Topics:

• Methods of Proof

• Intro to Sets.
Prove or Disprove

Example. Prove or disprove the following statement:

A product of any two rational numbers is rational.
Prove or Disprove

Example. Prove or disprove the following statement:

A product of any two irrational numbers is irrational.
Prove or Disprove

Example. Prove or disprove the following statement:

If $d$ is a positive integer, then $\sqrt{d}$ is irrational.
Prove or Disprove

Example. Prove or disprove the following statement:

If \( d \) is a positive integer that is not a perfect square, then \( \sqrt{d} \) is irrational.
Prove or Disprove

Example. Prove or disprove the following statement:

If \( n \) is a positive integer, then

\[
1 + 2 + 3 + \ldots + (n - 1) + n = \frac{n(n + 1)}{2}
\]
Prove or Disprove

Example. Prove or disprove the following statement:

If \( n \) is a positive integer and \( A \) is a real number not equal to one, then

\[
1 + A + A^2 + A^3 + \ldots + A^{n-1} + A^n = \frac{A^{n+1} - 1}{A - 1}
\]
SETS: notions and examples.

Notions

• \( a \in A \) denotes that \( a \) is an element of \( A \)

• Empty set \( \emptyset = \{ \} \)

• \( U \) is called a universal set or a universe

• Integers: \( \mathbb{Z} = \{ \ldots, -3, -2, -1, 0, 1, 2, 3, 4, \ldots \} \)

• Rational numbers:
  \( \mathbb{Q} = \{ \frac{n}{m} : n \text{ and } m \text{ are integers, and } m \neq 0 \} \)
• Real numbers: 
\[ \mathbb{R} = \{ \text{all values between } -\infty \text{ and } +\infty \} \]

• \( \overline{A} \) = all elements in the universe \( U \) that do not belong to \( A \)

• \( A \cap B \) = all elements in the universe \( U \) that belong to \( A \) and \( B \)

• \( A \cup B \) = all elements in the universe \( U \) that belong to \( A \) or \( B \), or to both sets, \( A \) and \( B \)
• $A - B = \text{all elements in the universe } U \text{ that belong to } A \text{ but do not belong to } B$

• $A \subseteq B \ (A \text{ is a subset of } B)$, i.e. all elements in $A$ also belong to $B$
SETS: notions and examples

Examples. Let the universe be the set of all digits

\[ U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \]

Let \( A = \{0, 1, 2, 3, 4, 5, 6\} \), \( B = \{2, 3, 5, 7, 9\} \), and \( E = \{0, 2, 4, 6, 8\} \). Then

1. \( \overline{A} = U - A = \{7, 8, 9\} \)
2. \( A \cup E = \{0, 1, 2, 3, 4, 5, 6, 8\} \)
3. \( A \cap E = \{0, 2, 4, 6\} \)
4. \( A - B = \{0, 1, 4, 6\} \)
5. \( \overline{A \cap E} \cap B = \{1, 3, 5, 7, 8, 9\} \cap B = \{3, 5, 7, 9\} \)
Let $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{0, 1, 2, 3, 4, 5, 6\}$, $B = \{2, 3, 5, 7, 9\}$, and $E = \{0, 2, 4, 6, 8\}$.

6. $\overline{A \cup E} \cap B = \{7, 9\} \cap B = \{7, 9\}$

7. $A \cap (B \cup E) = A \cap \{0, 2, 3, 4, 5, 6, 7, 8, 9\}$
   $= \{0, 2, 3, 4, 5, 6\}$

8. $(A \cap B) \cup E = \{2, 3, 5\} \cup E = \{2\}$

9. $A - (B \cup E) = A - \{0, 2, 3, 4, 5, 6, 7, 8, 9\}$
   $= \{1\}$

10. $(B \cup E) - A = \{0, 2, 3, 4, 5, 6, 7, 8, 9\} - A$
    $= \{7, 8, 9\}$

11. $(B \cap E) - A = \{2\} - A = \emptyset$