This print-out should have 9 questions. Check that it is complete before leaving the printer. Also, multiple-choice questions may continue on the next column or page: find all choices before making your selection.

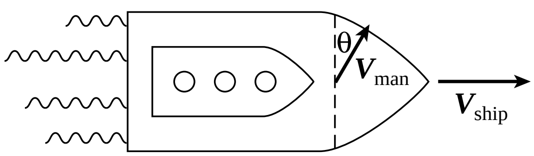
Note that only a few (usually 4) of the problems will have their scores kept for a grade. You may make multiple tries to get a problem right, although it's worth less each time. Worked solutions to a number of these problems (even some of the scored ones) may be found on the Ph211 home page.

Walking on a Ship

03:01, calculus, numeric, > 1 min.

001

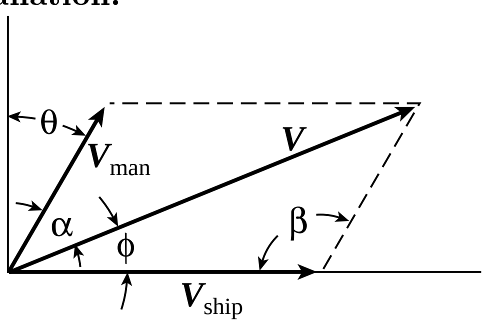
A ship cruises forward at 7 m/s relative to the water. On deck, a man walks diagonally toward the bow such that his path forms an angle $\theta = 16°$ with a line perpendicular to the boat's direction of motion. He walks at 4 m/s relative to the boat.



At what speed does he walk relative to the water?

Correct answer: 8.96859 m/s.

Explanation:



When you complete the parallelogram, the resultant velocity v with respect to the water is the side of the triangle opposite the obtuse angle, which has a measure of $\beta = 90° + \theta$. Let $\vec{v}_s$ be the velocity of the ship, $\vec{v}_m$ be the velocity of the man, and $\vec{v}$ be the resultant velocity of the man relative to the water (Earth). By the law of cosines

$$v^2 = v_m^2 + v_s^2 - 2\, v_m\, v_s \cos\beta$$

$$\begin{aligned} v &= \left[v_m^2 + v_s^2 - 2\, v_m\, v_s \cos\beta\right]^{1/2} \\ &= \big[(4 \text{ m/s})^2 + (7 \text{ m/s})^2 \\ &\quad -2(4 \text{ m/s})(7 \text{ m/s})\cos(106°)\big]^{1/2} \\ &= \big[(4 \text{ m/s})^2 + (7 \text{ m/s})^2 \\ &\quad +(15.4357 \text{ m}^2/\text{s}^2)\big]^{1/2} \\ &= 8.96859 \text{ m/s}\,. \end{aligned}$$

Alternatively: We can analyze the vector addition using the components of the vectors. *Note:*

$$\vec{v} = \vec{v}_s + \vec{v}_m\,,$$

or

$$v_x = v_s + v_m \sin\theta \quad \text{and}$$
$$v_y = v_m \cos\theta\,.$$

Hence the speed of the man with respect to the water is

$$\begin{aligned} v &= \sqrt{v_x^2 + v_y^2} \\ &= \sqrt{(8.10255 \text{ m/s})^2 + (3.84505 \text{ m/s})^2} \\ &= 8.96859 \text{ m/s}\,. \end{aligned}$$

002

At what angle to his intended path does the man walk with respect to the water?

Correct answer: 48.6134 °.

