf{P}_{II}| \end{aligned}$$

The magnitude of momentum is the same for both pucks.

Let's look at the other choices: ($m_I = 3m_{II}$)

Speed:

$$m_I v_I = P_I = P_{II} = m_{II} v_{II} \implies v_{II} = 3v_I.$$

Velocity:

$$\begin{aligned} m_I \mathbf{v}_I &= \mathbf{P}_I = -\mathbf{P}_{II} = -m_{II} \mathbf{v}_{II} \\ \implies \mathbf{v}_{II} &= -3\mathbf{v}_I. \end{aligned}$$

Acceleration:

$$\mathbf{a}_I = \frac{\mathbf{F}_I}{m_I} = \frac{-\mathbf{F}_{II}}{3m_{II}} = -\frac{1}{3} \frac{\mathbf{F}_{II}}{m_{II}} = -\frac{1}{3} \mathbf{a}_{II}.$$

Kinetic energy:

$$\begin{aligned} K_I &= \frac{1}{2} m_I v_I^2 \\ &= \frac{1}{2} (3m_{II}) \left(\frac{1}{3} v_{II} \right)^2 \\ &= \frac{1}{3} \left(\frac{1}{2} m_{II} v_{II}^2 \right) \\ &= \frac{1}{3} K_{II}. \end{aligned}$$

Collision With the Earth

09:04, calculus, numeric, > 1 min.

006

A 11.1 kg bowling ball initially at rest is dropped from a height of 4.32 m.

What is the speed of the Earth coming up to meet the ball just before the ball hits the ground? Use 5.98×10^{24} kg as the mass of the Earth.

Correct answer: 1.70802×10^{-23} m/s.

Explanation:

The speed of the ball just before impact is:

$$v = \sqrt{2gh} = 9.20174 \text{ m/s}$$

From conservation of momentum $\Delta p = 0$, or

$$M_e V + m_b v = 0$$

Therefore

$$\begin{aligned} |V| &= v m_b / M_e \\ &= (9.20174 \text{ m/s})(11.1 \text{ kg}) / 5.98 \times 10^{24} \text{ kg} \\ &= 1.70802 \times 10^{-23} \text{ m/s} \end{aligned}$$

Algorithm

$$g = 9.8 \text{ m/s}^2 \quad (1)$$

$$m = 11.1 \text{ kg} \left\{ \begin{matrix} 2.8 \\ 11.2 \end{matrix} \right\} \quad (2)$$

$$h = 4.32 \text{ m} \left\{ \begin{matrix} 1.2 \\ 4.8 \end{matrix} \right\} \quad (3)$$

$$M = 5.98 \times 10^{24} \text{ kg} \quad (4)$$

$$v = \sqrt{2.0 \text{ g h}} \quad (5)$$

$$= \sqrt{2.0 \langle 9.8 \rangle \langle 4.32 \rangle}$$

$$= 9.20174 \text{ m/s}$$

$$\langle \text{m/s} \rangle = \sqrt{\langle \rangle \langle \text{m/s}^2 \rangle \langle \text{m} \rangle} \quad \text{units}$$

$$V = \frac{m v}{M} \quad (6)$$

$$= \frac{\langle 11.1 \rangle \langle 9.20174 \rangle}{\langle 5.98 \times 10^{24} \rangle}$$

$$= 1.70802 \times 10^{-23} \text{ m/s}$$

$$\langle \text{m/s} \rangle = \frac{\langle \text{kg} \rangle \langle \text{m/s} \rangle}{\langle \text{kg} \rangle} \quad \text{units}$$

AP B 1993 MC 10

09:04, calculus, numeric, > 1 min.

009

Which of the following is true when an object of mass m moving on a horizontal frictionless surface hits and sticks to an object of mass $M > m$, which is initially at rest on the surface?

1. The speed of the objects that are stuck together will be less than the initial speed of the less-massive object. **correct**

2. All of the initial kinetic energy of the less-massive object is lost.

3. The momentum of the objects that are stuck together has a smaller magnitude than the initial momentum of the less-massive object.

