Data Loss Modeling and Analysis in Partially-Covered Delay-Tolerant Networks

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***Abstract*— We characterize some fundamental performance limits of partially covered, intermittently connected, delay-tolerant networks (DTNs) that are comprised of a hybrid mix of mobile and static nodes (i.e., access points). Specifically, we derive theoretic bounds on the expected hitting time between two consecutive visits of a mobile node to access points for both the square and hexagonal access point deployment structures. For each of these two models, we use the Poisson Clumping technique to derive theoretic bounds on the data loss rate, under the assumption that a mobile node has a finite buffer that overflows after a certain amount of time. Based of these obtained results, we provide asymptotic analysis of the expected hitting time in these partially covered DTNs. We test the applicability of the Poisson Clumping technique, and our hitting time results, with simulations.**

I. Introduction

Delay-tolerant networks (DTNs) are a class of networks that are, by nature, partially covered or intermittently connected. As a consequence, traditional end-to-end routing paradigms may not be the most effective in delivering data across nodes, due to the absence of multi-hop paths. In such sparse networks data delivery is only possible through the *store-carry-and-drop* routing approach, which relies on node mobility to carry data. Therefore, applications supported by these networks are typically delay insensitive/tolerant, as data packets are expected to experience some delay before reaching their destinations. DTNs have recently attracted significant interest in the context of mobile sensor networks (e.g., event/data collection [1–3], animal monitoring/tracking [4, 5], mobile ubiquitous LAN extensions [6, 7]), and continue to find new applications, for instance in vehicular networks (e.g., [8–10]).

Due to their importance and wide range of applications, there has been considerable research focus on DTNs, ranging from protocol design [11–14] to connectivity analysis [15–17] and delay modeling and characterization [16, 18–22]. The work in [16] uses continuum percolation theory [23] to show how delays in large wireless networks scale with the Euclidean distance between the sender and the receiver. Speed of information propagation has recently also been studied analytically for static [18, 19] as well as mobile [20–22] DTNs. The authors in [19] derived upper bounds on the maximum propagation speed in large-scale wireless networks, and those in [21] derived analytic upper bounds on information delay in large-scale DTNs with possible mobility and intermittent connectivity. Network connectivity has also been intensively studied, but mostly in the context of large-scale networks only. In [15], the authors derived an upper bound on the delay sufficient for disconnected networks to become connected through node mobility. The work in [16] derived the minimum node density required to ensure connectivity in large static networks.

In contrast, this work aims at modeling and characterizing some fundamental properties of partially covered, intermittently connected DTNs that are comprised of a hybrid mix of static and mobile nodes. Due to the limited coverage (the network is disconnected in the traditional sense), data delivery is only possible through mobile nodes, which store and carry data until they come close to a fixed node (henceforth referred to as an access point), where data is then immediately and fully downloaded. The focus of this work is then on partially connected networks, where both the node density and the coverage ratio are assumed to be low.

We investigate two two-dimensional (2-D) access point deployment structures: the square grid and the hexagonal grid. We model the path of a mobile node in both structures as a Brownian Motion and use standard results form this theory and the Poisson Clumping Heuristic [24] as the theoretical basis for deriving all of our analytic results. The main conditions for application and the results of this technique are given in Appendix A.

Specifically, we derive analytic bounds on the expected *hitting time*: the average time a mobile node spends between two consecutive visits to access points, in both scenarios/models, corresponding to the amount of time a mobile node spends without connectivity. Additionally, we provide an asymptotic analysis of the expected hitting time in these partially covered DTNs. Assuming a mobile node has a fixed, finite amount of memory and data is overwritten and lost when the buffer is full, the theoretical bounds on the hitting time allow us to derive theoretical bounds on the mean data loss rate.

More concisely, our contributions in this paper are:

- Development of mathematical models of partially covered, intermittently connected DTNs, consisting of both mobile and static nodes, by applying the Poisson Clumping technique.
- Derivation of a simple, closed-form function for the amount of time a mobile node spends outside the coverage range of an access point.
- Derivation of theoretical bounds on the expected hitting time of a mobile node, and its expected data loss rates.
- Asymptotic analysis of the expected hitting time of a mobile node as the coverage ratio decreases to zero.

