Math 464/564
Homework #1 - Due Monday, January 25th

1. (10 pts) Consider discrete random variables X and Y with a joint probability mass function

$$
\begin{array}{lll}
p(-1, 2) = 0.04 & p(2, 2) = 0.08 & p(4, 2) = 0.28 \\
p(-1, 1) = 0.01 & p(2, 1) = 0.02 & p(4, 1) = 0.07 \\
p(-1, 0) = 0.05 & p(2, 0) = 0.1 & p(4, 0) = 0.35
\end{array}
$$

Do the following parts.

i. (2 pts) Find the marginal probability mass functions p_x and p_y. Are X and Y independent?

ii. (4 pts) Find the covariance $Cov(X, Y)$.

iii. (4 pts) Find the correlation $\text{corr}(X, Y)$.

2. (15 pts) Consider continuous random variables X and Y with a joint probability density function

$$
f(x, y) = \begin{cases} 4xe^{-2xy} & \text{for } x \geq 0, \ y \geq 1, \\ 0 & \text{otherwise.} \end{cases}
$$

Do the following parts.

i. (3 pts) Verify that $f(x, y)$ integrates to one, i.e., $\iint_{\mathbb{R}^2} f(x, y)\, dx\, dy = 1$.

ii. (4 pts) Find the marginal probability density functions f_x and f_y. Are X and Y independent?

iii. (4 pts) Is the covariance $Cov(X, Y)$ well defined? If yes, find $Cov(X, Y)$. If no, explain why.

iv. (4 pts) Is the correlation $\text{corr}(X, Y)$ well defined? If yes, find $\text{corr}(X, Y)$. If no, explain why.

3. (15 pts) Consider continuous random variables X and Y with a joint probability density function

$$
f(x, y) = \begin{cases} 3xy^2 e^{-xy^3} & \text{for } x \geq 0, \ y \geq 1, \\ 0 & \text{otherwise.} \end{cases}
$$

Do the following parts.

i. (3 pts) Verify that $f(x, y)$ integrates to one, i.e., $\iint_{\mathbb{R}^2} f(x, y)\, dx\, dy = 1$.

ii. (4 pts) Find the marginal probability density functions f_x and f_y. Are X and Y independent?

iii. (4 pts) Is the covariance $Cov(X, Y)$ well defined? If yes, find $Cov(X, Y)$. If no, explain why.

iv. (4 pts) Is the correlation $\text{corr}(X, Y)$ well defined? If yes, find $\text{corr}(X, Y)$. If no, explain why.