Explanation:

The law of sines can be used to compute the requested angle α, which is the angle opposite the ship's path and velocity.

$$\frac{\sin\alpha}{v_s} = \frac{\sin\beta}{v}$$

$$\sin\alpha = \frac{v_s}{v}\sin\beta$$

$$\begin{aligned} \alpha &= \arcsin\left[\frac{v_s}{v}\sin\beta\right] \\ &= \arcsin\left[\frac{7 \text{ m/s}}{8.96859 \text{ m/s}}\sin(106°)\right] \\ &= 48.6134°\,. \end{aligned}$$

Alternative Solution: Using vector components from Part 1, we have

$$\begin{aligned} \phi &= \arctan\left(\frac{v_y}{v_x}\right) \\ &= \arctan\left(\frac{3.84505 \text{ m/s}}{8.10255 \text{ m/s}}\right) \\ &= 25.3866°\,. \end{aligned}$$

Therefore the angle between $\vec{v}_m$ and $\vec{v}$ is

$$\begin{aligned}\alpha &= 90^\circ - \theta - \phi \\ &= 90^\circ - (16^\circ) - (25.3866^\circ) \\ &= 48.6134^\circ\,.\end{aligned}$$

Algorithm

$$\langle\tfrac{\text{rad}}{\text{deg}}\rangle = 0.0174533\ \text{rad/deg} \tag{1}$$

$$\langle\tfrac{\text{deg}}{\text{rad}}\rangle = 57.2958\ \text{deg/rad} \tag{2}$$

$$v_s = 7\ \text{m/s}\ \left\{\begin{smallmatrix}5\\9\end{smallmatrix}\right\} \tag{3}$$

$$v_m = 4\ \text{m/s}\ \left\{\begin{smallmatrix}1\\4\end{smallmatrix}\right\} \tag{4}$$

$$\theta = 16^\circ\ \left\{\begin{smallmatrix}10\\70\end{smallmatrix}\right\} \tag{5}$$

$$\begin{aligned}\beta &= 90.0 + \theta \\ &= 90.0 + \langle 16\rangle \\ &= 106^\circ\end{aligned} \tag{6}$$

$$\langle^\circ\rangle = \langle\rangle + \langle^\circ\rangle \qquad \text{units}$$

$$\begin{aligned}\beta_u &= \beta\ \langle\tfrac{\text{rad}}{\text{deg}}\rangle \\ &= \langle 106\rangle\ \langle 0.0174533\rangle \\ &= 1.85005\end{aligned} \tag{7}$$

$$\langle\rangle = \langle^\circ\rangle\ \langle\text{rad/deg}\rangle \qquad \text{units}$$

$$\begin{aligned}\theta_u &= \theta\ \langle\tfrac{\text{rad}}{\text{deg}}\rangle \\ &= \langle 16\rangle\ \langle 0.0174533\rangle \\ &= 0.279253\end{aligned} \tag{8}$$

$$\langle\rangle = \langle^\circ\rangle\ \langle\text{rad/deg}\rangle \qquad \text{units}$$

$$\begin{aligned}v_x &= v_m\ \sin(\theta_u) + v_s \\ &= \langle 4\rangle\ \sin(\langle 0.279253\rangle) + \langle 7\rangle \\ &= 8.10255\ \text{m/s}\end{aligned} \tag{9}$$

$$\langle\text{m/s}\rangle = \langle\text{m/s}\rangle\ \sin(\langle\rangle) + \langle\text{m/s}\rangle \qquad \text{units}$$

$$\begin{aligned}v_y &= v_m\ \cos(\theta_u) \\ &= \langle 4\rangle\ \cos(\langle 0.279253\rangle) \\ &= 3.84505\ \text{m/s}\end{aligned} \tag{10}$$

$$\langle\text{m/s}\rangle = \langle\text{m/s}\rangle\ \cos(\langle\rangle) \qquad \text{units}$$

$$\begin{aligned}V &= \sqrt{v_x^{2.0} + v_y^{2.0}} \\ &= \sqrt{\langle 8.10255\rangle^{2.0} + \langle 3.84505\rangle^{2.0}} \\ &= 8.96859\ \text{m/s}\end{aligned} \tag{11}$$

$$\langle\text{m/s}\rangle = \sqrt{\langle\text{m/s}\rangle^{2.0} + \langle\text{m/s}\rangle^{2.0}} \qquad \text{units}$$

