1. Show that
\[ \sum_{k=0}^{n} \binom{n}{k} (-1)^k = 0 \]

2. Prove that
\[ \binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2 \]

Hint: Consider two experiments: filling in the first \( n \) spots, and filling in the last \( n \) spots. Use \( \binom{n}{k} = \binom{n}{n-k} \).

3. How many distinct 9 letter strings can be created using all the letters in the word \textsc{redresser} (that is one D, three E, three R, and two S)?

4. If 12 people are to be divided into 3 committees of respective sizes 3, 4, and 5, how many divisions are possible?

5. Find the number of integer solutions \((x_1, x_2, x_3, x_4)\) of
\[ x_1 + x_2 + x_3 + x_4 = 49 \]
such that \( x_1 \geq 1, x_2 \geq 1, x_3 \geq 1 \) and \( x_4 \geq 1 \).

6. Find the number of integer solutions \((x_1, x_2, x_3, x_4)\) of
\[ x_1 + x_2 + x_3 + x_4 = 49 \]
such that \( x_1 \geq 1, x_2 \geq 2, x_3 \geq 3 \) and \( x_4 \geq 4 \).
7. Consider a walk on the grid pictured below, originating at the point labeled A. Each time the walker can go one step up or one step to the right.

How many different paths from A to B are possible? Here is an example of such path:
Up-Right-Right-Up-Up-Right-Right-Up-Right-Right-Up-Right

8. Consider the paths from A to B as described in the previous problem. How many different paths from A to B go through C?
9. How many different ways are there of dealing 52 cards to four players (Player A, Player B, Player C and Player D) so that each player gets thirteen card, exactly one of which is an ace? Hint: First deal the aces, then the rest of the cards. Simplify.