Fitting: Interpolation

"data fitting" in some sense
interpolate table of data
later: "best" fit to data

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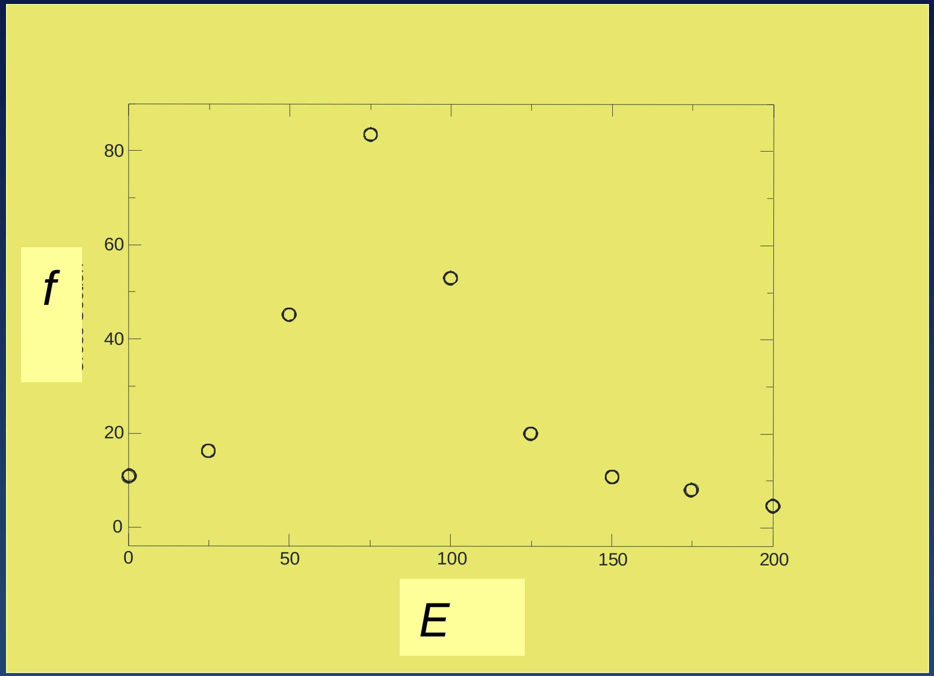
Problem: Interpolate Data

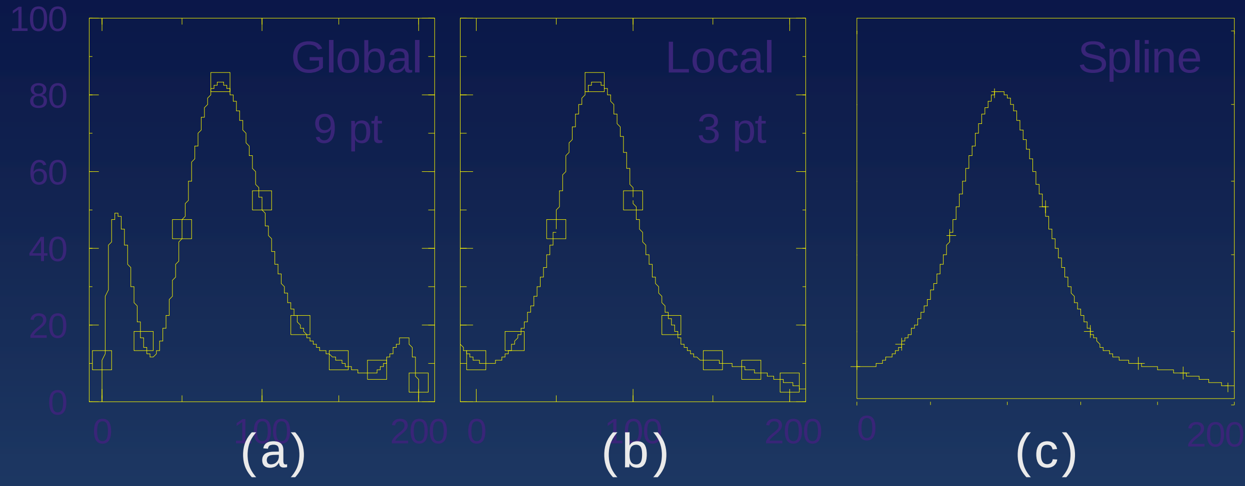
E	f(E)
0	10.6
25	16.0
50	45.0
75	83.5
100	52.8
125	19.9
150	10.8
175	8.25
200	4.7



- Tables = hard
- Look at data

- Determine *f (E)* for E's between entries
- Make table of numbers into a function
- Determine full width at half maximum

Method 1: Fit polynomial to Small Region



$$f(x) \simeq a_0 + a_1 x + a_2 x^2 + \cdots + a_{n-1} x^{n-1}, \qquad (x \simeq x_i) \tag{1}$$