$$\begin{aligned}\mathcal{V} &= (-2.0\ v_m\ v_s\ \cos(\beta_u)) \\ &= (-2.0\ \langle 4\rangle\ \langle 7\rangle\ \cos(\langle 1.85005\rangle)) \\ &= 15.4357\ \text{m}^2/\text{s}^2\end{aligned} \tag{12}$$

$$\langle\text{m}^2/\text{s}^2\rangle = (-\langle\rangle\ \langle\text{m/s}\rangle\ \langle\text{m/s}\rangle\ \cos(\langle\rangle)) \qquad \text{units}$$

$$\begin{aligned}v &= \sqrt{v_m^{2.0} + v_s^{2.0} + \mathcal{V}} \\ &= \sqrt{\langle 4\rangle^{2.0} + \langle 7\rangle^{2.0} + \langle 15.4357\rangle} \\ &= 8.96859\ \text{m/s}\end{aligned} \tag{13}$$

$$\langle\text{m/s}\rangle = \sqrt{\langle\text{m/s}\rangle^{2.0} + \langle\text{m/s}\rangle^{2.0} + \langle\text{m}^2/\text{s}^2\rangle} \qquad \text{units}$$

$$\begin{aligned}\phi_u &= \arctan2(v_y, v_x) \\ &= \arctan2(\langle 3.84505\rangle, \langle 8.10255\rangle) \\ &= 0.443079\end{aligned} \tag{14}$$

$$\langle\rangle = \arctan\langle\rangle\ (\langle\text{m/s}\rangle, \langle\text{m/s}\rangle) \qquad \text{units}$$

$$\begin{aligned}\phi &= \phi_u\ \langle\tfrac{\text{deg}}{\text{rad}}\rangle \\ &= \langle 0.443079\rangle\ \langle 57.2958\rangle \\ &= 25.3866^\circ\end{aligned} \tag{15}$$

$$\langle^\circ\rangle = \langle\rangle\ \langle\text{deg/rad}\rangle \qquad \text{units}$$

$$\begin{aligned}\alpha_u &= \arcsin\left(\frac{v_s\ \sin(\beta_u)}{v}\right) \\ &= \arcsin\left(\frac{\langle 7\rangle\ \sin(\langle 1.85005\rangle)}{\langle 8.96859\rangle}\right) \\ &= 0.848465\end{aligned} \tag{16}$$

$$\langle\rangle = \arcsin\left(\frac{\langle\text{m/s}\rangle\ \sin(\langle\rangle)}{\langle\text{m/s}\rangle}\right) \qquad \text{units}$$

$$\begin{aligned}\alpha &= \alpha_u\ \langle\tfrac{\text{deg}}{\text{rad}}\rangle \\ &= \langle 0.848465\rangle\ \langle 57.2958\rangle \\ &= 48.6134^\circ\end{aligned} \tag{17}$$

$$\langle^\circ\rangle = \langle\rangle\ \langle\text{deg/rad}\rangle \qquad \text{units}$$

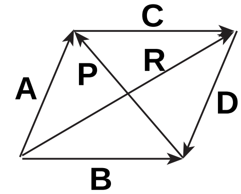
$$\begin{aligned}Alpha &= 90 - \theta - \phi \\ &= 90 - \langle 16\rangle - \langle 25.3866\rangle \\ &= 48.6134^\circ\end{aligned} \tag{18}$$

$$\langle^\circ\rangle = \langle\rangle - \langle^\circ\rangle - \langle^\circ\rangle \qquad \text{units}$$

Vector Addition 05

03:03, trigonometry, multiple choice, > 1 min.

