This print-out should have 8 questions. Check that it is complete before leaving the printer. Also, multiple-choice questions may continue on the next column or page: find all choices before making your selection.

You have till Friday to hand these in. But leave time to study for the final!

AP B 1993 MC 42

14:01, trigonometry, multiple choice, < 1 min.

001

Forces between two objects which are inversely proportional to the square of the distance between the objects include which of the following?

I. Gravitational force between two celestial bodies

II. Electrostatic force between two electrons

III. Nuclear force between two neutrons.

1. I only

2. III only

3. I and II only **correct**

4. II and III only

5. I, II, and III

Explanation:

Both the Gravitational force and Electrostatic force between two bodies is proportional to $1/r^2$. The nuclear force, or the force that holds the nucleus together, is very strong compared to the gravitational force and electrostatic force over a very short range. Outside of that range, however, it is negligible. It cannot be proportional to $1/r^2$. Our answer, then, is (3).

AP M 1993 MC 22

14:01, calculus, multiple choice, < 1 min.

002

A newly discovered planet has twice the mass of the Earth, but the acceleration due to gravity on the new planet's surface is exactly the same as the acceleration due to gravity on the Earth's surface.

The radius of the new planet in terms of the radius R of Earth is

1. $\frac{1}{2}\,R$

2. $\frac{\sqrt{2}}{2}\,R$

3. $2\,R$

4. $\sqrt{2}\,R$ **correct**

5. $4\,R$

Explanation:

From Newton's second law and the law of universal gravitation, the gravitational force near the surface is

$$F_g = mg = G\frac{Mm}{r^2}$$
$$g = \frac{GM}{r^2}$$

Now $M_p = 2M_e$ and $g_p = g_e$, so

$$\frac{GM_e}{R^2} = \frac{GM_p}{R_p^2} = \frac{2GM_e}{R_p^2}$$
$$\frac{1}{R^2} = \frac{2}{R_p^2}$$
$$R_p = \sqrt{2}\,R.$$

Circular Earth Orbit

14:01, trigonometry, numeric, > 1 min.

003

A satellite moves in a circular orbit around the Earth at a speed of 6 km/s. Determine the satellite's altitude above the surface of the Earth. Assume the Earth is a homogeneous sphere of radius $R_{earth} = 6370$ km and mass $M_{earth} = 5.98 \times 10^{24}$ kg . You will need $G = 6.67259 \times 10^{-11}\ \mathrm{N\,m^2/kg^2}$

Correct answer: 4713.91 km.

Explanation:

The gravitational force provides the centripetal acceleration.

$$\frac{G\,m\,M}{r^2} = \frac{m\,v^2}{r},$$

where M is the mass of the Earth and m is the mass of the satellite. Solving for r yields

$$r = \frac{GM}{v^2} = 1.10839 \times 10^7\ \mathrm{m}$$

Then the height of the satellite above the Earth's surface is

$$h = r - R_{\text{earth}} = 4713.91 \text{ km}$$

Algorithm

$$\langle{}^{\text{km}}_{\text{m}}\rangle = 0.001 \text{ km/m} \quad (1)$$
$$\langle{}^{\text{h}}_{\text{s}}\rangle = 0.000277778 \text{ h/s} \quad (2)$$
$$G = 6.67259\times10^{-11} \text{ Nm}^2/\text{kg}^2 \quad (3)$$
$$M_{earth} = 5.98\times10^{24} \text{ kg} \quad (4)$$
$$R_{earth} = 6.37\times10^{3} \text{ km} \quad (5)$$
$$v = 6 \text{ km/s}\{{}^{5}_{7}\} \quad (6)$$
$$r = \frac{G\langle{}^{\text{km}}_{\text{m}}\rangle^{2.0} M_{earth}}{v^{2.0}} \quad (7)$$
$$= \frac{\langle 6.67259\times10^{-11}\rangle\langle 0.001\rangle^{2.0}\langle 5.98\times10^{24}\rangle}{\langle 6\rangle^{2.0}}$$
$$= 1.10839\times10^{7} \text{ m}$$
$$\langle\text{m}\rangle = \frac{\langle\text{Nm}^2/\text{kg}^2\rangle\langle\text{km/m}\rangle^{2.0}\langle\text{kg}\rangle}{\langle\text{km/s}\rangle^{2.0}} \quad \text{units}$$
$$h = r\langle{}^{\text{km}}_{\text{m}}\rangle - R_{earth} \quad (8)$$
$$= \langle 1.10839\times10^{7}\rangle\langle 0.001\rangle - \langle 6370\rangle$$
$$= 4713.91 \text{ km}$$
$$\langle\text{km}\rangle = \langle\text{m}\rangle\langle\text{km/m}\rangle - \langle\text{km}\rangle \quad \text{units}$$

Moon Falling in Orbit

14:01, trigonometry, numeric, > 1 min.