4. The collision is elastic.

5. The direction of motion of the objects that are stuck together depends on whether the hit is a head-on collision.

Explanation:

The total momentum is conserved here (no friction).

$$m \mathbf{v}_i = \mathbf{P}_i = \mathbf{P}_f = (m + M) \mathbf{v}_f$$

$$\implies \mathbf{v}_f = \frac{m}{m + M} \mathbf{v}_i$$

$$0 < |\mathbf{v}_f| < |\mathbf{v}_i|$$

The speed of objects that are stuck together is

less than the initial speed of the less-massive object.

Let's look at the other choices:

It is obviously an inelastic collision since the two objects stick together after the collision.

$|\mathbf{v}_i| > 0 \implies K_f > 0$. Only part of the initial kinetic energy is lost.

The momentum is conserved. The final momentum has the same magnitude as the initial momentum.

$$\mathbf{v}_f = \frac{m}{m + M} \mathbf{v}_i$$

So the direction of the motion of the stuck-together is the same as the direction of the initial motion of the less-massive object. It does not depend on whether the hit is a head-on collision or not.

010

The mass of a star like our Sun is 513000 Earth masses, and the mean distance from the center of this star to the center of a planet like our Earth is 8.82×10^8 km.

Treating this planet and star as particles, with each mass concentrated at its respective geometric center, how far from the center of the star is the center of mass of the planet-star system?

Correct answer: 1719.29 km.

Explanation:

From

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$$

in this case, taking the origin to be the center of the star

$$x_{cm} = \frac{M_E L}{513000 M_E + M_E}$$

$$= \frac{L}{513000 + 1}$$

$$= \frac{(8.82 \times 10^8 \text{ km})}{(513000 + 1)}$$

$$= 1719.29 \text{ km.}$$

The center-of-mass of the planet-star system is well inside the star.

Algorithm

$$n = 513000 \left\{ \begin{matrix} 130000 \\ 520000 \end{matrix} \right\} \quad (1)$$

$$L = 8.82 \times 10^8 \text{ km} \left\{ \begin{matrix} 1.196 \times 10^8 \\ 1 \times 10^9 \end{matrix} \right\} \quad (2)$$

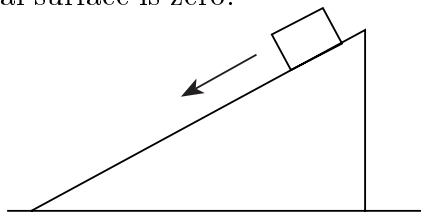
$$\begin{aligned} x_c &= \frac{L}{1.0 + n} & (3) \\ &= \frac{\langle 8.82 \times 10^8 \rangle}{1.0 + \langle 513000 \rangle} \\ &= 1719.29 \text{ km} \\ \langle \text{km} \rangle &= \frac{\langle \text{km} \rangle}{\langle \rangle + \langle \rangle} \quad \text{units} \end{aligned}$$

Block on a Wedge

09:05, trigonometry, multiple choice, > 1 min.

012

A large wedge (with a rough inclined surface and a smooth lower surface) rests on a horizontal frictionless surface. A block with a rough bottom starts from rest and slides down the inclined surface of the wedge. The coefficient for friction between the block and inclined wedge surface is $.1 < \mu_{\text{block incline}} < .5$ and that between the lower wedge surface and horizontal surface is zero.



While the block is sliding down the inclined surface, the combined center of mass of the block and wedge:

1. does not move
2. moves vertically with increasing speed **correct**
3. moves horizontally with constant speed
4. moves horizontally with increasing speed
5. moves vertically with constant speed
6. moves both horizontally and vertically with constant speed

7. moves both horizontally and vertically with increasing speed

8. cannot be determined

Explanation:

Basic Concepts: Momentum

The motion of the center of mass of a system is only affected by the external forces acting on the system, and not by any internal forces such as the friction between the block and wedge. The net external force on the system acts only in the vertical direction, so the center of mass will not move in a horizontal direction. The fact that the block starts from rest tells us that it must be accelerating. Since one component of the system (the block) has a component of acceleration in the vertical direction and the other component (the wedge) does not, we know that the center of mass must be accelerating. The net external force on this system acts only in the vertical direction. Therefore, while the block is moving, the center of mass of the block and wedge must be moving downward with increasing speed.

