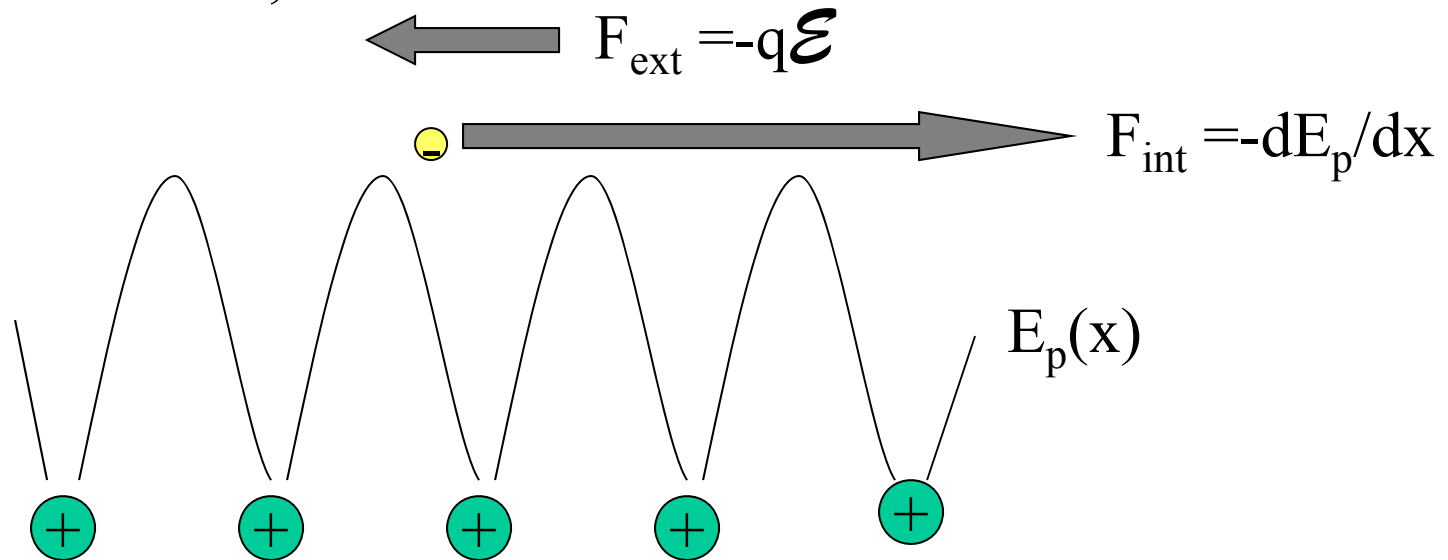


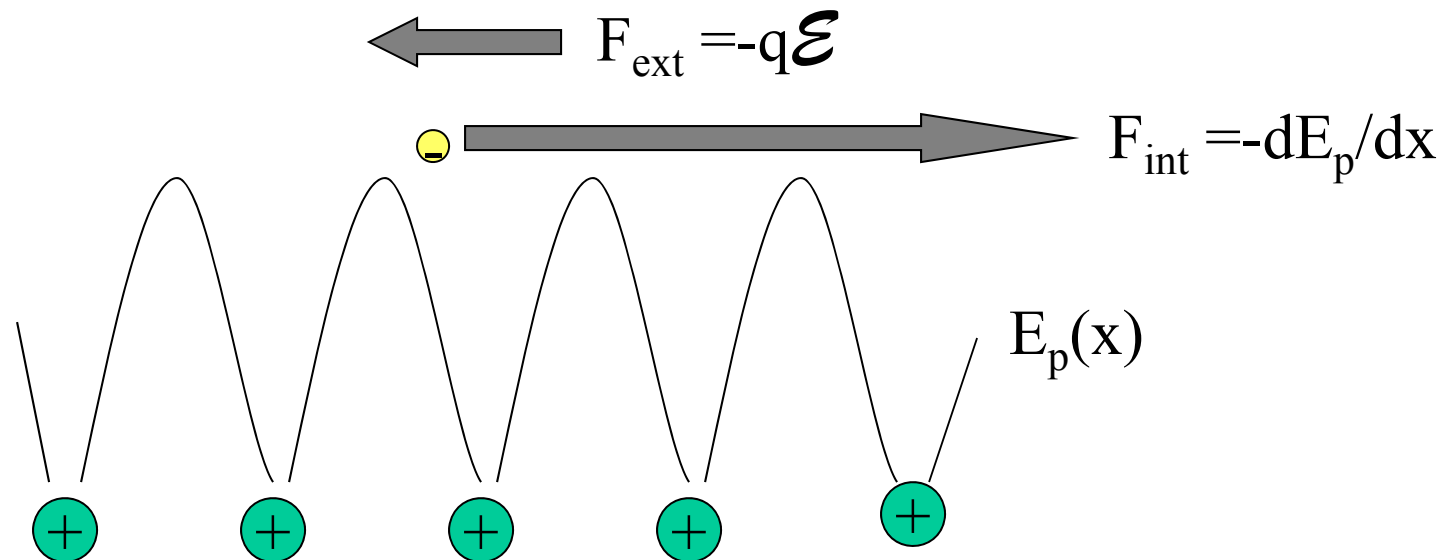
# Interpretation

- The electron is subject to internal forces from the lattice (ions and core electrons) AND external forces such as electric fields
- In a crystal lattice, the net force may be opposite the external force, however:



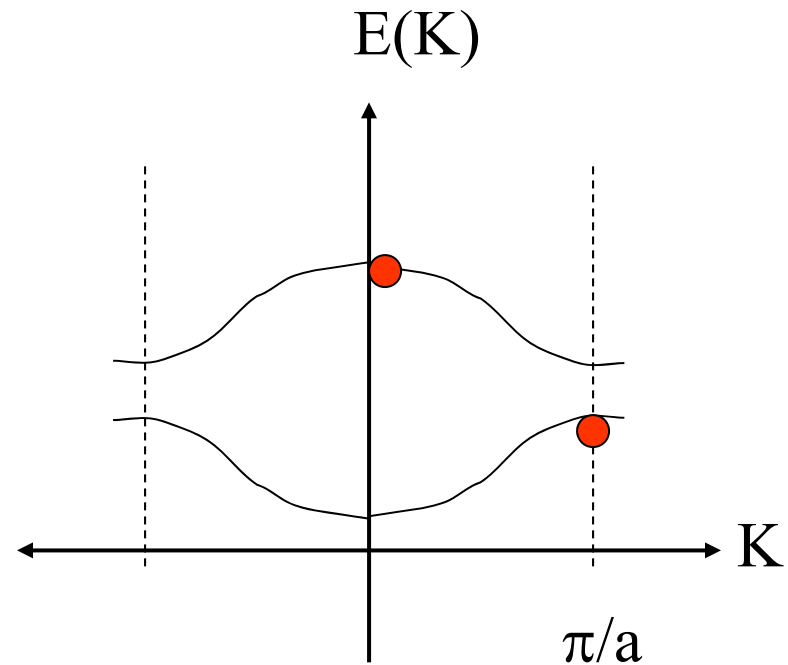
# Interpretation

- electron acceleration is not equal to  $F_{\text{ext}}/m_e$ , but rather...
- $a = (F_{\text{ext}} + F_{\text{int}})/m_e == F_{\text{ext}}/m^*$
- The dispersion relation  $E(K)$  compensates for the internal forces due to the crystal and allows us to use *classical* concepts for the electron as long as its mass is taken as  $m^*$



# The Hole

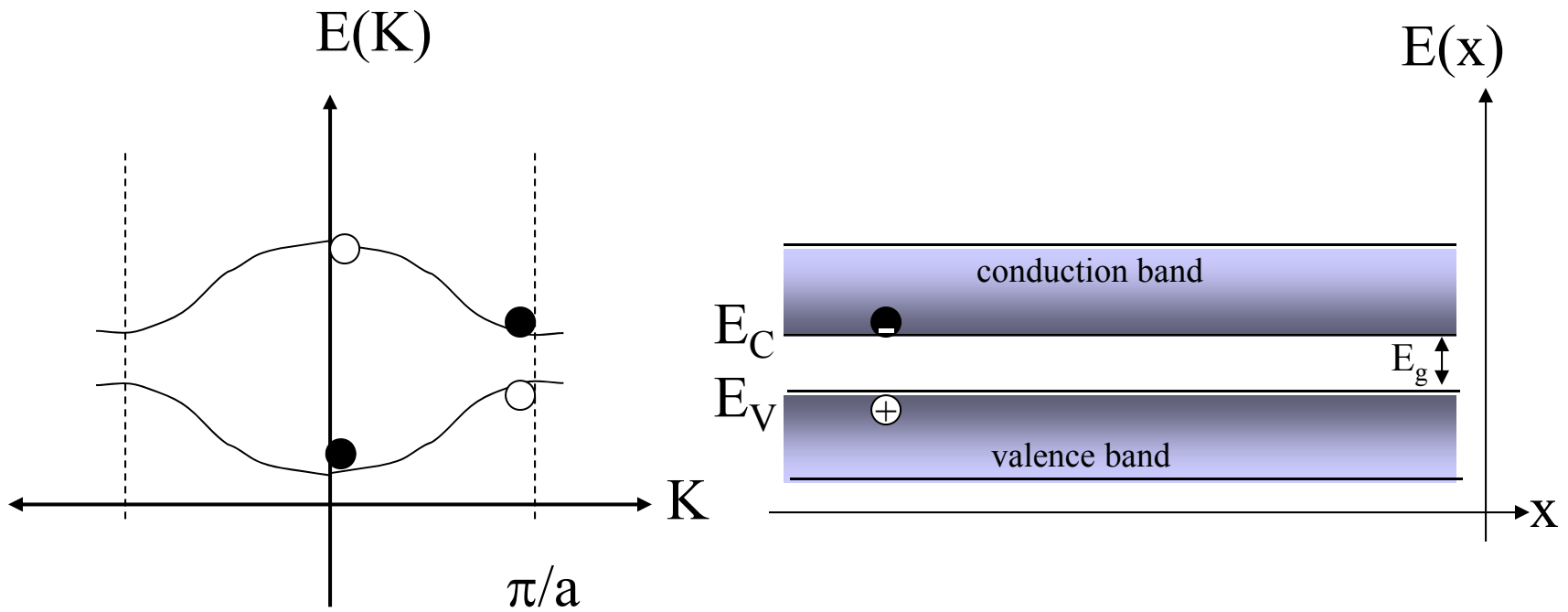
- The hole can be understood as an electron with negative effective mass
- An electron near the top of an energy band will have a negative effective mass
- A negatively charged particle with a negative mass will be accelerated like a positive particle with a positive mass (a hole!)