003



The vector $\vec{R}$ shown in the sketch may be expressed in terms of $\vec{A}$, $\vec{B}$, $\vec{C}$, and $\vec{D}$, which are the edges of a parallelogram, as

1. $\vec{R} = \vec{B} + \vec{D}$

2. $\vec{R} = \vec{B} - \vec{A}$

3. $\vec{R} = \vec{A} - \vec{B}$

4. $\vec{R} = \vec{C} + \vec{B}$

5. $\vec{R} = \vec{C} + \vec{D}$

6. $\vec{R} = \vec{A} + \vec{D}$

7. $\vec{R} = \vec{A} - \vec{D}$

8. $\vec{R} = \vec{D} - \vec{A}$

9. $\vec{R} = \vec{A} - \vec{C}$

10. $\vec{R} = \vec{B} + \vec{A}$ **correct**

Explanation:

The resultant, or sum of several vectors, is the single vector leading from the tail of the first vector to the head of the last vector when the vectors are drawn tail to head in any order.

004

The vector $\vec{P}$ shown in the sketch may be expressed in terms of $\vec{A}$, $\vec{B}$, $\vec{C}$, and $\vec{D}$ as

1. $\vec{P} = \vec{A} - \vec{B}$ **correct**

2. $\vec{P} = \vec{B} - \vec{A}$

3. $\vec{P} = \vec{A} + \vec{D}$

4. $\vec{P} = \vec{C} + \vec{B}$

5. $\vec{P} = \vec{C} + \vec{D}$

6. $\vec{P} = \vec{B} + \vec{A}$

7. $\vec{P} = \vec{A} - \vec{D}$

8. $\vec{P} = \vec{D} - \vec{A}$

9. $\vec{P} = \vec{C} - \vec{A}$

10. $\vec{P} = \vec{B} + \vec{D}$

Explanation:

Wind and a Jet

03:04, calculus, numeric, > 1 min.

006

What is the direction of the aircraft (use counterclockwise from due East as the positive angular direction, between the limits of $-180°$ and $+180°$)?

Correct answer: 11.0616 °.

Explanation:

The direction is

$$\theta = \arctan\left(\frac{v_y}{v_x}\right).$$

Algorithm

$$\begin{aligned}
\left\langle\tfrac{\text{rad}}{\text{deg}}\right\rangle &= 0.0174533 \text{ rad/deg} && (1)\\
\left\langle\tfrac{\text{deg}}{\text{rad}}\right\rangle &= 57.2958 \text{ deg/rad} && (2)\\
v_1 &= 488 \text{ mph } \left\{{100 \atop 990}\right\} && (3)\\
v_2 &= 905 \text{ mph } \left\{{100 \atop 990}\right\} && (4)\\
\theta &= 17° \left\{{10 \atop 80}\right\} && (5)\\
\theta_r &= \theta \left\langle\tfrac{\text{rad}}{\text{deg}}\right\rangle && (6)\\
&= \langle 17\rangle \langle 0.0174533\rangle \\
&= 0.296706 \text{ rad} \\
\langle\text{rad}\rangle &= \langle°\rangle \langle\text{rad/deg}\rangle && \text{units}\\
v_x &= v_1 + v_2 \cos(\theta_r) && (7)\\
&= \langle 488\rangle + \langle 905\rangle \cos(\langle 0.296706\rangle) \\
&= 1353.46 \text{ mph} \\
\langle\text{mph}\rangle &= \langle\text{mph}\rangle + \langle\text{mph}\rangle \cos(\langle\text{rad}\rangle) && \text{units}\\
v_y &= v_2 \sin(\theta_r) && (8)\\
&= \langle 905\rangle \sin(\langle 0.296706\rangle) \\
&= 264.596 \text{ mph} \\
\langle\text{mph}\rangle &= \langle\text{mph}\rangle \sin(\langle\text{rad}\rangle) && \text{units}\\
v &= \sqrt{v_x^{2.0} + v_y^{2.0}} && (9)\\
&= \sqrt{\langle 1353.46\rangle^{2.0} + \langle 264.596\rangle^{2.0}} \\
&= 1379.08 \text{ mph} \\
\langle\text{mph}\rangle &= \sqrt{\langle\text{mph}\rangle^{2.0} + \langle\text{mph}\rangle^{2.0}} && \text{units}\\
\alpha_r &= \text{arctan2}(v_y, v_x) && (10)\\
&= \text{arctan2}(\langle 264.596\rangle, \langle 1353.46\rangle) \\
&= 0.193062 \text{ rad} \\
\langle\text{rad}\rangle &= \arctan\langle\rangle(\langle\text{mph}\rangle, \langle\text{mph}\rangle) && \text{units}\\
\alpha &= \left\langle\tfrac{\text{deg}}{\text{rad}}\right\rangle \alpha_r && (11)\\
&= \langle 57.2958\rangle \langle 0.193062\rangle
\end{aligned}$$