AP M 1993 MC 20

09:06, calculus, numeric, > 1 min.

013

A turntable that is initially at rest is set in motion with a constant angular acceleration α .

What is the angular velocity of the turntable after it has made one complete revolution?

1. $\sqrt{2\alpha}$
2. $\sqrt{2\pi\alpha}$
3. $\sqrt{4\pi\alpha}$ **correct**
4. 2α
5. $4\pi\alpha$

Explanation:

Similar to uniform linear accelerated motion, we have

$$\omega_f^2 = \omega_0^2 + 2\alpha\theta.$$

$\omega_0 = 0$ and $\theta = 2\pi$, so

$$\omega_f^2 = 2\alpha(2\pi) = 4\pi\alpha$$

Thus $\omega_f = \sqrt{4\pi\alpha}$

Wheels in Contact

09:06, calculus, numeric, > 1 min.

014

A small wheel of radius 1.7 cm drives a large wheel of radius 14.7 cm by having their circumferences pressed together.

If the small wheel turns at 414 rad/s, how fast does the larger one turn?

Correct answer: 47.8776 rad/s.

Explanation:

Basic Concept: Linear and angular velocity are related by

$$v = r \omega$$

where ω is in radians per unit time.

Solution: The rim of the small wheel moves at

$$v_1 = r_1 \omega_1 .$$

Since their circumferences are in contact, the rim of the larger one is forced to travel at the same linear speed, and

$$v_2 = r_2 \omega_2$$

$$r_1 \omega_1 = r_2 \omega_2 .$$

Thus

$$\begin{aligned} \omega_2 &= \frac{r_1 \omega_1}{r_2} \\ &= \frac{(1.7 \text{ cm})(414 \text{ rad/s})}{(14.7 \text{ cm})} \\ &= 47.8776 \text{ rad/s} . \end{aligned}$$

Algorithm

$$r_1 = 1.7 \text{ cm} \left\{ \begin{matrix} 1.2 \\ 2.3 \end{matrix} \right\} \quad (1)$$

$$r_2 = 14.7 \text{ cm} \left\{ \begin{matrix} 12 \\ 15 \end{matrix} \right\} \quad (2)$$

$$\omega_1 = 414 \text{ rad/s} \left\{ \begin{matrix} 400 \\ 550 \end{matrix} \right\} \quad (3)$$

$$\omega_2 = \frac{r_1 \omega_1}{r_2} \quad (4)$$

$$= \frac{\langle 1.7 \rangle \langle 414 \rangle}{\langle 14.7 \rangle}$$

$$= 47.8776 \text{ rad/s}$$

$$\langle \text{rad/s} \rangle = \frac{\langle \text{cm} \rangle \langle \text{rad/s} \rangle}{\langle \text{cm} \rangle} \quad \text{units}$$

Rotation of Tires

09:07, calculus, multiple choice, > 1 min.

015

A car accelerates uniformly from rest and reaches a speed of 20.3 m/s in 13.4 s. The diameter of a tire is 29.8 cm. Find the number of revolutions the tire makes during this motion, assuming no slipping.

Correct answer: 145.28 rev.

Explanation:

The distance the car traveled during the time interval is,

$$s = \bar{v} t = \frac{1}{2} v t$$

Thus, the angle of rotation is, $\theta = \frac{s}{R}$ from the rolling without slipping condition. The final angular velocity is given by, $\omega = \frac{v}{R}$.

Rotation of Tires

10:01, calculus, multiple choice, < 1 min.

016

What is final rotational speed of a tire in revolutions per second?

Correct answer: 21.6835 rev/s.