The rest of the paper is organized as follows. In Section II, we state our network model and overview the main results. We then establish a fundamental theorem, Theorem 3.1, in order to derive analytic bounds on the hitting times and data loss rates, which we provide in Section III. In Section IV, we test our theoretical hitting time bounds and Poisson Clumping technique via simulation. Finally, we conclude the paper in Section V. Appendix A briefly describes the assumptions needed to apply the Poisson Clumping technique, and its main results. The full proof of Theorem 3.1 is included in Appendix B.

II. Network Model

The focus of this work is on the performance modeling and analysis of delay-tolerant sensor networks for tracking and monitoring animals in open spaces, such as forests and grazing areas. These networks are typically partially-covered, delay-tolerant, and consist of access points laid out on a grid and a set of mobile nodes (e.g., animals). We assume that mobile nodes in these sensor networks continuously generate data at a rate, c, independent of one another. For example, an animal would be continuously recording its position and speed. Whenever a mobile node comes by an access point, it then immediately and completely downloads its generated data. Each mobile node is assumed to be equipped with a memory chip that has a buffer with limited size of B bits, and when the buffer is full the newly generated data is dropped. Let $\tau = B/c$, representing the amount of time required to overflow the buffer of the mobile node. Also, let ϵ denote the data loss rate threshold that mobile nodes can tolerate.

In these sensor networks, the access points typically cover the network only partially and the coverage ratio is relatively low. Here, mobile nodes rely on their mobility to maintain connectivity with the access points. As mobile nodes move, they will eventually traverse an area covered by an access point, allowing them to download all of their buffered data. Throughout this paper, we assume that the *coverage ratio*[1] is low; all the mathematical analysis and derivation provided in this paper depend on this assumption.

[1]The coverage ratio is defined as the fraction of the area covered by access points' communication ranges to that of the network area.

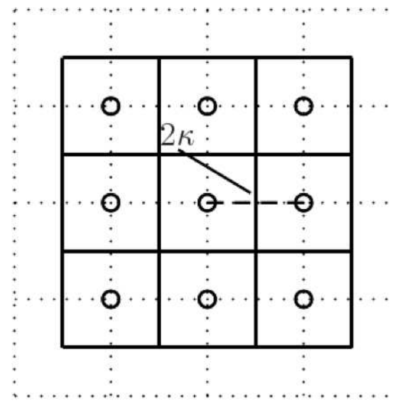

Fig. 1. In the square grid deployment structure, each access point has a communication disk of radius ρ surrounding it, and is distance 2κ away from four other access points. We draw a square about the access points, representing all those points closest to each specific access point.

We consider node deployment structures where access points are placed via a grid structure, and mobile nodes are free to move within the plane – their paths are modeled by 2-D Brownian Motion. In this paper, we study two node deployment structures: the square grid and the hexagonal grid. In the square grid structure, shown in Fig. 1, each access point is surrounded by four access points from each direction, each 2κ distance away. We can identify an arbitrary point's closest access point by drawing a square of side length 2κ about each access point. We assume that each access point is surrounded by a communication disk of radius ρ, yielding node density ν and coverage ratio η equal to $1/(4\kappa^2)$ and $\pi\rho^2/(4\kappa^2)$ respectively.

In the hexagonal grid structure, shown in Fig. 2, access points are placed in the plane to form a hexagonal grid – each acces point is surrounded by six access points in each direction, each 2κ away. We identify an arbitrary point's closest access point by drawing a hexagon around each access point. We assume that each access point is surrounded by a small communication disk of radius ρ, and that each hexagon has *apothem* κ, where the *apothem* is defined to be the length of the shortest line from the center to an edge (the radius is the length of the longest such line). In the hexagonal grid structure, the node density ν and the coverage ratio η can be expressed as $1/(2\sqrt{3}\kappa^2)$ and $\pi\rho^2/(2\sqrt{3}\kappa^2)$.

