1. Consider events $E$, $F$, and $G$. Find the expressions in $E$, $F$ and $G$ for the following events.

(a) At least one of the three events occurs.
(b) At most one of the three events occurs.
(c) Exactly two of them occur.
(d) At most two of the three events occur.
(e) All three events occur.
(f) None of the three events occurs.
(g) At most three of the events occur.
(h) $E$ or $F$, but not $G$ occur.
(i) Both $E$ and $F$, but not $G$ occur.
(j) Exactly one of the three events occurs.

*Example:* The event that “only $G$ occurs” is expressed as $E \cap F \cap G$.

2. Consider a sample space $S = \{a, b, c\}$. Suppose $P(\{a, b\}) = 0.68$, and $P(\{b, c\}) = 0.76$. Compute the probabilities of $\{a\}$, $\{b\}$, and $\{c\}$.

3. Suppose $A$ and $B$ are disjoint events with $P(A) = 0.25$ and $P(B) = 0.57$. Find $P(\overline{A \cap B})$.

4. Prove the inclusion-exclusion formula for three events stated as follows

$$P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(E \cap F) - P(F \cap G) - P(E \cap G) + P(E \cap F \cap G)$$

*Hint:* Take $A = E$ and $B = F \cup G$. Use $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

5. Consider rolling a swindler’s die that turns up a ‘6’ three times likelier than a ‘1’, and suppose ‘2’, ‘3’, ‘4’ and ‘5’ are each as likely to appear as ‘1’.

(a) Find the probabilities for each outcome.
(b) Find the probability of the event that the die turns up an *even* digit.
(c) Conditioning on the event that the die turns up an even digit, how likely for it to be a ‘6’?
(d) Conditioning on the event that the die turns up an odd digit, how likely for it to be a ‘5’?
(e) Suppose your friend rolled the swindler’s die and informed you (truthfully) that
the outcome was either ‘4’, ‘5’, or ‘6’ . It is assumed you know the probabilities in
part (a), but you were in a different room and did not see the experiment. How
likely it is for the outcome to be a ‘6’ ?

(f) Are the events \( E = \{ \text{the outcome is odd} \} \) and \( F = \{4, 5\} \) independent?

6. Show that if \( P(E) = 0.9 \) and \( P(F) = 0.4 \), then \( P(E \cap F) \geq 0.3 \).
   Hint: Use Bonferroni’s inequality, \( P(E \cap F) \geq P(E) + P(F) - 1 \)

7. Show that the probability that exactly one of the two events, \( E \) or \( F \), occurs is
   \[ P(E) + P(F) - 2P(E \cap F) \]

8. Let \( A \) and \( B \) be events of positive probability. Prove that \( P(A|B) > P(A) \) if and only
   if \( P(B|A) > P(B) \).

9. Consider two urns, one containing 2 green and 1 black marbles, the other 2 green and
   3 black marbles. An urn is selected at random with probability 1/2 each, and a marble
   is drawn at random from a selected urn. What is the probability that the marble is
   green? What is the probability that the first urn was selected given that the marble is
green?

10. (Extra credit) An urn contains \( n \) green and \( m \) black balls. The balls are withdrawn
    one at a time until only those of the same color are left. Show that with probability
    \( \frac{n}{n+m} \) they are all green.