Explanation:

Algorithm

$$\langle \frac{\text{m}}{\text{cm}} \rangle = 0.01 \text{ m/cm} \quad (1)$$

$$\langle \frac{\text{rev}}{\text{rad}} \rangle = 0.159155 \text{ rev/rad} \quad (2)$$

$$v = 20.3 \text{ m/s} \left\{ \begin{matrix} 8.8 \\ 35.2 \end{matrix} \right\} \quad (3)$$

$$t = 13.4 \text{ s} \left\{ \begin{matrix} 3.6 \\ 14.4 \end{matrix} \right\} \quad (4)$$

$$D = 29.8 \text{ cm} \left\{ \begin{matrix} 23.2 \\ 92.8 \end{matrix} \right\} \quad (5)$$

$$R = \frac{D}{2.0} \quad (6)$$

$$= \frac{\langle 29.8 \rangle}{2.0}$$

$$= 14.9 \text{ cm}$$

$$\langle \text{cm} \rangle = \frac{\langle \text{cm} \rangle}{\langle \rangle} \quad \text{units}$$

$$R_u = R \langle \frac{\text{m}}{\text{cm}} \rangle \quad (7)$$

$$= \langle 14.9 \rangle \langle 0.01 \rangle$$

$$= 0.149 \text{ m}$$

$$\langle m \rangle = \langle \text{cm} \rangle \langle \text{m/cm} \rangle \quad \text{units}$$

$$s = \frac{vt}{2.0} \quad (8)$$

$$= \frac{\langle 20.3 \rangle \langle 13.4 \rangle}{2.0}$$

$$= 136.01 \text{ m}$$

$$\langle m \rangle = \frac{\langle \text{m/s} \rangle \langle \text{s} \rangle}{\langle \rangle} \quad \text{units}$$

$$\theta = \frac{s}{R_u} \quad (9)$$

$$= \frac{\langle 136.01 \rangle}{\langle 0.149 \rangle}$$

$$= 912.819 \text{ rad}$$

$$\langle \text{rad} \rangle = \frac{\langle \text{m} \rangle}{\langle \text{m} \rangle} \quad \text{units}$$

$$\theta_u = \theta_{\langle \text{rad} \rangle}^{\langle \text{rev} \rangle} \quad (10)$$

$$= \langle 912.819 \rangle \langle 0.159155 \rangle$$

$$= 145.28 \text{ rev}$$

$$\langle \text{rev} \rangle = \langle \text{rad} \rangle \langle \text{rev/rad} \rangle \quad \text{units}$$

$$\omega = \frac{v}{R_u} \quad (11)$$

$$= \frac{\langle 20.3 \rangle}{\langle 0.149 \rangle}$$

$$= 136.242 \text{ rad/s}$$

$$\langle \text{rad/s} \rangle = \frac{\langle \text{m/s} \rangle}{\langle \text{m} \rangle} \quad \text{units}$$

$$\omega_u = \omega_{\langle \text{rad} \rangle}^{\langle \text{rev} \rangle} \quad (12)$$

$$= \langle 136.242 \rangle \langle 0.159155 \rangle$$

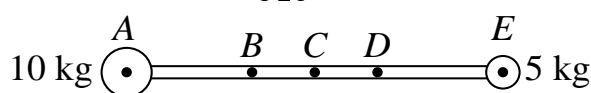
$$= 21.6835 \text{ rev/s}$$

$$\langle \text{rev/s} \rangle = \langle \text{rad/s} \rangle \langle \text{rev/rad} \rangle \quad \text{units}$$

AP M 1993 MC 29 30

10:01, trigonometry, numeric, > 1 min.

019



A 5-kilogram sphere is connected to a 10-kilogram sphere by a rigid rod of negligible mass, as shown above.

Which of the five lettered points represents the center of mass of the sphere-rod combination?

1. A

2. D

3. C

4. B correct

5. E

Explanation:

Basic formula:

$$X_c = \frac{X_1 \times m_1 + X_2 \times m_2}{m_1 + m_2}$$

AP M 1993 MC 29 30

10:02, calculus, numeric, > 1 min.

020

The sphere-rod combination can be pivoted about an axis that is perpendicular to the plane of the page and that passes through one of the five lettered points. Through which point should the axis pass for the moment of inertia of the sphere-rod combination about this axis to be greatest?