III. Performance Analysis

We are interested in calculating the amount of time, on average, that a mobile node spends outside of an access point's communication range. Before proceeding with our derivation and analysis, we observe that if the path of a mobile node reaches the edge of a square or hexagon

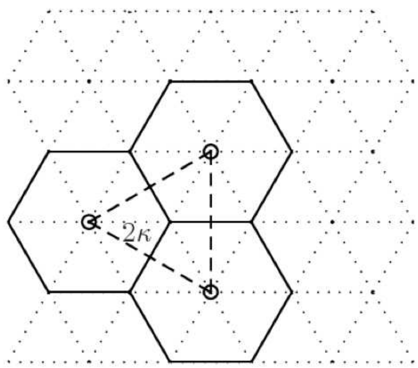

Fig. 2. In the hexagonal grid deployment structure, each access point has a communication disk of radius ρ surrounding it, and is distance 2κ away from six other access points. We draw a hexagon about the identified access points, representing all those points closest to each specific access point.

it makes no difference whether it returns to the same communication disk or proceeds to another one, in terms of the amount of time spent outside of communication range, so the problem is symmetric. Hence, studying one square in the case of square grid deployment, or one hexagon in the case of hexagonal grid deployment, suffices.

We note that we model the mobile node's movement with 2-D Brownian Motion and that the *Bessel process*, which is the one-dimensional process obtained from looking only at the Euclidean distance of the Brownian Motion, r, from the origin, has outward drift given by $\mu(r) = 1/(2r)$. This implies that it is more likely for a Brownian Motion to cross an outer circle of radius R than to return to a small circle centered at the origin. This, together with the aforementioned symmetry, means that we may calculate the hitting time of a general mobile node by looking at just one access point (surrounded by either a square or hexagonal boundary) and calculate the time it takes the mobile node to leave from the edge of a communication disk, hit the edge of the surrounding square or hexagon, and then return to the communication disk. This is an approximation of course, but one whose accuracy increases as the radius of the communication disk, ρ, decreases. As we study only sparsely covered networks – those where the communication disks are relatively small – we proceed with the approximation.

A communication disk has a circular shape and our boundary region has either a square or hexagonal shape, so we do not directly calculate the hitting time. Instead, we find a lower bound on the hitting time by considering a mobile node traveling in the circle inscribed in the square or hexagon with radius κ and centered at our access point. Similarly, we find an upper bound on the hitting time by considering a mobile node traveling in the outer circle that circumscribes the square or hexagon. In the case of the square grid, this circle has radius $\sqrt{2}\kappa$ and in the case of the hexagonal grid, this circle has radius $2\kappa/\sqrt{3}$. For the square boundary region, this process is depicted in Fig. 3, and Fig. 4 similarly demonstrates this in the case of the hexagonal boundary region.

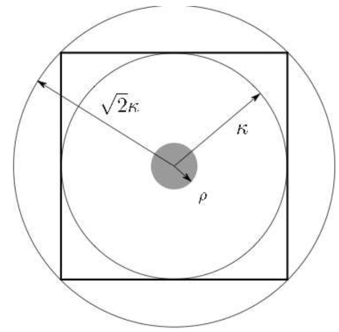

Fig. 3. We approximate the square by two circles, one inscribed in the square, and the other circumscribing the square, in order to calculate bounds on the hitting time.

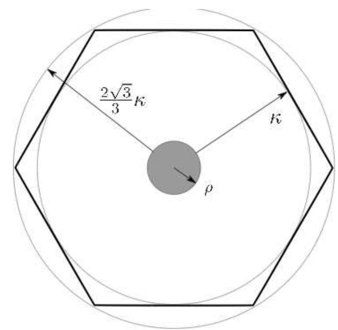

Fig. 4. We approximate the hexagon by two circles, one inscribed in the hexagon, and the other circumscribing the hexagon, in order to calculate bounds on the hitting time.

A. Hitting Times for Centered Disks Case

In this section we develop the main theorem of this paper, which is an equation for the hitting time of a mobile node, as modeled by a Bessel process (the stochastic process described by the Euclidean distance of a 2-D Brownian Motion from the origin), on a communication disk of radius ρ centered in a disk of radius $R > \rho$. We assume that a mobile node starts inside the disk of radius R, and it bounces back when it hits the boundary. This setup is shown in Fig. 5.

