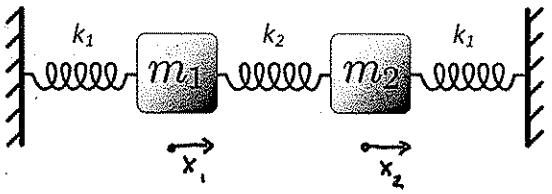
PHY 427 Worksheet Activity #1 – Differential Equations for Coupled Systems

Figures below depict five different systems of particles. For each system, write down differential equations of motion by inspection. Assume that the masses experience only a small perturbation in their position.

System 1:

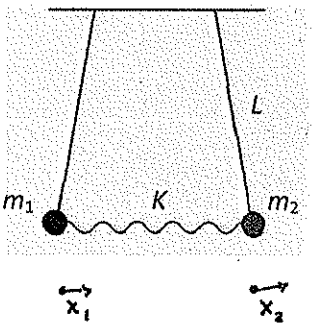


$m_1\ddot{x}_1 = -K_1x_1 + K_2(x_2 - x_1)$

$m_2\ddot{x}_2 = -K_1x_2 + K_2(x_1 - x_2)$

Trick: if the system is symmetric swapping $x_1 \leftrightarrow x_2$ must give you the second equation

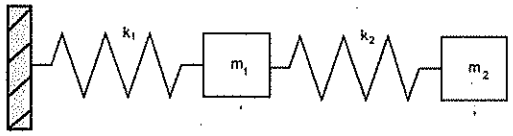
System 2:



$m_1\ddot{x}_1 = \frac{-mg}{\ell}x_1 - Kx_1 + Kx_2 = \frac{-mg}{\ell}x_1 - K(x_1 - x_2)$

$m_2\ddot{x}_2 = \frac{-mg}{\ell}x_2 - Kx_2 + Kx_1 = \frac{-mg}{\ell}x_2 - K(x_2 - x_1)$

System 3:



$m_1\ddot{x}_1 = -K_1x_1 + K_2(x_2 - x_1)$

$m_2\ddot{x}_2 = -K_2x_2 + K_2x_1 = -K_2(x_2 - x_1)$

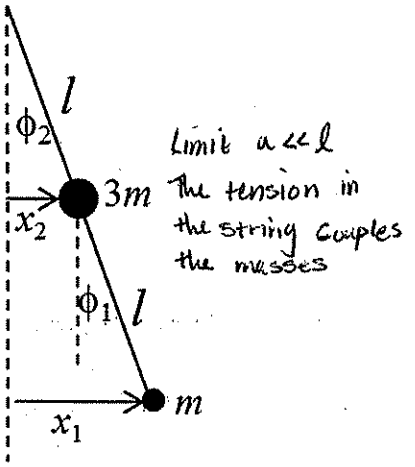
Recall one pendulum is $m\ddot{x} = -\frac{mg}{\ell} x$

System 4:

$$m\ddot{x}_1 = \frac{mg}{\ell}(x_2 - x_1)$$

$$3m\ddot{x}_2 = \frac{mg}{\ell}(x_1 - x_2) - \frac{4mg}{\ell} x_2$$

↓ restoring tension in 1st length of string is ~ 4mg

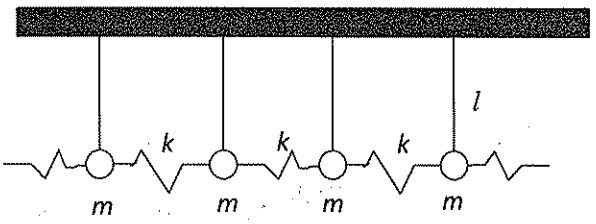


Limit $a \ll \ell$
The tension in the string couples the masses

System 5:

$$m\ddot{x}_n = \frac{-mg}{\ell} x_n - K(x_n - x_{n-1}) + \ldots$$

$m\ddot{x}_{n+1} =$ → exercise for you ←



System 6:

$$m\ddot{x}_1 = -K(x_1 - x_2) + K(x_3 - x_1)$$

$$m\ddot{x}_2 = +K(x_1 - x_2) - K(x_2 - x_3)$$

$$m\ddot{x}_3 = +K(x_2 - x_3) - K(x_3 - x_1)$$