004

An apple on the surface of the earth falls 4.9 m in its first second of fall from rest. The moon is about 60 times farther away from the center of the earth than is the apple. How far does the moon fall toward the earth each second?

Correct answer: 0.00136111 m.

Explanation:

In uniformly accelerated motion in a straight line,

$$s = \frac{1}{2}\, a\, t^2 \quad \Rightarrow s \propto a$$

The acceleration is due to the universal gravitation by the earth,

$$a = G\,\frac{M_{earth}}{r^2} \quad \Rightarrow a \propto \frac{1}{r^2}$$

For the same time interval, we have:

$$\frac{s_m}{s_a} = \frac{a_m}{a_a} = \left(\frac{r_a}{r_m}\right)^2 = \left(\frac{r_a}{60\, r_a}\right)^2 = \left(\frac{1}{60}\right)^2$$

$$s_m = \left(\frac{1}{60}\right)^2 s_a$$
$$= \left(\frac{1}{60}\right)^2 4.9 \text{ m}$$
$$= 0.00136111 \text{ m}$$

Algorithm

$$s_a = 4.9 \text{ m} \quad (1)$$
$$s_m = \left(\frac{1.0}{60.0}\right)^{2.0} s_a \quad (2)$$
$$= \left(\frac{1.0}{60.0}\right)^{2.0} \langle 4.9\rangle$$
$$= 0.00136111 \text{ m}$$
$$\langle\text{m}\rangle = \left(\frac{\langle\rangle}{\langle\rangle}\right)^{2.0} \langle\text{m}\rangle \quad \text{units}$$

Mutual Attraction

14:01, trigonometry, numeric, > 1 min.

005

Tom has a mass of 69.4 kg and Sally has a mass of 58.1 kg. Tom and Sally are standing 17.4 m apart on the dance floor. Sally looks up and she sees Tom. She feels an attraction. If the attraction is gravitation, find its magnitude. Assume both can be replaced by spherical masses.

Correct answer: 8.88652×10^{-10} N.

Explanation:

By Newton's Universal Law of Gravitation,

$$F = G\,\frac{m_T\, m_S}{d^2}$$
$$= 6.67259 \times 10^{-11}\, \text{N m}^2/\text{kg}^2 \times \frac{69.4 \text{ kg} \times 58.1 \text{ kg}}{(17.4 \text{ m})^2}$$
$$= 8.88652 \times 10^{-10} \text{ N}$$

Algorithm

$$G = 6.67259\times10^{-11} \text{ Nm}^2/\text{kg}^2 \quad (1)$$
$$m_T = 69.4 \text{ kg}\{{}^{65}_{75}\} \quad (2)$$
$$m_S = 58.1 \text{ kg}\{{}^{40}_{60}\} \quad (3)$$
$$d = 17.4 \text{ m}\{{}^{15}_{40}\} \quad (4)$$

$$F = \frac{G m_T m_S}{d^{2.0}} \quad (5)$$
$$= \frac{\langle 6.67259\times10^{-11}\rangle\langle 69.4\rangle\langle 58.1\rangle}{\langle 17.4\rangle^{2.0}}$$
$$= 8.88652\times10^{-10}\ \text{N}$$
$$\langle \text{N}\rangle = \frac{\langle \text{Nm}^2/\text{kg}^2\rangle\langle \text{kg}\rangle\langle \text{kg}\rangle}{\langle \text{m}\rangle^{2.0}} \quad \text{units}$$

Apollo Astronauts

14:03, calculus, numeric, > 1 min.

006

On the way to the moon the Apollo astronauts reach a point where the Moon's gravitational pull is stronger than that of Earth's. Determine the distance of this point from the center of the Earth. The masses of the Earth and the Moon are respectively 6.13×10^{24} kg and 7.36×10^{22} kg. The distance from the Earth to the Moon is 4×10^8 m.

Correct answer: 3.60499×10^8 m.

Explanation:

If r_e is the distance from this point to the center of the Earth and r_m is the distance from this point to the center of the Moon, then from the formula

$$\frac{G\, m\, M_e}{r_e^2} = \frac{G\, m\, M_m}{r_m^2}$$

we obtain

$$q = \frac{r_m}{r_e} = \sqrt{\frac{M_m}{M_e}} = \sqrt{\frac{7.36 \times 10^{22}\ \text{kg}}{6.13 \times 10^{24}\ \text{kg}}} = 0.109574\,.$$

On the other hand,

$$r_e + r_m = R\,.$$

Eliminating r_m from the last two equalities, we obtain

$$r_e = \frac{R}{q+1} = \frac{4 \times 10^8\ \text{m}}{0.109574 + 1} = 3.60499 \times 10^8\ \text{m}\,.$$

012

An artificial satellite circling the Earth completes each orbit in 119 minutes. What is the value of g at the location of this satellite?

