Our 1-2-3 Stepwise Program to *Conquer Time-Dependent Perturbation Theory*

Goal: Find the probability of being in a final state $| f \rangle$ at given time *t* or $\left| c_f(t) \right|^2$.

Let the wave of a multilevel system be $|\psi\rangle = \sum_k c_k(t) e^{-iE_k t/\hbar} |k\rangle$.

Step 1: Operate on both sides of the above equation with $\left[i\hbar \frac{\partial}{\partial t} - \left(H_o + H' \right) \right]$

Note: The Schrodinger Equation tell us that $\left[i\hbar \frac{\partial}{\partial t} - \left(H_o + H' \right) \right] |\psi\rangle = 0$

Step 2: Take the dot product of Step 1 with a final bra state, $\langle f(t) | \cdot [step1] = \langle f | e^{iE_f t/\hbar} \cdot [step1]$,

to show we get the ODE equation, $\dot{c}_f(t) = \frac{-i}{\hbar} \sum_k c_k(t) \langle f | H' | k \rangle e^{i(E_f - E_k)t/\hbar}$

Step 3: You now want to determine the transition probability to this final state *f*, given by $\left| c_f(t) \right|^2$

Solve the above ODE iteratively.
To get the 1st order solution sub-in $c_k(t) = \delta_{fk}$ and integrate both sides from 0 to t.

Note: This comes from 0^{th} order solution must be $c_f(t) = \delta_{fk}$ as clearly $c_f(t<0) = 0$ and $c_f(t>0) = 1$
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