1. A

2. B

3. C

4. E correct

5. D

Explanation:

The parallel-axis theorem is

$$I = Mh^2 + I_{cm},$$

where I_{cm} is the moment of inertia about an axis through the center of mass. Since I_{cm} and M is fixed, maximum h will give the maximum moment of inertia. Therefore, the axis should pass the point E .

021

Consider the disk, ring, and square all have equal mass, and are shown in the figure. The disk has continuous mass, the ring has its mass only on its circumference, and the square has its mass only on its perimeter.

fig[1,180] Which of the following conditions is correct?

1. $I_{square}^{cm} > I_{ring}^{cm} > I_{disk}^{cm}$ **correct**
2. $I_{disk}^{cm} > I_{square}^{cm} > I_{ring}^{cm}$
3. $I_{ring}^{cm} > I_{disk}^{cm} > I_{square}^{cm}$
4. $I_{ring}^{cm} > I_{square}^{cm} > I_{disk}^{cm}$
5. $I_{square}^{cm} > I_{disk}^{cm} > I_{ring}^{cm}$
6. $I_{disk}^{cm} > I_{ring}^{cm} > I_{square}^{cm}$

Explanation:

The moment of inertia is $I \equiv \int r^2 dm$. Consequently, the object with the mass concentrated farthest from the center of mass will have the largest moment of inertia.

$$I_{square}^{cm} > I_{ring}^{cm} > I_{disk}^{cm}$$

The moment of inertia of the disk about its center of mass is

$$I_{disk}^{cm} = \frac{1}{2} m r^2,$$

as given in the textbook.

The moment of inertia of the ring about its center of mass is

$$I_{ring}^{cm} = m r^2,$$

as given in the textbook.

The moment of inertia of the square about its center of mass is

$$I_{square}^{cm} = \frac{3}{4} m r^2.$$

This can be shown by considering only one side of the square which is a rod. The moment of inertia of a rod about its center of mass is

$$I_{rod} = \frac{1}{12} \left(\frac{m}{4} \right) (2r)^2 = \frac{1}{3} \left(\frac{m}{4} \right) r^2,$$

and the parallel axis theorem gives the moment of inertia about the center of the square

$$\begin{aligned} I_{center} &= \left(\frac{1}{3} \right) \left(\frac{m}{4} \right) r^2 + \left(\frac{m}{4} \right) r^2 \\ &= \left(\frac{1}{3} + 1 \right) \left(\frac{m}{4} \right) r^2 \\ &= \frac{1}{3} m r^2. \end{aligned}$$

Since there are four sides to the square, the moment of inertia of the square about its center of mass is

$$I_{square}^{cm} = \frac{4}{3} m r^2.$$

Therefore

$$\frac{4}{3} m r^2 > m r^2 > \frac{1}{2} m r^2$$

$$I_{square}^{cm} > I_{ring}^{cm} > I_{disk}^{cm}.$$

Decelerated Grinding Wheel

10:04, calculus, numeric, > 1 min.

022

The motor driving a grinding wheel with a rotational inertia of 0.45 kg m^2 is switched off when the wheel has a rotational speed of 38 rad/s . After 6.6 s , the wheel has slowed down to 30.4 rad/s .

What is the absolute value of the constant torque exerted by friction to slow the wheel down?

Correct answer: 0.518182 N m .

Explanation:

We have

$$\tau \Delta t = \Delta L = \Delta(I \omega),$$

so that

$$\begin{aligned} |\tau| &= \frac{I |\omega_1 - \omega_0|}{\Delta t_1} \\ &= \frac{(0.45 \text{ kg m}^2) (38 \text{ rad/s} - 30.4 \text{ rad/s})}{6.6 \text{ s}} \\ &= 0.518182 \text{ N m}. \end{aligned}$$

023

If this torque remains constant, how long after the motor is switched off will the wheel come to rest?

Correct answer: 33 s .