The proof is supplied in the Appendix. In short, we compute the result by applying standard results of Brownian Motion to calculate two components of the hitting time: the time it takes a mobile node to leave the communication disk and hit the boundary, and the time it takes a mobile node to hit the communication disk, starting at the edge of the disk of radius R. The result is given in the following theorem:

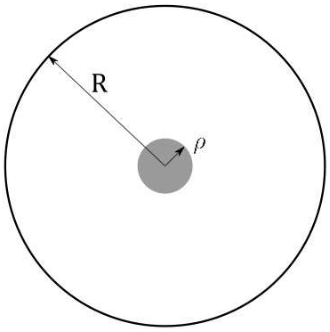

Fig. 5. Communication disk of radius ρ centered in a disk of radius R.

Theorem 3.1: For a communication disk of radius ρ centered in the region disk of radius R, the expected hitting time is $h(\rho, R)$, where

$$h(x, y) = y^2 \ln \left| \frac{y}{x} \right| \tag{1}$$

Proof: See Appendix B. ■

B. Hitting Times

We now use Theorem 3.1 to provide upper and lower bounds on the expected hitting times when considering square and hexagonal grid deployments. For a communication disk of radius ρ with access points separated from their nearest neighbors by distance 2κ, the expected hitting time is bounded as stated in the following propositions, depending on the type of deployment.

Proposition 3.2: For sufficiently small η, the expected hitting time $\bar{\sigma}$ in the square grid deployment is bounded above and below as

$$\frac{1}{8\nu} \ln \left(\frac{1}{4\nu\rho^2} \right) \leq \bar{\sigma} \leq \frac{1}{4\nu} \ln \left(\frac{1}{2\nu\rho^2} \right)$$

Proof: From Theorem 3.1, it follows that the expected hitting time is lower bounded by $h(\rho, \kappa)$ and upper bounded by $h(\rho, \sqrt{2}\kappa)$, where $h(x, y)$ is given in Eq. (1). Now, using the fact that the node density is $\nu = 1/(4\kappa^2)$ in the case of square grid deployment, replacing κ by its expression $1/(2\sqrt{\nu})$ leads to the stated result. ■

We apply the same theory to derive lower and upper bounds on the expected hitting time for the case of a communication disk of radius ρ with access points laid out on a hexagonal grid, separated from their nearest neighbors by distance 2κ.

Proposition 3.3: For sufficiently small η, the expected hitting time $\bar{\sigma}$ in the hexagonal grid deployment is bounded from above and below as

$$\frac{1}{4\sqrt{3}\nu} \ln \left(\frac{1}{2\sqrt{3}\nu\rho^2} \right) \leq \bar{\sigma} \leq \frac{1}{3\sqrt{3}\nu} \ln \left(\frac{2}{3\sqrt{3}\nu\rho^2} \right).$$

Proof: Applying Theorem 3.1, the expected hitting time is lower bounded by $h(\rho, \kappa)$ and upper bounded by $h(\rho, 2\kappa/\sqrt{3})$, where $h(x, y)$ is given in Eq. (1). Now given $\nu = 1/(2\sqrt{3}\kappa^2)$ in the case of square grid deployment, replacing κ^2 by its expression $1/(2\sqrt{3}\nu)$ gives the stated bounds. ■

To analyze the behavior of the expected hitting time as the coverage ratio decreases to zero, we use Propositions 3.2 and 3.3 to derive the following asymptotic behaviors.

Corollary 3.4: For a sufficiently small η, the expected hitting time in both grid deployments is $\Theta(\frac{\ln \nu}{\nu})$ as $\nu \to 0$. We interpret this to mean the expected hitting time grows asymptotically as fast as the ratio of the natural log of the node density to the node density, as the density approaches zero. That is, the expected time increases asymptotically as the density goes to zero, but not slower than $\ln \nu/\nu$.

C. Data Loss Rates

Recall that the buffer of each mobile node is of size B bits, and that it takes τ seconds to overflow the buffer, resulting in data packet drop/loss. The following results for the square and hexagonal grid deployments provide upper and lower bounds on the data packet loss given the design parameter τ.

Proposition 3.5: For sufficiently small η in the square grid, a mobile node experiences an average data loss rate $\bar{\epsilon}$ satisfying

$$\frac{\eta}{1-\eta} - \frac{8\nu\tau}{\ln(4\nu\rho^2)} \leq \ln \bar{\epsilon} \leq \frac{\eta}{1-\eta} - \frac{4\nu\tau}{\ln(2\nu\rho^2)}.$$

Proof: For the purpose of our Brownian Motion model, it suffices to consider the motion of a mobile node on a single square of the grid. Let C be the random variable representing the amount of time the mobile node spends in the communication disk and let T be the random variable representing the total amount of time spent in the grid. For small coverage, we approximate $\mathbb{E}C/(\mathbb{E}C + \bar{\sigma}) \approx \eta$. Simple algebra yields $\mathbb{E}C \approx \bar{\sigma}/(1/\eta - 1)$.

