Matrix Computations (cont)
Testing Matrix Calls

Exercises to keep handy

Before you try it big, try it small

Hard to get calling procedure perfect

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"*Engaging People in Cyber Infrastructure*"

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Try These!

1) Find the inverse

$$[A] = \begin{bmatrix} +4 & -2 & +1 \\ +3 & +6 & -4 \\ +2 & +1 & +8 \end{bmatrix}$$

2) Check in both directions

$$[A][A]^{-1} = [A]^{-1}[A] = [I]$$

3) Verify

$$[A]^{-1} = \frac{1}{263} \begin{bmatrix} +52 & +17 & +2 \\ -32 & +30 & +19 \\ -9 & -8 & +30 \end{bmatrix}.$$

Try These (cont)!

4) Same [A], solve 3 sets simultaneous linear equations

$$[A]\vec{x} = \vec{b}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Know vector [b], Solve for [x], for 3 known [b]'s:

$$b_1 = \begin{bmatrix} +12 \\ -25 \\ +32 \end{bmatrix}, \quad b_2 = \begin{bmatrix} +4 \\ -10 \\ +22 \end{bmatrix}, \; b_3 = \begin{bmatrix} +20 \\ -30 \\ +40 \end{bmatrix}.$$

Solution

$$x_1 = \begin{bmatrix} +1 \\ -2 \\ +4 \end{bmatrix}, \; x_2 = \begin{bmatrix} +0.312 \\ -0.038 \\ +2.677 \end{bmatrix}, \; x_3 = \begin{bmatrix} +2.319 \\ -2.965 \\ +4.790 \end{bmatrix}.$$

Some Eigenvalue Problems $[A]\vec{x} = \lambda\vec{x}$

5)

$$[A] = \begin{matrix} \alpha & \beta \\ -\beta & \alpha \end{matrix}\Bigg]$$

(normalization = ?)

$$\vec{x}_{1,2} = [\, +1 \text{¨} \; i \,], \qquad \lambda_{1,2} = \alpha \text{¨} \; i\beta$$

6) Multiple eigenvalues (degeneracy)

$$\mathbf{A} = \begin{bmatrix} -2 & +2 & -3 \\ +2 & +1 & -6 \\ -1 & -2 & +0 \end{bmatrix}$$

$$\Rightarrow \quad \lambda_1 = 5 \qquad \vec{x}_1 = \frac{1}{\sqrt{6}} \begin{bmatrix} -1 \\ -2 \\ +1 \end{bmatrix}$$

$\Rightarrow \quad \lambda_2 = \lambda_3 = -3$ Double root $\Rightarrow$ combo of eigenvectors

$$\vec{x}_2,\ \vec{x}_3 = \frac{1}{\sqrt{5}} \begin{bmatrix} -2 \\ +1 \\ +0 \end{bmatrix}, \quad \frac{1}{\sqrt{10}} \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}.$$

Solve N = 100 Linear Equations for $\vec{y}$

$$\begin{aligned} a_{11}y_1 + a_{12}y_2 + \cdots + a_{1N}y_N &= b_1, \\ a_{21}y_1 + a_{22}y_2 + \cdots + a_{2N}y_N &= b_2, \\ &\cdots \\ a_{N1}y_1 + a_{N2}y_2 + \cdots + a_{NN}y_N &= b_N \end{aligned}$$

$$[a]\vec{y} = \vec{b}$$

[a] = Hilbert matrix

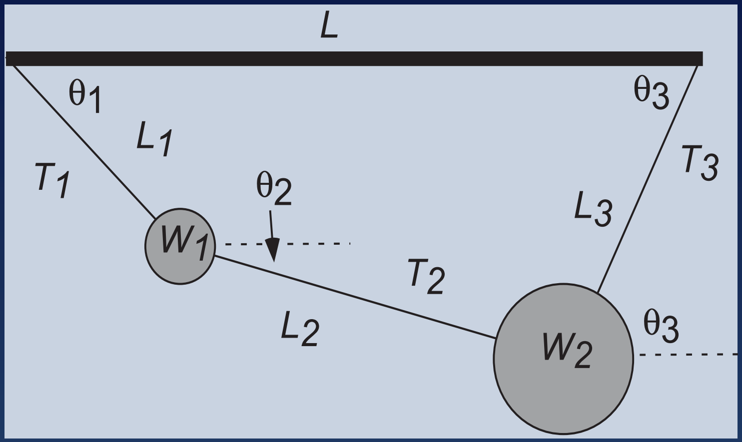
$$[a_{ij}] = \left[\frac{1}{i+j-1}\right] = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \cdots & \frac{1}{100} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \cdots & \frac{1}{101} \\ \ddots & & & & & \\ \frac{1}{100} & \frac{1}{101} & \cdots & & \cdots & \frac{1}{199} \end{bmatrix}$$

[b] = 1st row

$$\mathbf{b} = \left[\frac{1}{i}\right] = \begin{bmatrix} 1 \\ \frac{1}{2} \\ \frac{1}{3} \\ \ddots \\ \frac{1}{100} \end{bmatrix}$$

Verify $\Rightarrow \begin{bmatrix} y_1 \\ y_2 \\ \ddots \\ y_N \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \ddots \\ 0 \end{bmatrix}$

At last! Solve Masses on String



$$\begin{bmatrix} \partial f_1/\partial x_1 & \cdots & \partial f_1/\partial x_N \\ \partial f_2/\partial x_1 & \cdots & \partial f_2/\partial x_9 \\ & \vdots & \\ \cdots & \cdots & \partial f_9/\partial x_9 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_9 \end{bmatrix} = - \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_9 \end{bmatrix}_o$$

1. Is solution physical?
2. Try various weights & lengths
3. Deduced tensions > 0, proportional to weights?
4. sin θ, cos θ : sensible (sketch)?
5. Determine when initial guess not close enough.
6. Solve similar 3-m problem*.