$$= 11.0616^\circ$$
$$\langle^\circ\rangle = \langle \text{deg/rad} \rangle \langle \text{rad} \rangle \qquad \text{units}$$

007

During a baseball game, a batter hits a pop-up to a fielder 90 m away.

If the ball remains in the air for 7 s, how high does it rise?

Correct answer: 60.025 m.

Explanation:

The distance to the fielder is extraneous information and can be ignored.

Consider the motion as the ball rises to its maximum height h: $\Delta t = \dfrac{t}{2}$. The height is defined by

$$h = \frac{1}{2} g (\Delta t)^2 = \frac{1}{2} g \frac{t^2}{4} = \frac{g t^2}{8}$$

Algorithm

$$t = 7 \text{ s} \left\{ {5 \atop 7} \right\} \qquad (1)$$
$$d = 90 \text{ m} \left\{ {60 \atop 95} \right\} \qquad (2)$$
$$g = 9.8 \text{ m/s}^2 \qquad (3)$$
$$h = \frac{g\, t^{2.0}}{8.0} \qquad (4)$$
$$= \frac{\langle 9.8 \rangle \langle 7 \rangle^{2.0}}{8.0}$$
$$= 60.025 \text{ m}$$
$$\langle \text{m} \rangle = \frac{\langle \text{m/s}^2 \rangle \langle \text{s} \rangle^{2.0}}{\langle\rangle} \qquad \text{units}$$

Dropping Medical Supplies

04:03, trigonometry, numeric, > 1 min.

009

A plane drops a hamper of medical supplies from a height of 5940 m during a practice run over the ocean. The plane's horizontal velocity was 143 m/s at the instant the hamper was dropped.

What is the magnitude of the overall velocity of the hamper at the instant it strikes the surface of the ocean?

Correct answer: 369.964 m/s.

Explanation:

Basic Concepts Motion in gravity field

$$v^2 = 2\, g\, h$$

Solution:

This is a projectile motion problem. The motion of the dropping hamper consists of two parts: horizontally, it moves with the initial velocity of the plane, i.e. $v_h = v = 143$ m/s ; vertically, due to Gravity, it moves as a freely falling body. Applying the equation above gives the vertical velocity as

$$v_v = \sqrt{2\, g\, h}$$
$$= 341.21 \text{ m/s}$$

Thus the overall velocity at the instant the hamper strikes the surface of the ocean is

$$v_f = \sqrt{v_v^2 + v_h^2}$$
$$= \sqrt{(341.21 \text{ m/s})^2 + (143 \text{ m/s})^2}$$
$$= 369.964 \text{ m/s}$$

Algorithm

$$g = 9.8 \text{ m/s}^2 \qquad (1)$$
$$h = 5940 \text{ m} \left\{ {2500 \atop 6000} \right\} \qquad (2)$$
$$v = 143 \text{ m/s} \left\{ {100 \atop 150} \right\} \qquad (3)$$
$$v_h = v \qquad (4)$$
$$= 143 \text{ m/s}$$
$$v_v = \sqrt{2.0\, g\, h} \qquad (5)$$
$$= \sqrt{2.0 \langle 9.8 \rangle \langle 5940 \rangle}$$
$$= 341.21 \text{ m/s}$$
$$\langle \text{m/s} \rangle = \sqrt{\langle\rangle \langle \text{m/s}^2 \rangle \langle \text{m} \rangle} \qquad \text{units}$$
$$v_f = \sqrt{v_v^{2.0} + v_h^{2.0}} \qquad (6)$$
$$= \sqrt{\langle 341.21 \rangle^{2.0} + \langle 143 \rangle^{2.0}}$$
$$= 369.964 \text{ m/s}$$
$$\langle \text{m/s} \rangle = \sqrt{\langle \text{m/s} \rangle^{2.0} + \langle \text{m/s} \rangle^{2.0}} \qquad \text{units}$$