Buffer overflow occurs when T, minus the time spent in the coverage area, $\mathbb{E}C$, exceeds τ. Calculating the probability of this event, $P(T - \mathbb{E}C > \tau) = P(T > \bar{\sigma}/(1/\eta - 1)) + \tau)$, and, since we assume the hitting time to have an exponential distribution with parameter $1/\bar{\sigma}$,

$$P(T > \mathbb{E}C + \tau) = \exp \left\{ -\left(\frac{\eta}{1-\eta} + \frac{\tau}{\bar{\sigma}} \right) \right\}. \tag{2}$$

Substituting in the bounds derived in Proposition 3.2 and taking the natural logarithm yields the result for the square grid. ■

Proposition 3.6: For sufficiently small η in the hexagonal grid, a mobile node experiences an average data loss rate $\bar{\epsilon}$ satisfying

$$\frac{\eta}{1-\eta} - \frac{4\sqrt{3}\nu\tau}{\ln(2\sqrt{3}\nu\rho^2)} \leq \ln \bar{\epsilon} \leq \frac{\eta}{1-\eta} - \frac{3\sqrt{3}\nu\tau}{\ln(3\sqrt{3}/2\nu\rho^2)}$$

Proof: Eq. (2) does not depend on the shape of the boundary, so it likewise describes the probability of buffer

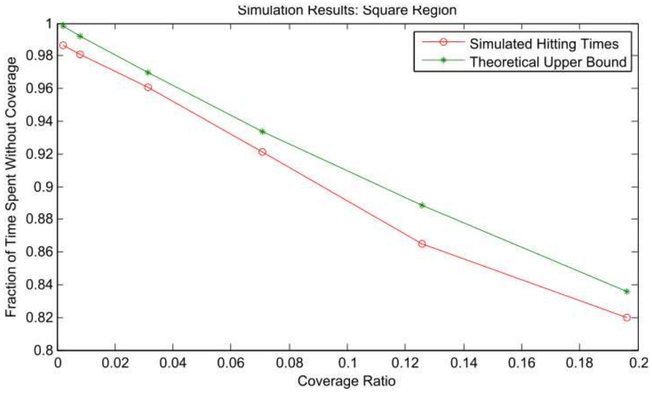

Fig. 6. The measured and the derived percentage (upper bound) of time a mobile node spends without communication coverage in the 2-d square region when varying the coverage ratio from 0.0019 to 0.19. “Fraction of Time Spent Without Coverage” is calculated as the ratio of time spent without coverage to the total cycle time (a cycle corresponds to a mobile node entering the communication disk, leaving the communication disk, reaching the endpoint, and then returning to the communication disk).

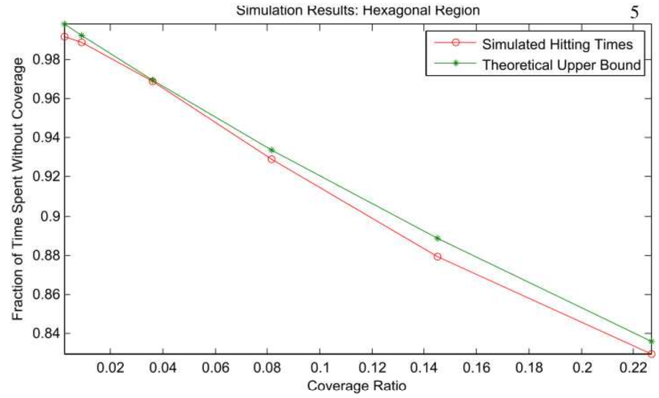

Fig. 7. The measured and the derived percentage of time a mobile node spends without communication coverage in the hexagonal boundary region when varying the coverage ratio from 0.0022 to 0.23.

overflow in the hexagonal grid deployment. We apply Eq. (2) by substituting in the bounds derived in Proposition 3.3. Taking the natural logarithm yields the result. ■

IV. Validation

In this section, we validate our theoretical hitting time bounds, as well as the assumption that the frequency distribution of hitting times follows an exponential distribution. We do so by running simulations of a Brownian Motion on the square and hexagonal grid, measuring the hitting times, and comparing the averages (for varying communication disk radii) with our derived theoretical results.

