MTH 355 - Lecture 9

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Topics:

• Intro to Sets.

• Venn diagrams.

• De Morgan’s laws.
SETS: notions and examples.

Notions

• $a \in A$ denotes that $a$ is an element of $A$

• Empty set $\emptyset = \{\}$

• $U$ is called a universal set or a universe

• Integers: $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, 4, \ldots \}$

• Rational numbers: $\mathbb{Q} = \{ \frac{n}{m} : n \text{ and } m \text{ are integers, and } m \neq 0 \}$
• Real numbers:
\[ \mathbb{R} = \{ \text{all values between } -\infty \text{ and } +\infty \} \]

• \( A \) = all elements in the universe \( U \) that do not belong to \( A \)

• \( A \cap B \) = all elements in the universe \( U \) that belong to \( A \) \text{ and } B

• \( A \cup B \) = all elements in the universe \( U \) that belong to \( A \) \text{ or } B, \text{ or to both sets, } A \text{ and } B
• $A - B = \text{all elements in the universe } U \text{ that belong to } A \text{ but do not belong to } B$

• $A \subseteq B$ ($A$ is a subset of $B$), i.e. all elements in $A$ also belong to $B$
**SETS: notions and examples**

**Example.**

Let the universe be the set of all digits

\[ U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \]

Let \( A = \{0, 1, 2, 3, 4, 5, 6\} \), \( B = \{2, 3, 5, 7, 9\} \), and \( E = \{0, 2, 4, 6, 8\} \). Then

- \( \overline{A} = U - A = \{7, 8, 9\} \)
- \( A \cup E = \{0, 1, 2, 3, 4, 5, 6, 8\} \)
- \( A \cap E = \{0, 2, 4, 6\} \)
- \( A - B = \{0, 1, 4, 6\} \)
**SETS: notions and examples**

**Example.**

Let the universe be the set of all digits

\[ U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \]

Let \( A = \{0, 1, 2, 3, 4, 5, 6\} \), \( B = \{2, 3, 5, 7, 9\} \), and \( E = \{0, 2, 4, 6, 8\} \). Then

- \( \overline{A \cap E} \cap B = \{1, 3, 5, 7, 8, 9\} \cap B = \{3, 5, 7, 9\} \)
- \( \overline{A \cup E} \cap B = \{7, 9\} \cap B = \{7, 9\} \)
- \( A \cap (B \cup E) = A \cap \{0, 2, 3, 4, 5, 6, 7, 8, 9\} = \{0, 2, 3, 4, 5, 6\} \)
- \( (A \cap B) \cup E = \{2, 3, 5\} \cup E = \{0, 2, 3, 4, 5, 6, 8\} \)
SETS: notions and examples

Example.
Let the universe be the set of all digits

\[ U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \]

Let \( A = \{0, 1, 2, 3, 4, 5, 6\} \), \( B = \{2, 3, 5, 7, 9\} \), and \( E = \{0, 2, 4, 6, 8\} \). Then

- \( A - (B \cup E) = A - \{0, 2, 3, 4, 5, 6, 7, 8, 9\} = \{1\} \)
- \( (B \cup E) - A = \{0, 2, 3, 4, 5, 6, 7, 8, 9\} - A = \{7, 8, 9\} \)
- \( (B \cap E) - A = \{2\} - A = \emptyset \)
VENN DIAGRAMS

- Shade $A \cap \overline{B}$

$A \cap \overline{B} = A - B$ represents all elements in the universe $U$ that belong to the set $A$, but do not belong to $B$. 

\[ A \cap \overline{B} = A - B \]
VENN DIAGRAMS

- Shade \((A \cup B) - (A \cap B)\)

\((A \cup B) - (A \cap B)\) represents all elements in the universe \(U\) that belong to \(A\), or \(B\), but do not belong to both \(A\) and \(B\).
VENN DIAGRAMS

- Shade $A \cup \overline{B}$

$$A \cup \overline{B} = \overline{(B - A)}$$ represents all elements in the universe $U$ that belong to $A$, **or** do not belong to $B$, **or** both belong to $A$ and do not belong to $B.
VENN DIAGRAMS

• Shade \((A \cap \overline{B}) \cup C\)

Here \((A \cap \overline{B}) \cup C = (A - B) \cup C\) represents all elements in the universe \(U\) that belong to \(C\), or that belong to \(A\) and do not belong to \(B\).
VENN DIAGRAMS

