Partial Differential Equations (PDEs)

Introductory Generalities

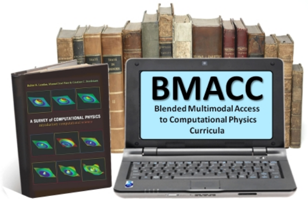
Rubin H Landau

Sally Haerer, Producer-Director

Based on *A Survey of Computational Physics* by Landau, Páez, & Bordeianu

with Support from the National Science Foundation

Course: **Computational Physics II**



PDE Generalities

When Ordinary?, When Partial?

- Field $U(x, y, z, t)$ describe
- Physical quantities (T, P) vary continuously in x & t
- Changes in $U(x, y, z, t)$ affect U nearby
- $\Rightarrow$ Dynamic equations in partial derivatives: PDEs
- *vs* Ordinary differential equations

General Forms of PDES

$$A\frac{\partial^2 U}{\partial x^2} + 2B\frac{\partial^2 U}{\partial x \partial y} + C\frac{\partial^2 U}{\partial y^2} + D\frac{\partial U}{\partial x} + E\frac{\partial U}{\partial y} = F$$

Elliptic	Parabolic	Hyperbolic
$d = AC - B^2 > 0$	$d = AC - B^2 = 0$	$d = AC - B^2 < 0$
$\nabla^2 U(x) = -4\pi\rho(x)$	$\nabla^2 U(\mathbf{x}, t) = a\,\partial U/\partial t$	$\nabla^2 U(\mathbf{x}, t) = c^{-2}\partial^2 U/\partial t^2$
Poisson's	Heat	Wave

- Elliptic PDE: All 2nd O, same signs
- Parabolic PDE: 1st-O derivative + 2nd O
- Hyperbolic PDE: All 2nd O, opposite signs

Relation Boundary Conditions & Uniqueness

Boundary Condition	Elliptic (*Poisson*)	Hyperbolic (*Wave*)	Parabolic (*Heat*)
Dirichlet open S	Under	Under	*Unique & stable (1-D)*
Dirichlet closed S	*Unique & stable*	Over	Over
Neumann open S	Under	Under	*Unique & Stable (1-D)*
Neumann closed S	*Unique & stable*	Over	Over
Cauchy open S	Nonphysical	*Unique & stable*	Over
Cauchy closed S	Over	Over	Over

- Initial Conditions $(x(0), x'(0), \ldots)$: always requisite
- Boundary Conditions: sufficient for unique solution
- Dirichlet: value on surrounding closed S
- Neumann: value normal derivative on surrounding S
- Cauchy: both solution & derivative on closed boundary

Solving PDEs & ODEs Is Different

No Standard PDE Solver

- Standard form for ODE

$$\frac{d\mathbf{y}(t)}{dt} = \mathbf{f}(\mathbf{y}, t)$$

- Single independent variable $\Rightarrow$ standard algorithm (`rk4`)
- PDEs: several independent variables: $\rho(x, y, z, t)$
- $\Rightarrow$ Complicated: algorithm simultaneously, independently
- More variables $\Rightarrow$ more equations $\Rightarrow$ > ICs, BCs
- Each PDE: particular BCs $\Rightarrow$ particular algorithm