We use Matlab to simulate a 2-d Brownian Motion in a bounded, square region and in a bounded, hexagonal regions. We do so by simulating two normal random variables at each time step for the distance in the x- and y-directions the mobile node travels in a unit time interval. The simulation also stipulates that whenever the mobile node crosses the square or hexagonal boundary, its position is reverted to its position the last time it was within the boundary, effectively simulating a reflection at the boundary. We measure the hitting times as previously defined: the time it takes the Brownian Motion to leave a communication disk, having radius $\rho = 1$ and centered at the origin, hit the boundary of the square or hexagon, and return to the communication disk. In both regions, $\kappa = 20$.

We compare the recorded hitting times in our simulation with the theoretical bounds of the expected hitting times, given by Eq. (1). In both cases we normalize by the total amount of time spent in a cycle, where a cycle corresponds to a mobile node entering the communication disk, leaving the communication disk, reaching the endpoint, and then returning to the communication disk. Our simulated hitting times lie between the two bounds, as seen in Fig. 6 and Fig. 7 respectively.

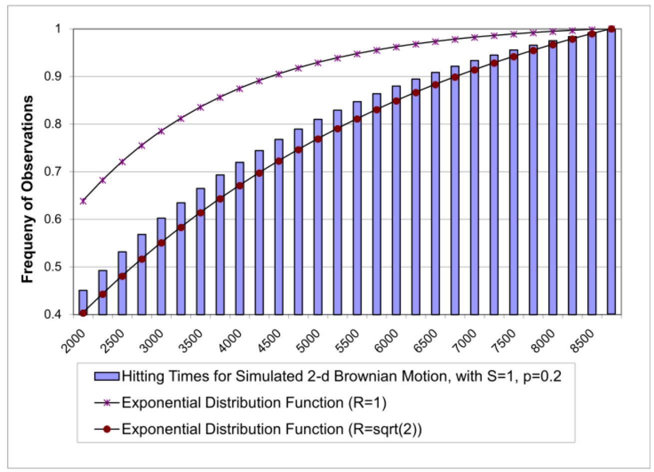

Fig. 8. We compare the measured and theoretical cumulative distributions of hitting times for the square grid deployment. The exponential distribution function for a square with $\rho = .1$ and $\kappa = 1$ corresponds to the theoretical lower bound distribution, whereas, the exponential distribution function (with $\rho = .1$ and $\kappa = \sqrt{2}$) corresponds to the theoretical upper bound distribution.

In Fig. 8, we compare the cumulative distribution of hitting times given from the simulation with the exponential distribution functions representing the theoretical lower and upper bounds on the hitting time distribution. The probability of the hitting time being less than some number, as given by the cumulative distribution of the frequency distribution of the simulated hitting times, is bounded above and below by the estimates calculated through the theoretical results that we derived, so the theoretical bounds apply to the simulated data.

V. Conclusion

In this paper, we studied the performances of partially covered, intermittently connected, hybrid DTNs under two models of access point deployment: the square grid, and the hexagonal grid. Specifically, we investigated the relationship between the coverage ratio in each deployment and the resulting mean amount of time a mobile node spends without coverage, as well as the rate at which data is lost, due to infrequent access point delivery. By modeling the path of the mobile node as a Brownian Motion, we derived a simple function for the average time a mobile node spends between consecutive access point visits and derived theoretical upper and lower bounds on the hitting time. We then applied the Poisson Clumping technique to calculate the expected rate of data loss.

We showed that for partially covered networks under either the square or hexagonal grid deployment, the expected hitting time grows asymptotically as the node density approaches zero, but not slower than the ratio of the natural log of the node density to the node density. We tested, via simulation, our assumption that the hitting times of a 2-D Brownian Motion are exponentially distributed, and similarly used simulations to validate the theoretical bounds on the hitting times in both deployments.