Explanation:

When the wheel comes to rest, its angular speed is $\omega_2 = 0$; hence

$$\begin{aligned} \Delta t_2 &= \left| \frac{I (\omega_0 - \omega_2)}{\tau} \right| \\ &= \frac{I \omega_0}{|\tau|} \\ &= \frac{(0.45 \text{ kg m}^2) (38 \text{ rad/s})}{(0.518182 \text{ N m})} \\ &= 33 \text{ s} . \end{aligned}$$

Algorithm

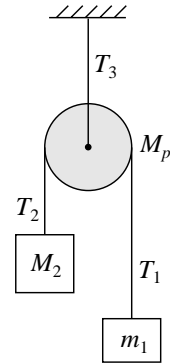
$$\begin{aligned} I &= 0.45 \text{ kg m}^2 \left\{ \begin{matrix} 0.1 \\ 0.9 \end{matrix} \right\} & (1) \\ \omega_0 &= 38 \text{ rad/s} \left\{ \begin{matrix} 20 \\ 40 \end{matrix} \right\} & (2) \\ t_1 &= 6.6 \text{ s} \left\{ \begin{matrix} 6 \\ 12 \end{matrix} \right\} & (3) \\ \omega_1 &= 0.8 \omega_0 & (4) \\ &= 0.8 \langle 38 \rangle \\ &= 30.4 \text{ rad/s} \\ \langle \text{rad/s} \rangle &= \langle \rangle \langle \text{rad/s} \rangle & \text{units} \\ \tau &= \frac{I (\omega_0 - \omega_1)}{t_1} & (5) \\ &= \frac{\langle 0.45 \rangle (\langle 38 \rangle - \langle 30.4 \rangle)}{\langle 6.6 \rangle} \\ &= 0.518182 \text{ N m} \\ \langle \text{N m} \rangle &= \frac{\langle \text{kg m}^2 \rangle (\langle \text{rad/s} \rangle - \langle \text{rad/s} \rangle)}{\langle \text{s} \rangle} & \text{units} \\ t_2 &= \frac{I \omega_0}{\tau} & (6) \\ &= \frac{\langle 0.45 \rangle \langle 38 \rangle}{\langle 0.518182 \rangle} \\ &= 33 \text{ s} \\ \langle \text{s} \rangle &= \frac{\langle \text{kg m}^2 \rangle \langle \text{rad/s} \rangle}{\langle \text{N m} \rangle} & \text{units} \end{aligned}$$

Atwood with Massive Pulley

10:05, calculus, multiple choice, < 1 min.

025

A pulley (in the form of a uniform disk) with mass M_p and a radius R_p is attached to the ceiling, in a uniform gravitational field g , and rotates with no friction about its pivot. Mass M_2 is larger than mass m_1 . These masses are connected by a massless inextensible cord. T_1 , T_2 , and T_3 are magnitudes of the tensions.



The tension $T_1 = m_1 g$

1. true

2. false **correct**

3. Not enough information is available.

Explanation:

The equations of motion using free body diagrams are

$$\begin{aligned} \sum F_y : \quad T_1 - m_1 g &= m_1 a \\ \sum F_y : \quad M_2 g - T_2 &= M_2 a \\ \sum \tau_o : \quad I \alpha &= (T_2 - T_1) r . \end{aligned}$$

Thus, since $\alpha \equiv \frac{a}{r}$

$$T_1 = m_1 (g + a) \tag{1}$$

$$T_2 = M_2 (g - a) \tag{2}$$

$$I \alpha = [M_2 (g - a) - m_1 (g + a)] r$$

$$I \frac{a}{r} = [(M_2 - m_1) g - (M_2 + m_1) a] r .$$

for a disk, $I = \frac{1}{2} M_p r^2$, so

$$[M_p + 2 (M_2 + m_1)] a = 2 (M_2 - m_1) g .$$

Solving for a , we have

$$a = \frac{2 (M_2 - m_1)}{M_p + 2 (M_2 + m_1)} g . \tag{3}$$

Equation (1) shows that $T_1 \neq m_1 g$.

