*Trial-and-Error Searching**
(Part II, N-Dimensions)

(science at last!)

Rubin H Landau
With
Sally Haerer and Scott Clark

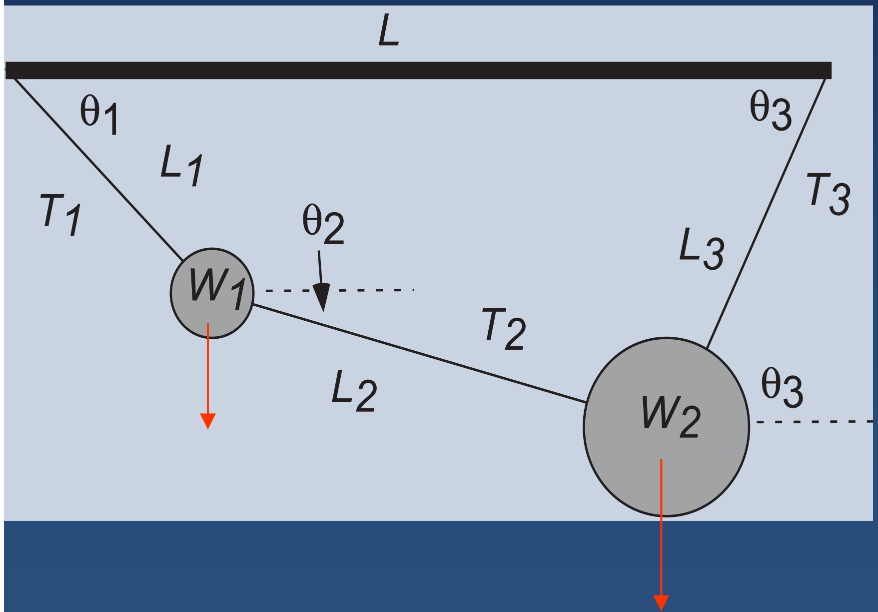
Computational Physics for Undergraduates
BS Degree Program: Oregon State University

"*Engaging People in Cyber Infrastructure*"
Support by EPICS/NSF & OSU

Weights on a String; Roots of Simultaneous Nonlinear Equations

1. Problem (6 unknowns):

$T_i = ?,\quad \theta_i = ?,\quad i = 1, 6$



(simple can be hard)

2. Geometric constraints:

$$L_1 \cos\theta_1 + L_2 \cos\theta_2 + L_3 \cos\theta_3 = L \quad (1)$$
$$L_1 \sin\theta_1 + L_2 \sin\theta_2 - L_3 \sin\theta_3 = 0 \quad (2)$$

3. $\sum$ forces$_{x,y}$ = 0:

$$T_1 \sin\theta_1 - T_2 \sin\theta_2 - W_1 = 0_{y,1} \quad (3)$$
$$T_1 \cos\theta_1 - T_2 \cos\theta_2 = 0_{x,1} \quad (4)$$
$$T_2 \sin\theta_2 + T_3 \sin\theta_3 - W_2 = 0_{y,2} \quad (5)$$
$$T_2 \cos\theta_2 - T_3 \cos\theta_3 = 0_{x,2} \quad (6)$$

4. Trigonometry (6 → 9 unknowns):

$$\sin^2\theta_i + \cos^2\theta_i = 1, \quad i = 1, 2, 3 \;\; (7-9)$$

Multi-D Newton-Raphson Search

- No analytic solution
- Can solve *f (x) = 0*
- 9 simultaneous nonlinear equations
- Rename variables to vector [x]

$$[\mathbf{x}] = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{bmatrix} = \begin{bmatrix} \sin\theta_1 \\ \sin\theta_2 \\ \sin\theta_3 \\ \cos\theta_1 \\ \cos\theta_2 \\ \cos\theta_3 \\ T_1 \\ T_2 \\ T_3 \end{bmatrix},$$

$$\begin{aligned}
f_1(\mathsf{x}) &= 3x_4 + 4x_5 + 4x_6 - 8 = 0 \\
f_2(\mathsf{x}) &= 3x_1 + 4x_2 - 4x_3 = 0 \\
f_3(\mathsf{x}) &= x_7x_1 - x_8x_2 - 10 = 0 \\
f_4(\mathsf{x}) &= x_7x_4 - x_8x_5 = 0 \\
f_5(\mathsf{x}) &= x_8x_2 + x_9x_3 - 20 = 0 \\
f_6(\mathsf{x}) &= x_8x_5 - x_9x_6 = 0 \\
f_7(\mathsf{x}) &= x_1^2 + x_4^2 - 1 = 0 \\
f_8(\mathsf{x}) &= x_2^2 + x_5^2 - 1 = 0 \\
f_9(\mathsf{x}) &= x_3^2 + x_6^2 - 1 = 0
\end{aligned}$$

sin θ, cos θ = independent → f_7, f_8, f_9

Solve Matrix Equation for 9 Δx_j

(roots) $$f_i(x_1^n, x_2^n, \ldots, x_9^n) = 0, \qquad x_i^n = x_i^{n-1} + \Delta x_i,$$ (guess) (1)

$$f_i(x_1^n, x_2^n, \ldots, x_9^n) \simeq f_i(x_1^{n-1}, x_2^{n-1}, \ldots, x_9^{n-1}) + \sum_j^9 \frac{\partial f_i}{\partial x_j}(x_i^{n-1})\Delta x_j$$ (2)

