Yet Another Interval-Division Problem

Abstract: Many authors have considered problems arising from repeated division of the unit interval. The problem considered here may be viewed as a one-sided version of a two-sided car parking problem first studied by Rényi. It may be represented as follows:

Let a sequence of independent Uniform(0,1) random variables, $U_1, U_2, \ldots$, be chosen. At the kth stage, let

$$M_k = max(1 - U_1, U_1(1 - U_2), U_1U_2(1 - U_3), \ldots, U_1U_2 \cdots U_{k-1}(1 - U_k)).$$

Set $M = sup_{k \geq 1} M_k$. The problem is to determine the distribution of M, and at least its first moment. We can (almost) do this. In addition, interpreting this problem as a one-sided car parking problem, we get the asymptotic distribution of the scaled "walking distance."