026

The relationship between the magnitudes of the acceleration is

1. $a_{M_2} > a_{m_1}$

2. $a_{M_2} < a_{m_1}$

3. $a_{M_2} = a_{m_1}$ **correct**

4. Not enough information is available.

Explanation:

The rope does not stretch, so the acceleration of both masses must be equal.

Atwood with Massive Pulley

10:05, trigonometry, multiple choice, < 1 min.

027

The center of mass of $m_1 + M_2$

1. accelerates up.

2. accelerates down. **correct**

3. does not accelerate.

4. Not enough information is available.

Explanation:

The larger mass accelerates down and the smaller mass accelerates up, so the center of mass of the system accelerates down.

The acceleration of the center of mass is

$$a_{cm} \equiv \frac{M_2 a_{M_2} + m_1 a_{m_1}}{M_2 + m_1}.$$

Using “up” as the positive direction, then $a_{M_2} = -a$ and $a_{m_1} = a$.

From Eq. (3), we have

$$\begin{aligned} a_{cm} &= -2 \frac{M_2 (M_2 - m_1) - m_1 (M_2 - m_1)}{(M_2 + m_1) [M_p + 2 (M_2 + m_1)]} g \\ &= -2 \frac{(M_2 - m_1)^2}{(M_2 + m_1) [M_p + 2 (M_2 + m_1)]} g. \end{aligned}$$

The negative sign indicates the center of mass acceleration is down.

Atwood with Massive Pulley

10:06, trigonometry, numeric, > 1 min.

028

The relationship between the magnitudes of the tension T_2 and T_1 is

1. $T_2 > T_1$ **correct**

2. $T_2 < T_1$

3. $T_2 = T_1$

4. Not enough information is available.

Explanation:

Substituting a from Eq. (3) into Eq. (1) gives

$$\begin{aligned} T_1 &= m_1 (g + a) \\ &= \left[\frac{m_1 M_p + 4 m_1 M_2}{M_p + 2 (M_2 + m_1)} \right] g \end{aligned} \quad (4)$$

and substituting a into Eq. (2) gives

$$\begin{aligned} T_2 &= M_2 (g - a) \\ &= \left[\frac{M_2 M_p + 4 m_1 M_2}{M_p + 2 (M_2 + m_1)} \right] g. \end{aligned} \quad (5)$$

Therefore $T_2 > T_1$ by the amount

$$\Delta T = T_2 - T_1 = \left[\frac{(M_2 - m_1) M_p}{M_p + 2 (M_2 + m_1)} \right] g.$$

Alternative Solution: The torque equation is

$$\sum \tau_o : \quad I \alpha = (T_2 - T_1) r.$$

Since α is positive, $T_2 > T_1$; however, α is positive since $T_2 > T_1$, so this argument is circular.

029

The relationship between the magnitudes of the tension T_3 , T_2 , and T_1 is

1. $T_1 + T_2 > T_3$

2. Not enough information is available.

3. $T_1 + T_2 = T_3$

4. $T_1 + T_2 < T_3$ **correct**

Explanation:

Using a free body diagram for the pulley alone and one sees

$$\begin{aligned} T_3 &= T_1 + T_2 + M_p g \\ &> T_1 + T_2. \end{aligned} \quad (6)$$

Atwood with Massive Pulley

10:07, calculus, numeric, > 1 min.

030

The relations between the magnitudes of the tension T_3 and m_1 , M_2 , and M_p is

1. $T_3 > (m_1 + M_2 + M_p) g$
2. $T_3 = (m_1 + M_2 + M_p) g$
3. $T_3 < (m_1 + M_2 + M_p) g$ **correct**
4. Not enough information is available.

Explanation:

Using Eqs. (4), (5), & (6), we have

$$\begin{aligned} T_3 &= T_1 + T_2 + M_p g \\ &= (M_2 + m_1 + M_p) g - (M_2 - m_1) a \\ &< (M_2 + m_1 + M_p) g . \end{aligned}$$

Since $M_2 > m_1$ the center of mass of the blocks is accelerating down and the tension T_3 is reduced.