(linear approx)

$$f_i(x_1^{n-1}, x_2^{n-1}, \ldots, x_9^{n-1}) + \sum_j^9 \frac{\partial f_i}{\partial x_j}(\{x_i^{n-1}\})\,\Delta x_j = 0,$$ (3)

(unknown)

- Matrix form: Standard form for linear equations

$$\begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_9 \end{bmatrix}_o + \begin{bmatrix} \partial f_1/\partial x_1 & \cdots & \partial f_1/\partial x_N \\ \partial f_2/\partial x_1 & \cdots & \partial f_2/\partial x_9 \\ & \vdots & \\ \cdots & \cdots & \partial f_9/\partial x_9 \end{bmatrix}_o \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_9 \end{bmatrix} = 0$$ (4)

(known) (unknowns)

$$\frac{\partial f_i}{\partial x_j} \simeq \frac{f_i(x_j + \Delta x_j) - f_i(x_j)}{\delta x_j}$$ (5)

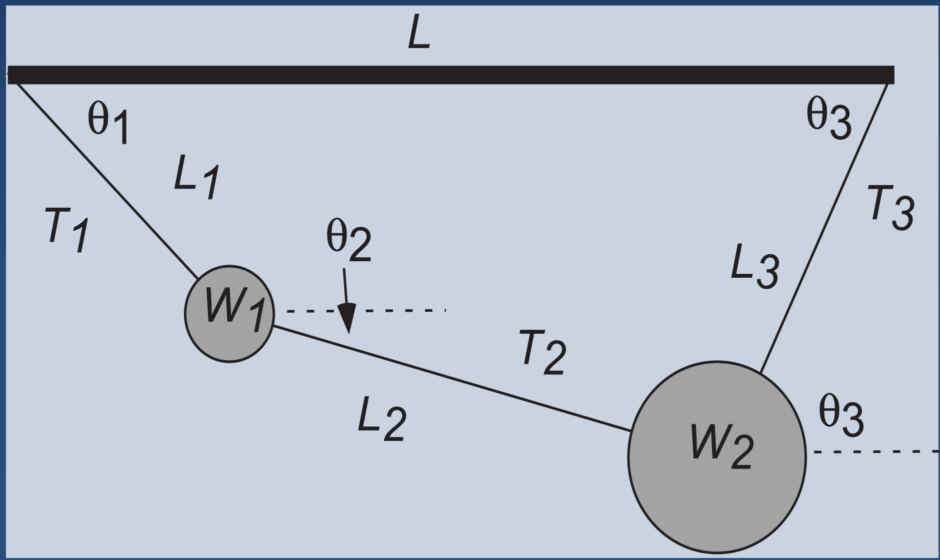
- Now use matrix library program (JAMA)

Assessment

1. Check reasonableness of W_1, W_2 solution
   a .various m, L
   b. deduced Tensions > 0, ≈ W
   c. deduced angles physical (sketch)

2. See how bad initial guess fails

3. * *3* masses (hard)



Relation of 1D and 9D Methods

- **1-D:**

$$\Delta x = -\frac{f}{f'} = -\frac{1}{f'} f \qquad (1)$$

- **N-D: Linear Equations:**

$$\begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_9 \end{bmatrix}_o + \begin{bmatrix} \partial f_1/\partial x_1 & \cdots & \partial f_1/\partial x_N \\ \partial f_2/\partial x_1 & \cdots & \partial f_2/\partial x_9 \\ & \vdots & \\ \cdots & \cdots & \partial f_9/\partial x_9 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_9 \end{bmatrix} = 0 \qquad (2)$$

- **Write solution as (formal)**

$$\begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_9 \end{bmatrix} = - \begin{bmatrix} \partial f_1/\partial x_1 & \cdots & \partial f_1/\partial x_N \\ \partial f_2/\partial x_1 & \cdots & \partial f_2/\partial x_9 \\ & \vdots & \\ \cdots & \cdots & \partial f_9/\partial x_9 \end{bmatrix}^{-1} \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_9 \end{bmatrix} \qquad (3)$$