Moon Orbit

04:03, trigonometry, numeric, > 1 min.

010

The orbit of a Moon about its planet is approximately circular, with a mean radius of 6.07×10^8 m. It takes 39.3 days for the Moon to complete one revolution about the planet.

Find the mean orbital speed of the Moon.
Correct answer: 1123.21 m/s.
Explanation:
Dividing the length $C = 2\pi r$ of the trajectory of the Moon by the time

$$T = 39.3 \text{ days } = 3.39552 \times 10^6 \text{ s}$$

of one revolution (in seconds!), we obtain that the mean orbital speed of the Moon is

$$\begin{aligned} v &= \frac{C}{T} = \frac{2\,\pi\,r}{T} \\ &= \frac{2\,\pi\,(6.07 \times 10^8 \text{ m})}{3.39552 \times 10^6 \text{ s}} \\ &= 1123.21 \text{ m/s} \,. \end{aligned}$$

Moon Orbit
04:03, trigonometry, numeric, > 1 min.
011
Find the Moon's centripetal acceleration.
Correct answer: 0.00207843 m/s^2.
Explanation:
Since the magnitude of the velocity is constant, the tangential acceleration of the Moon is zero. For the centripetal acceleration we use the formula

$$\begin{aligned} a_c &= \frac{v^2}{r} \\ &= \frac{(1123.21 \text{ m/s})^2}{6.07 \times 10^8 \text{ m}} \\ &= 0.00207843 \text{ m/s}^2 \,. \end{aligned}$$

Algorithm

$$\begin{aligned} \langle \tfrac{\text{s}}{\text{day}} \rangle &= 86400 \text{ s/day} && (1) \\ r &= 6.07 \times 10^8 \text{ m} \left\{ \begin{smallmatrix} 1.536\times 10^8 \\ 6.144\times 10^8 \end{smallmatrix} \right\} && (2) \\ T &= 39.3 \text{ days} \left\{ \begin{smallmatrix} 10.92 \\ 43.68 \end{smallmatrix} \right\} && (3) \\ T_u &= \langle \tfrac{\text{s}}{\text{day}} \rangle\, T && (4) \\ &= \langle 86400 \rangle \langle 39.3 \rangle \\ &= 3.39552 \times 10^6 \text{ s} \\ \langle \text{s} \rangle &= \langle \text{s/day} \rangle \langle \text{days} \rangle && \text{units} \\ v &= \frac{2.0\,\pi\,r}{T_u} && (5) \\ &= \frac{2.0\,\langle 3.1415926 \rangle \langle 6.07 \times 10^8 \rangle}{\langle 3.39552 \times 10^6 \rangle} \\ &= 1123.21 \text{ m/s} \\ \langle \text{m/s} \rangle &= \frac{\langle \rangle \langle \rangle \langle \text{m} \rangle}{\langle \text{s} \rangle} && \text{units} \\ a_c &= \frac{v^{2.0}}{r} && (6) \\ &= \frac{\langle 1123.21 \rangle^{2.0}}{\langle 6.07 \times 10^8 \rangle} \\ &= 0.00207843 \text{ m/s}^2 \\ \langle \text{m/s}^2 \rangle &= \frac{\langle \text{m/s} \rangle^{2.0}}{\langle \text{m} \rangle} && \text{units} \end{aligned}$$