- Local: different poly each interval
- Low *O* polynomial (small *n*), but many: (b) (c)
- Full region: global fit higher poly (large *n*)
  - probably bad representation (a)
  - even if perfect fit!
- Do not extrapolate

n Point Lagrange Interpolation = *n-1*th *O* Poly

$$f(x) \simeq f_1\lambda_1(x) + f_2\lambda_2(x) + \cdots + f_n\lambda_n(x) \quad (1)$$

i	f_i
1	-12
2	-12
3	-24
4	-60

$$\lambda_i(x) = \prod_{j(\neq i)=1}^{n} \frac{x - x_j}{x_i - x_j} = \frac{x - x_1}{x_i - x_1}\frac{x - x_2}{x_i - x_2}\cdots\frac{x - x_n}{x_i - x_n} \quad (2)$$

- E.g. 4-points, 3rd degree polynomial to table:

$$f(x) = (-12)\frac{(x-1)(x-2)(x-4)}{(0-1)(0-2)(0-4)} + (-12)\frac{x(x-2)(x-4)}{(1-0)(1-2)(1-4)} \quad (3)$$

$$+(-24)\frac{x(x-1)(x-4)}{(2-0)(2-1)(2-4)} + (-60)\frac{x(x-1)(x-2)}{(4-0)(4-1)(4-2)}$$

$$\Rightarrow f(x) = x^3 - 9x^2 + 8x - 12 \quad (4)$$

- Check: $f(4) = 4^3 - 94^2 + 32 - 12 = -60$ (5)

Method 2: Cubic Splines

$$f(x) \simeq f_i(x) = f_i + f_i^{(1)}(x - x_i) + \frac{1}{2} f_i^{(2)}(x - x_i)^2 + \frac{1}{6} f_i^{(3)}(x - x_i)^3$$

- 3rd degree polynomial in each interval: most eye-pleasing
- Continuous 1st, 2nd derivatives
- Spline: flexible drafting tool
- Guaranteed: integrable, differentiable (*F* =-*dV*/*dx*)
- Complex procedure: easy for computers
- Popular for graphics

Real & Digital Demos!

Try out Applet

Spline.html

Cubic Splines Implementation

$$f(x) \simeq f_i(x) = f_i + f_i^{(1)}(x - x_i) + \tfrac{1}{2} f_i^{(2)}(x - x_i)^2 + \tfrac{1}{6} f_i^{(3)}(x - x_i)^3 \qquad (1)$$

1. Match f_i adjoining intervals Eq(2)
2. Match $f^{(1)}$, $f^{(2)}$ adjoining intervals Eq(3)
3. Match $f^{(3)}$ (forward difference) Eq(4)
4. Boundary Conditions: input $f^{(1)}$
   - natural spline: $f^{(2)}(a) = f^{(2)}(b) = 0$

$$f_i(x_{i+1}) = f_{i+1}(x_{i+1}), \qquad i = 1, N-1 \qquad (2)$$

$$f_{i-1}^{(1)}(x_i) = f_i^{(1)}(x_i), \qquad f_{i-1}^{(2)}(x_i) = f_i^{(2)}(x_i) \qquad (3)$$

$$f_i^{(3)} \simeq \frac{f_{i+1}^{(2)} - f_i^{(2)}}{x_{i+1} - x_i} \qquad (4)$$

Implementation: SplineAppl.java

```
public class SplineAppl  {
   public static void main(   {
     double x[] = { 0., 1.2, 2.5, 3.7, 5., 6.2, 7.5, 8.7, 9.9 };         // input
     double y[] = {0. ,0.93,. 6, -0.53, -0.96, -0.08, 0.94,0.66,-0.46};
     yp1 = (y[1] - y[0]) / (x[1] - x[0]) - (y[2] - y[1]) / (x[2] - x[1])
            + (y[2] -   y[0]) / (x[2]-x[0]);
     ypn = (y[n-1] - y[n-2]) / (x[n-1]-x[n-2]) - (y[n-2]-y[n-3])
                 / (x[n-2] - x[n-3]) + (y[n-1] - y[n-3]) / (x[n-1] - x[n-3]);
     y2[0] = u[0] = 0. ;                                                  // Natural
     for ( i=1;  i <= n-2;  i++ )  {      ...}                 // Decomposition loop
    qn = un = 0. ;                                                     // Natural
    y2[n-1] = (un-qn*u[n-2]) / (qn*y2[n-2] + 1.);
    for ( k = n-2;  k>= 0; k--)  y2[k] = y2[k] * y2[k + 1] + u[k];
    for ( i=1;  i <= Nfit;  i++ )  {                        }     // initialization
    xout = x[0] + ( x[n-1] - x[0] ) * (i - 1) / (Nfit);
     ...
    yout = ( a*y[klo] + b*y[khi] + ( (a*a*a-a)*y2[klo]
             +  ( b*b*b -  b ) * y2[khi]) * (h*h)/6. );
      } } }
```

Go try these out!