Appendix

A. Application of the Poisson Clumping Heuristic

Given a time-dependent stochastic process, and a set A, if the process intersects the set A rarely, then we can approximate the behavior of this process' arrivals to the set by the Poisson Process. The Poisson Process states that the inter-arrival times of the mobile node are exponentially distributed, with parameter λ, and that the number of times the mobile node has hit an access point up to time t is Poisson distributed, with parameter λt. In the language of the heuristic, λ is called the clump rate, so named because the random sets of times that the process spends in A, denoted C, appear to "clump" together. The approximations given by the Poisson Clumping Heuristic improve if the process is unlikely to return to A immediately after leaving A; there should be some drift away from A. Let $\pi(A)$ be the probability (of the stationary distribution) that the process is in A. Then, the main result of the heuristic is:

$$\pi(A) = \lambda \mathbb{E} C. \tag{3}$$

The assumption that the interarrival times follow an exponential distribution additionally gives us that $\lambda = 1/\bar{\sigma}$, where $\bar{\sigma}$ is the hitting time.[24]

In our model, the distribution of hitting times satisfies the assumptions above regarding the rarity with which the mobile node hits the coverage area because of the assumption of a small coverage ratio, and the fact that the drift for the radial part of Brownian Motion, given by the Bessel Process, has drift $\mu(r) = 1/(2r)$, where r is the Euclidean distance of the Brownian Motion from the origin. Since symmetry allows us to calculate the expected hitting time and expected data loss rate on a single square or hexagonally bounded region, we set the center of the communication disk at the origin and observe a strong drift away from the communication disk, assuming a small coverage ratio.

B. Proof of Theorem 3.1

We now provide the proof of our main theorem: the equation for the hitting time of a mobile node to a communication disk of radius ρ centered at the origin, on a larger disk with radius $R > \rho$, also centered at the origin. The hitting time is defined as the sum of the amount of time the mobile node spends traveling from the edge of the communication disk to the boundary of the larger disk (with expected value denoted $\mathbb{E}_\rho[T_R]$), and the time spent returning from the edge of the larger disk to the boundary of the communication disk (with expected value denoted $\mathbb{E}_R[T_\rho]$). We calculate these two means in the following lemmas.

Lemma A.1: $\mathbb{E}_\rho[T_R] = \frac{1}{2}\left(R^2 - \rho^2\right)$.

Proof: Consider the radial interval $[a, R]$, where ρ is in this interval. Let $\mathbb{E}_\rho[\min\{T_a, T_R\}]$ be the expected time that it takes to hit either a or R given that we start at ρ. Because the Bessel process has a drift away from the origin that becomes large as we approach the origin, then the expected time $\mathbb{E}_\rho[T_R]$ it takes to hit the boundary given we are at ρ is equal to $\lim_{a\to 0} \mathbb{E}_\rho[\min\{T_a, T_R\}]$.

From Eq. (15.3.12) in [25], it follows that the expected hitting time $\mathbb{E}_\rho[\min\{T_a, T_R\}]$ equals

$$\int_a^\rho 2m(y)\frac{(S(R)-S(\rho))(S(y)-S(a))}{S(R)-S(a)}\, dy + \int_\rho^R 2m(y)\frac{(S(\rho)-S(a))(S(R)-S(y))}{S(R)-S(a)} dy$$

where $S(x) = \ln|x|$ and $m(x) = x$ are the speed function and speed density of the Bessel process. Integrating the above expression yields $\mathbb{E}_\rho[\min\{T_a, T_R\}]$ to be equal to

$$\frac{2}{\ln(R/a)}\left[\ln(R/\rho)a^2\left(\frac{1}{2}\left(\frac{\rho}{a}\right)^2\left(\ln(\rho/a)-\frac{1}{2}\right)+\frac{1}{4}\right) - \ln(\rho/a)R^2\left(-\frac{1}{4}-\frac{1}{2}\left(\frac{\rho}{R}\right)^2\left(\ln(\rho/R)-\frac{1}{2}\right)\right)\right].$$

When a goes to 0, the first term goes to $\rho^2\ln\left(\frac{R}{\rho}\right)$ and the second term goes to $R^2\left(\frac{1}{2}+\left(\frac{\rho}{R}\right)^2\left[\ln\left(\frac{\rho}{R}\right)-\frac{1}{2}\right]\right)$. Since $\mathbb{E}_\rho[T_R] = \lim_{a\to 0}\mathbb{E}_\rho[\min\{T_a, T_R\}]$, it then follows that $\mathbb{E}_\rho[T_R] = \frac{1}{2}(R^2-\rho^2)$. ■

Once we hit the boundary at R, we can find the time that it takes to hit the communication disk again through a similar process.

