Problems

For the following questions, define a one-dimensional molecule to be one electron in one-dimensional (z) potential of length L. The potential energy is infinite beyond the ends of the molecule and constant within. Choose L such that the wavelength of the first transition, $n = 1 \rightarrow n = 2$, is 200 nm. Use the usual wavefunctions for this quantum system, but change the energies to be more realistic. Use the usual expression for the energy of the $n = 1$ and $n = 2$ states, but for $n \geq 3$ assume that the energy difference between states is equal to the energy difference between states 1 and 2.

1. (a) Calculate the polarizability matrix $\alpha(\omega)$ of a one-dimensional molecule with one electron. Include only states 1, 2, 3 and 4.

   (b) Assume that a single sheet of molecules is arranged in a periodic array with spacing equal to L. The molecules are aligned perpendicular to this plane. A three-dimensional solid consists of sheets separated by the distance L. Use the Clausius-Mosotti relation to determine the index of refraction n as a function of wavelength for $300 < \lambda < 1000$ nm.

2. (a) Calculate the third-order polarizability tensor $\gamma(\omega_3 = 2\omega_1 - \omega_2)$ of a one-dimensional molecule with one electron. Include only states 1, 2, 3 and 4. Use $\lambda_1 = 500$ and $\lambda_2 = 600$ nm.

   (b) Using your result for the index of refraction $n(\omega)$, determine the directions of propagation for phase-matching.

   (c) Now suppose that the molecules are oriented at 45° with repect to the plane. What is γ? What is the phase-matching condition?

3. Add a small perturbation to the molecules, that is, the potential energy is not constant but rather $PE = az$. This breaks inversion symmetry along the z axis.

   (a) Calculate the second-order polarizability tensor $\beta(2\omega = \omega + \omega)$ of a one-dimensional molecule with one electron. Include only states 1, 2, 3 and 4. Use $\lambda = 600$ nm.

   (b) Using your result for the index of refraction $n(\omega)$, determine the directions of propagation for phase-matching.

   (c) Now suppose that the molecules are oriented at 45° with repect to the plane. What is β? What is the phase-matching condition?