MTH 463/563 - Lecture 29

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Topics:

- Examples.
- Review.

Review.

Problem 5 on p.71. Suppose you are watching a radioactive source that emits particles at a rate described by the exponential density

$$f(x) = \lambda e^{-\lambda x} \qquad (x \geq 0),$$

where $\lambda = 1$, so that the probability $P(0 \leq X \leq T)$ that a particle will appear in the next T seconds is $P(0 \leq X \leq T) = \int_0^T \lambda e^{-\lambda x} dx$

Find the probability that a particle will appear

(a) within the next second.

(b) within the next 3 seconds.

(c) between 3 and 4 seconds from now.

(d) after 4 seconds from now.

Review.

Problem 1 on p.278. Let X be a random variable with range $[-1,1]$ and let $f_{\mathrm{x}}(x)$ be the density function of X. Find $E[X]$ and $Var(X)$ if, for $|x| < 1$,

(a) $f_{\mathrm{x}}(x) = \frac{1}{2}$

(b) $f_{\mathrm{x}}(x) = |x|$

(c) $f_{\mathrm{x}}(x) = 1 - |x|$

(d) $f_{\mathrm{x}}(x) = \frac{3}{2}x^2$

Review.

Problem 3 on p.278. The lifetime, measured in hours, of the ACME super light bulb is a random variable T with density function $f(x) = \lambda^2 x e^{-\lambda x}$, where $\lambda = 0.05$. What is the expected lifetime of this light bulb? What is its variance?

Review.

Problem 4 on p.278. Let X be a random variable with range $[-1, 1]$ and density function $f_X(x) = ax + b$ if $|x| < 1$,

(a) Show that if $\int_{-1}^{1} f_X(x)dx = 1$, then $b = \frac{1}{2}$

(b) Show that if $f_X(x) \geq 0$, then $-\frac{1}{2} \leq a \leq \frac{1}{2}$.

(c) Show that $E[X] = \frac{2}{3}a$, and hence that $-\frac{1}{3} \leq E[X] \leq \frac{1}{3}$.

(d) Show that $Var(X) = \frac{2}{3}b - \frac{4}{9}a^2 = \frac{1}{3} - \frac{4}{9}a^2$.

Review.

Problem. Let X and Y be two independent random variables, each exponential with the same parameter $\lambda > 0$. Show that their sum, $X+Y$ is distributed via the following density function

$$f_{\mathrm{x+y}}(x) = \lambda^2 x e^{-\lambda x} \qquad (x \geq 0)$$

Review.

Problem 2 on p.219. Choose a number U from the interval $[0, 1]$ with uniform distribution. Find the cumulative distribution and density for the random variables

(a) $Y = \frac{1}{U+1}$

(b) $Y = \log(U + 1)$

Review.

Problem 10 on p.220. Let U, V be random numbers chosen independently from the interval $[0, 1]$. Find the cumulative distribution and density for the random variables

(a) $Y = \max(U, V)$

(b) $Y = \min(U, V)$

Review.

Problem 16 on p.221. Let X be a random variable with density function

$$f_X(x) = \begin{cases} cx(1-x) & \text{if } 0 < x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

(a) What is the value of c?

(b) What is the cumulative distribution function F_X for X?

(c) What is the probability that $X < \frac{1}{4}$?