Fractals & Statistical Models by Example II

Nonlinear Computational Science in Action

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Based on *A Survey of Computational Physics* by Landau, Páez, & Bordeianu

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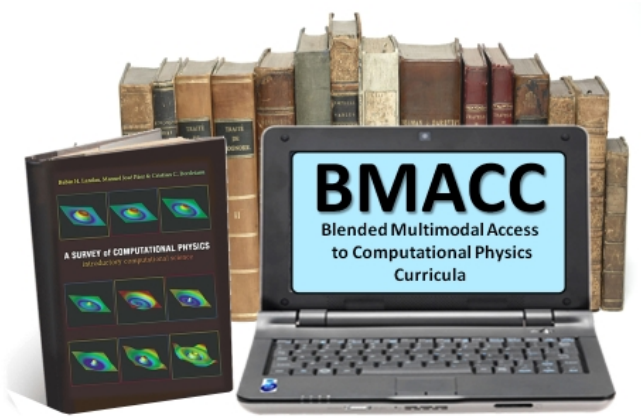
Course: **Computational Physics II**



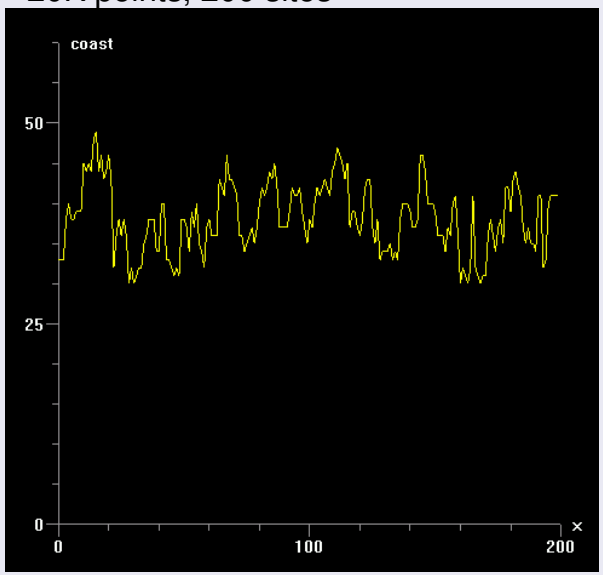
Break Over, Back to Work

E.G. 3: Ballistic Deposition of Film

CODE

Grow Film Particle-by-Particle

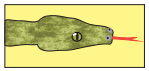
20K points, 200 sites



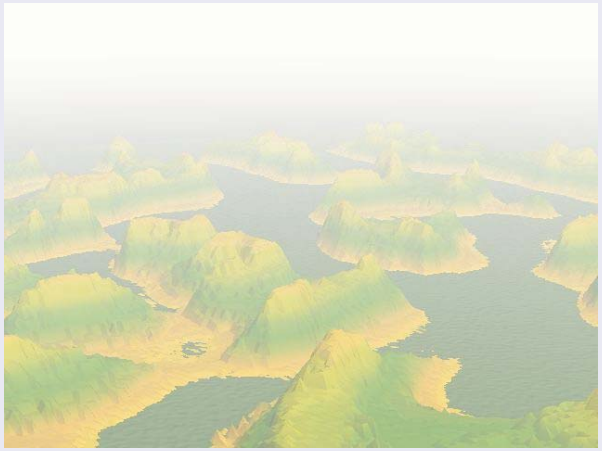
- Hot filament $\Rightarrow$ random
- Evaporated particles stick
- Line L, 200 sites
- $0 \leq x_i = r_i \leq L$
- Sticks, grows if h > neighbors
- Fills in hole $\Rightarrow$ max neighbors

$$h_r = \begin{cases} h_r + 1, & \text{if } h_r \geq h_{r-1}, \quad h_r > h_{r+1}, \\ \max[h_{r-1}, h_{r+1}], & \text{if } h_r < h_{r-1}, \quad h_r < h_{r+1}. \end{cases}$$

Length of the British Coastline?



1967 Mandelbrot: "How long is the coast of Britain?"

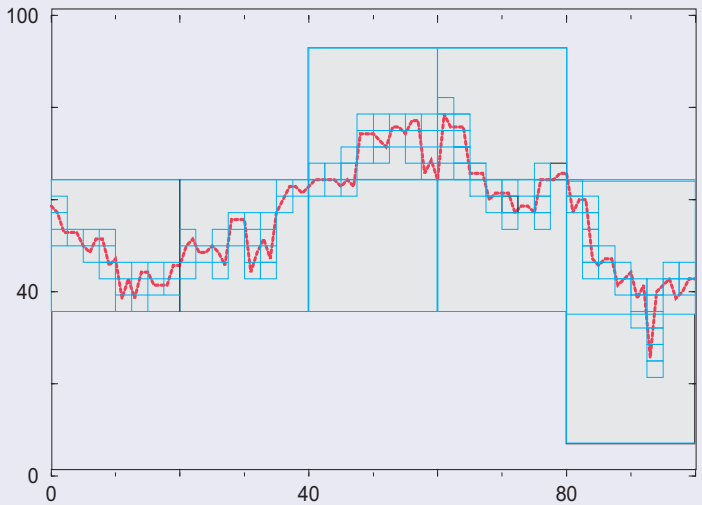


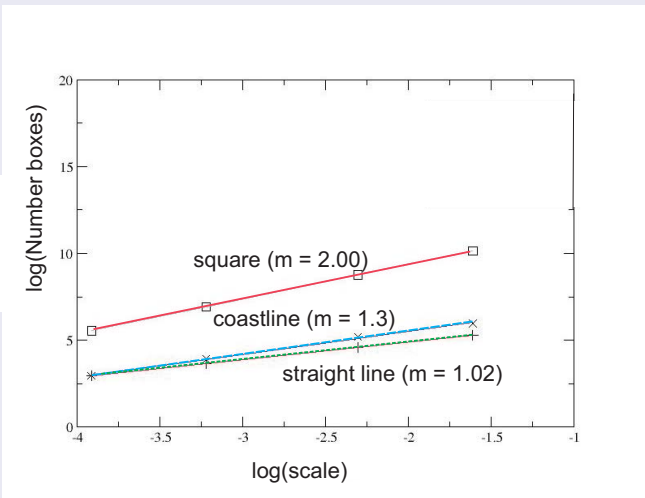
- Colorado, Wyoming = easy
- 2D = finite perimeter
- Coasts = geographic
- Coasts $\neq$ geometric
- Appear self similar
- Model coast = fractal

- $\Rightarrow$ Length = perimeter
- **Problem:** d_f = ?
- Length self similar = ?
- Map maker: ruler r
- $\Rightarrow L_r \simeq Nr$
- Geometric: $L_r \rightarrow L$
- Nature: $L(r) = Mr^{1-d_f}$
- $d_f > 1$, $L \rightarrow_{r \rightarrow 0} \infty$
- Finite size, ∞ perimeter
- Math $\neq$ physics: quantum, Compton sizes

Box Counting Algorithm

Cover Perimeter L with boxes Length $r \to 0$ (Area, Volume)





$$N(r) = C\left(\frac{1}{r}\right)^{d_f} = C'\, s^{d_f} \qquad (1)$$

$$\log N(r) = \log C - d_f \log r \qquad (3)$$

$$s \propto \frac{1}{r} = \text{scale} \qquad (2)$$

$$\Rightarrow \quad d_f = -\lim_{r \to 0} \frac{\Delta N(r)}{\Delta r} \qquad (4)$$

Get to Work!

Do this yourself please.