Math 464/564
Homework #4 - Due Monday, March 8th

1. (10 pts) Given a random variable X such that $Y = \ln X$ is a normal random variable with mean μ and variance σ^2. Use the moment generating function of Y to find the mean and variance of X.

2. (10 pts) Suppose the moment generating function of a random variable X is

$$M_X(s) = e^{-s+s^2} \quad \text{for all} \quad s \in \mathbb{R}.$$

Complete the following steps.

i. (5 pts) Find the distribution (p.d.f. or p.m.f.) of X. Use the moment generating function to compute $E[X^3]$.

ii. (5 pts) Minimize Chernoff bound to obtain an upper bound on $P(X \geq 3)$.

Hint: Recall that Chernoff bound states $P(X \geq a) \leq e^{-sa} M_X(s)$ for all $s > 0$.

3. (10 pts) Suppose X is a Poisson random variable with parameter $\lambda = 5$. We want to find the best upper bound on $P(X \geq 15)$. Use the following three inequalities and identify the best bound.

i. (2 pts) Use Markov inequality to obtain an upper bound on $P(X \geq 15)$.

ii. (3 pts) Use one-sided Chebyshev inequality to obtain an upper bound on $P(X \geq 15)$.

iii. (5 pts) Apply Chernoff bound. Minimize Chernoff bound to obtain an upper bound on $P(X \geq 15)$.

4. (5 pts) Let X be a nonnegative random variable. Show that

$$\left(E[X^a]\right)^{\frac{1}{a}} \leq \left(E[X^b]\right)^{\frac{1}{b}}$$

for any $0 < a < b$.

Hint: Let $Y = X^a$, then use Jensen's inequality.