Lemma A.2: $\mathbb{E}_R(T_\rho) = R^2\ln|\frac{R}{\rho}| - \frac{1}{2}\left(R^2-\rho^2\right)$.

Proof: Let Δ be small and assume that whenever our process hits R that it jumps instantaneously to $R-\Delta$. Taking the limit as Δ approaches zero will make R a reflecting boundary. By letting

$$\begin{aligned} p_\Delta &= P_{R-\Delta}(T_\rho < T_R) \\ a_\Delta &= \mathbb{E}_{R-\Delta}(T_\rho | T_\rho < T_R) \\ b_\Delta &= \mathbb{E}_{R-\Delta}(T_R | T_R < T_\rho) \end{aligned}$$

one can write $\mathbb{E}_{R-\Delta}(T_\rho \wedge T_R) = p_\Delta a_\Delta + (1-p_\Delta) b_\Delta$.

By Wald's Theorem, we find that $\mathbb{E}_{R-\Delta}(T_\rho)$

$$\begin{aligned} &= p_\Delta a_\Delta + (1-p_\Delta) p_\Delta (a_\Delta + b_\Delta) \\ &\quad + (1-p_\Delta)^2 p_\Delta (a_\Delta + 2b_\Delta) + \cdots \\ &= a_\Delta + \frac{b_\Delta (1-p_\Delta)}{p_\Delta} \\ &= \frac{1}{p_\Delta} \mathbb{E}_{R-\Delta}(T_\rho \wedge T_R) \end{aligned}$$

From Eq. (15.3.10) [25], it follows

$$P_{R-\Delta}(T_\rho < T_R) = \frac{S(R) - S(R-\Delta)}{S(R) - S(\rho)}.$$

and from Eq. (15.3.12) [25], it follows $\mathbb{E}_{R-\Delta}(T_\rho \wedge T_R)$ equals

$$\int_\rho^{R-\Delta} \frac{2(S(R) - S(R-\Delta))(S(y) - S(\rho))}{S(R) - S(\rho)} m(y) dy + \int_{R-\Delta}^{R} \frac{2(S(R-\Delta) - S(\rho))(S(R) - S(y))}{S(R) - S(\rho)} m(y) dy.$$

From this, we can write $\mathbb{E}_{R-\Delta}(T_\rho)$ as

$$2 \int_\rho^{R-\Delta} (S(y) - S(\rho)) m(y) dy + \frac{2(S(R-\Delta) - S(\rho))}{S(R) - S(R-\Delta)} \int_{R-\Delta}^{R} (S(R) - S(y)) m(y) dy.$$

Substituting in our functions and solving, we can write $\mathbb{E}_{R-\Delta}(T_\rho)$ as

$$\frac{\rho^2 \ln|\frac{R-\Delta}{R}| \left[(\frac{R-\Delta}{\rho})^2 (\ln|\frac{R-\Delta}{\rho}| - \frac{1}{2}) + \frac{1}{2} \right]}{\ln|\frac{R-\Delta}{R}|} - \frac{\ln|\frac{R-\Delta}{\rho}| \left[(\frac{R-\Delta}{R})^2 (\ln|\frac{R-\Delta}{R}| - \frac{1}{2}) + \frac{1}{2} \right]}{\ln|\frac{R-\Delta}{R}|}.$$

Now, by taking the limit as Δ goes to zero, we find $\mathbb{E}_R(T_\rho) = R^2 \ln|\frac{R}{\rho}| - \frac{1}{2}\left(R^2 - \rho^2\right)$. ■

The expected hitting time is then the sum of $\mathbb{E}_\rho[T_R]$ and $\mathbb{E}_R[T_\rho]$, derived respectively in Lemmas A.1 and A.2, and is as stated in the following proposition.

Theorem A.3 (3.1): For a communication disk of radius ρ centered in the region disk of radius R, the expected hitting time is $h(\rho, R)$, where

$$h(x, y) = y^2 \ln\left|\frac{y}{x}\right|.$$

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