Day 5: Dispersion, sound and phonons. Oh MY!

Last time: (m) — (m) — (m) springs α, masses $n-1$, n, $n+1$ ...

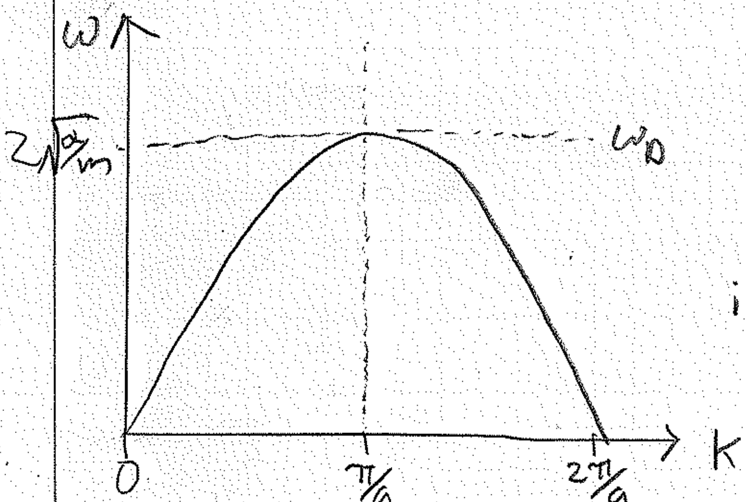
① $m\ddot{x}_n = -\alpha(x_n - x_{n-1}) - \alpha(x_n - x_{n+1})$

② ansatz: $x_n(t) = Ae^{i\omega t}e^{ikna}$ → put in ①

$$-m\omega^2 = \alpha(-2 + e^{ika} + e^{-ika})$$

$$\Rightarrow \boxed{\omega(k) = 2\sqrt{\alpha/m}\,\left|\sin(ka/2)\right|}$$ k & ω are now continuous

$\omega(k)$ is the 1D acoustic phonon dispersion relation, which gives all natural frequencies allowed in our system.



Definitions

i. Debye frequency, ω_D: the highest natural frequency supported by a periodic system

ii. vibrational band: the frequencies for which a normal mode exists.

iii. 1st Brillouin zone: the smallest range of k values that describe all normal modes

i.e. $k = 0$ to π/a

iv. speed of sound: $\left.\frac{d\omega}{dk}\right|_{k=k_0}$

For small ka: $\omega(k) = 2\sqrt{\frac{\alpha}{m}} \sin\left(\frac{ka}{2}\right)$

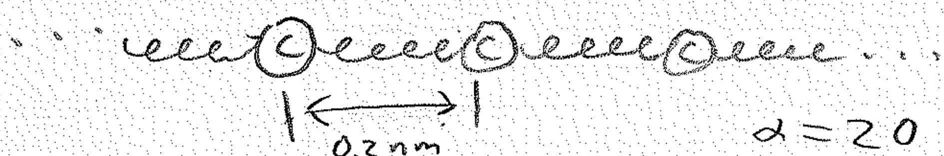
$$\simeq 2\sqrt{\frac{\alpha}{m}} \left(\frac{a}{2}k\right)$$

$$= \sqrt{\frac{\alpha}{m}}\, a k, \quad \frac{d\omega}{dk} \cong \sqrt{\frac{\alpha}{m}}\, a \equiv v_s$$

$\therefore$ compare vs. engineer's v_s definition

$$v_s = \sqrt{\frac{K}{\rho}} \quad K \rightarrow \text{stiffness}, \quad \rho \rightarrow \text{density}$$

Ex consider a 1D diamond:



$\alpha = 20$ N/m

$m = 12\, m_N$

$= 12\,(2\times10^{-27}$ kg$)$

i What is ω_D?

ii. What is the speed of sound in diamond?

(assume low energy normal modes carry the sound).

$$v_s \cong \sqrt{\frac{\alpha}{m}}\, a$$

$= 6000$ m/s (actual value in 3D diamond is $\sim 12{,}000$ m/s)

each normal mode is supported by a phonon.

phonon: is a quantized vibrational wavepacket that travels all the way through a perfect crystal with momentum $\hbar k$