- Shade $A \cap B \cap C$

Here $A \cap B \cap C$ represents all elements in the universe $U$ that belong to $A$ and $B$ and $C$ altogether.
VENN DIAGRAMS

• Shade $A \cap (B \cup C)$

Here $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ represents all elements in the universe $U$ that belong to $A$, and to at least one of the two other sets, $B$ or $B$. 
VENN DIAGRAMS

• Shade \((A \cap B) \cup (A \cap C) \cup (B \cap C)\)

Here \((A \cap B) \cup (A \cap C) \cup (B \cap C)\) represents all elements in the universe \(U\) that belong to at least two of the three sets, \(A\), \(B\) and \(C\).
Rules of set theory:

- Commutative laws:
  \[ E \cup F = F \cup E \quad E \cap F = F \cap E \]

- Associative laws:
  \[ (E \cup F) \cup G = E \cup (F \cup G) \quad (E \cap F) \cap G = E \cap (F \cap G) \]

- Distributive laws:
  \[ (E \cup F) \cap G = (E \cap G) \cup (F \cap G) \]
  \[ (E \cap F) \cup G = (E \cup G) \cap (F \cup G) \]
Propositions and sets

If we compare propositions to sets we will observe some similarities. Namely,

\[ \neg p \] is similar to \[ A \]

\[ p \land q \] is similar to \[ A \cap B \]

\[ p \lor q \] is similar to \[ A \cup B \]

and \[ p \rightarrow q \] is similar to \[ A \subseteq B \]

The **truth tables** are similar to Venn diagrams.
TRUTH TABLES

• Given proposition $p$, here is the truth table for $\neg p$

<table>
<thead>
<tr>
<th>$p$</th>
<th>$\neg p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>$F$</td>
</tr>
<tr>
<td>$F$</td>
<td>$T$</td>
</tr>
</tbody>
</table>

• Given propositions $p$ and $q$, here is the truth table for $p \lor q$

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \lor q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>$T$</td>
<td>$T$</td>
</tr>
<tr>
<td>$T$</td>
<td>$F$</td>
<td>$T$</td>
</tr>
<tr>
<td>$F$</td>
<td>$T$</td>
<td>$T$</td>
</tr>
<tr>
<td>$F$</td>
<td>$F$</td>
<td>$F$</td>
</tr>
</tbody>
</table>

• Given propositions $p$ and $q$, here is the truth table for $p \land q$

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \land q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>$T$</td>
<td>$T$</td>
</tr>
<tr>
<td>$T$</td>
<td>$F$</td>
<td>$F$</td>
</tr>
<tr>
<td>$F$</td>
<td>$T$</td>
<td>$F$</td>
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<tr>
<td>$F$</td>
<td>$F$</td>
<td>$F$</td>
</tr>
</tbody>
</table>
Notations:

\[ \bigcup_{j=1}^{n} A_j = A_1 \cup A_2 \cup \ldots \cup A_n \]

\[ \bigcap_{j=1}^{n} A_j = A_1 \cap A_2 \cap \ldots \cap A_n \]

Example. \[ \bigcup_{j=1}^{3} A_j = A_1 \cup A_2 \cup A_3 \]

Example. \[ \bigcap_{j=1}^{5} A_j = A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5 \]
De Morgan’s laws:

Consider sets $E_1, E_2, \ldots, E_n$, then

$$\left( \bigcup_{j=1}^{n} E_j \right) = \bigcap_{j=1}^{n} \overline{E_j}$$

and

$$\left( \bigcap_{j=1}^{n} E_j \right) = \bigcup_{j=1}^{n} \overline{E_j}$$
De Morgan’s laws:

\[
\left( \bigcup_{j=1}^{n} E_j \right) = \bigcap_{j=1}^{n} \overline{E}_j
\]

Proof:

\[
x \in \left( \bigcup_{j=1}^{n} E_j \right) \iff x \notin \bigcup_{j=1}^{n} E_j \iff x \notin E_j \text{ for all } j = 1, \ldots, n
\]

\[
\iff x \in \overline{E}_j \text{ for all } j = 1, \ldots, n \iff x \in \bigcap_{j=1}^{n} \overline{E}_j
\]

Hence \( \left( \bigcup_{j=1}^{n} E_j \right) = \bigcap_{j=1}^{n} \overline{E}_j \)