Correct answer: $6.20832\ \text{m/s}^2$.

Explanation:

$$\omega = \frac{2\,\pi}{T} = \frac{2\,(3.1415926\,)}{119\ \text{minutes}} = \frac{2\,(3.1415926\,)}{7140\ \text{s}} = 0.000879998\ \text{rad/s}.$$

$$F_c = m\, r\, \omega^2 = \frac{G\, M_{earth}\, m}{r^2}, \quad \text{or}$$

$$r = \left(\frac{G\, M_{earth}}{\omega^2}\right)^{1/3} = \left(\frac{(6.67259 \times 10^{-11}\ \text{N m}^2/\text{kg}^2)}{(0.000879998\ \text{rad/s})^2}\right)^{1/3} \times (5.98 \times 10^{24}\ \text{kg})^{1/3} = 8.01698 \times 10^6\ \text{m}$$

is the radius of the satellite's orbit. At this distance from the Earth,

$$F = \frac{G\, M_{earth}\, m}{r^2} = m\, g, \qquad \text{or}$$

$$g = \frac{G\, M_{earth}}{r^2} = \frac{(6.67259 \times 10^{-11}\ \text{N m}^2/\text{kg}^2)}{(8.01698 \times 10^6\ \text{m})^2} \times (5.98 \times 10^{24}\ \text{kg}) = 6.20832\ \text{m/s}^2.$$

Algorithm

$$\langle \tfrac{\text{s}}{\text{min}}\rangle = 60\ \text{s/min} \quad (1)$$
$$G = 6.67259\times10^{-11}\ \text{Nm}^2/\text{kg}^2 \quad (2)$$
$$M_{earth} = 5.98\times10^{24}\ \text{kg} \quad (3)$$
$$t = 119\ \text{minutes}\left\{{}^{90}_{150}\right\} \quad (4)$$
$$t_1 = t\langle \tfrac{\text{s}}{\text{min}}\rangle \quad (5)$$
$$= \langle 119\rangle\langle 60\rangle$$
$$= 7140\ \text{s}$$
$$\langle \text{s}\rangle = \langle \text{minutes}\rangle\langle \text{s/min}\rangle \quad \text{units}$$
$$\omega = \frac{2.0\pi}{t_1} \quad (6)$$

$$= \frac{2.0\langle 3.1415926\rangle}{\langle 7140\rangle}$$
$$= 0.000879998 \text{ rad/s}$$
$$\langle \text{rad/s}\rangle = \frac{\langle\rangle\langle\rangle}{\langle \text{s}\rangle} \quad \text{units}$$
$$r = \left(\frac{GM_{earth}}{\omega^{2.0}}\right)^{1.0/3.0} \quad (7)$$
$$= \left(\frac{\langle 6.67259\times10^{-11}\rangle\langle 5.98\times10^{24}\rangle}{\langle 0.000879998\rangle^{2.0}}\right)^{1.0/3.0}$$
$$= 8.01698\times10^{6} \text{ m}$$
$$\langle \text{m}\rangle = \left(\frac{\langle \text{Nm}^2/\text{kg}^2\rangle\langle \text{kg}\rangle}{\langle \text{rad/s}\rangle^{2.0}}\right)^{\langle\rangle/\langle\rangle} \quad \text{units}$$
$$g_1 = \frac{GM_{earth}}{r^{2.0}} \quad (8)$$
$$= \frac{\langle 6.67259\times10^{-11}\rangle\langle 5.98\times10^{24}\rangle}{\langle 8.01698\times10^{6}\rangle^{2.0}}$$
$$= 6.20832 \text{ m/s}^2$$
$$\langle \text{m/s}^2\rangle = \frac{\langle \text{Nm}^2/\text{kg}^2\rangle\langle \text{kg}\rangle}{\langle \text{m}\rangle^{2.0}} \quad \text{units}$$

Satellite Periods

14:03, trigonometry, numeric, > 1 min.

013

Two satellites A and B, where B has twice the mass of A, orbit the earth in circular orbits. The distance of satellite B from the earth's center is twice the distance of satellite A from the earth's center. What is the ratio of the orbital period of satellite B to that of satellite A?

1. 1/8

2. $\sqrt{8}$ **correct**

3. $\sqrt{1/8}$

4. 1/2

5. $\sqrt{1/2}$

6. $\sqrt{2}$

7. 1/4

8. 1

9. 8

10. 2

Explanation:

According to Kepler's third law, the square of the orbital period is proportional to the cube of the orbital radius. Therefore,

$$\frac{T_A^2}{R_A^3} = \frac{T_B^2}{R_B^3}.$$

Therefore,

$$\frac{T_B}{T_A} = \left(\frac{R_B}{R_A}\right)^{3/2} = 2^{3/2} = \sqrt{8}.$$