Math 463/563
Homework #1 - Due Friday, October 2

1. Show that
$$\sum_{k=0}^{n} \binom{n}{k} (-1)^k = 0$$

2. Prove that
$$\binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2$$
Hint: Consider two experiments: filling in the first n spots, and filling in the last n spots. Use $\binom{n}{k} = \binom{n}{n-k}$.

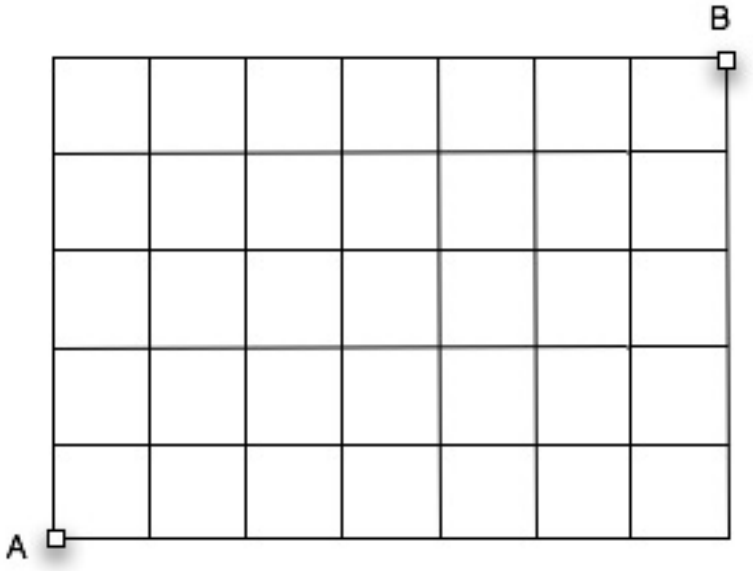
3. How many distinct 9 letter strings can be created using all the letters in the word **REDRESSER** (that is one D, three E, three R, and two S) ?

4. If 12 people are to be divided into 3 committees of respective sizes 3, 4, and 5, how many divisions are possible?

5. Find the number of integer solutions (x_1, x_2, x_3, x_4) of
$$x_1 + x_2 + x_3 + x_4 = 49$$
subject to $x_1 \geq 1$, $x_2 \geq 1$, $x_3 \geq 1$ and $x_4 \geq 1$.

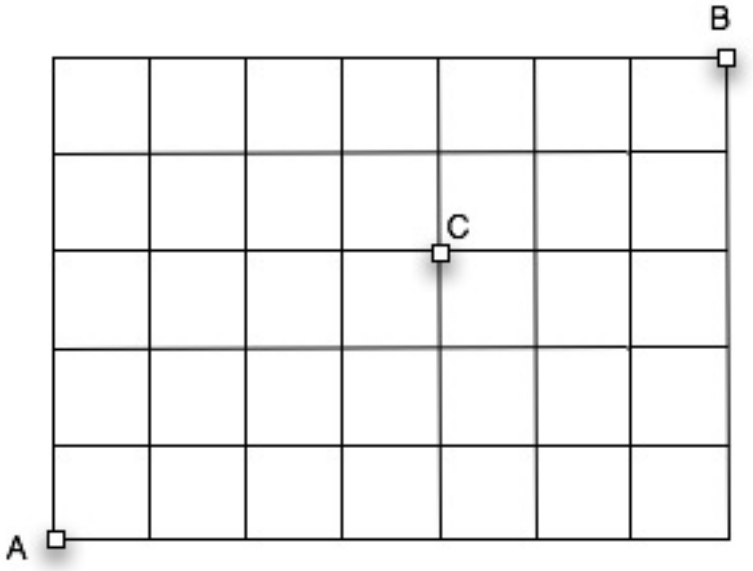
6. Find the number of integer solutions (x_1, x_2, x_3, x_4) of
$$x_1 + x_2 + x_3 + x_4 = 49$$
subject to $x_1 \geq 1$, $x_2 \geq 2$, $x_3 \geq 3$ and $x_4 \geq 4$.

7. Consider a walk on the grid pictured below, originating at the point labeled **A**. Each time the walker can go one step up or one step to the right.



How many different paths from **A** to **B** are possible? Here is an example of such path:
Up-Right-Right-Up-Up-Right-Right-Up-Right-Right-Up-Right

8. Consider the paths from **A** to **B** as described in the previous problem. How many different paths from **A** to **B** go through **C**?



9. How many different ways are there of dealing 52 cards to four players (Player A, Player B, Player C and Player D) so that each player gets thirteen card, exactly one of which is an ace? Hint: First deal the aces, then the rest of the cards. Simplify.