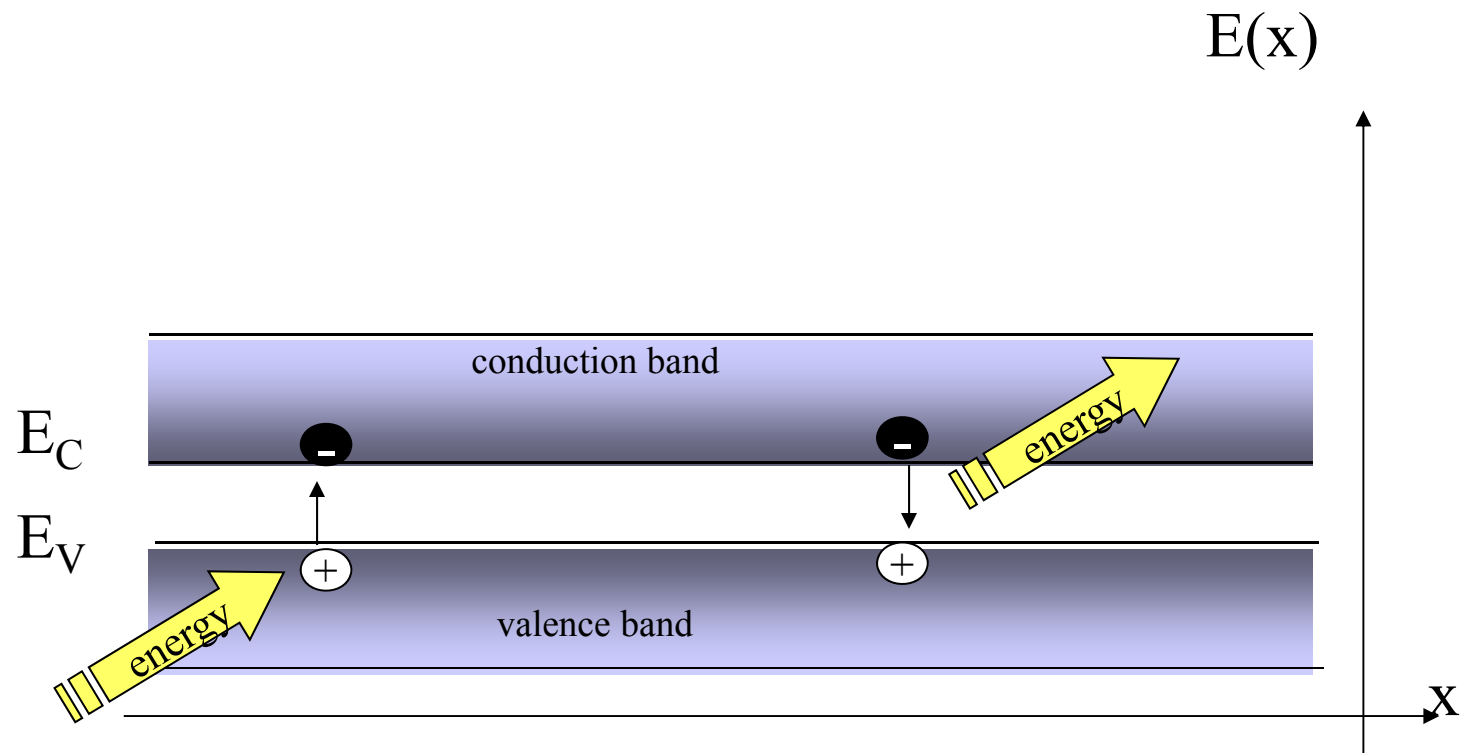
$$F = m^* a = Q\mathcal{E}$$

Without the crystal lattice, the hole cannot exist. It is an artifact of the periodic potential ( $E_p$ ) created by the crystal.

# $E(K)$ and $E(x)$

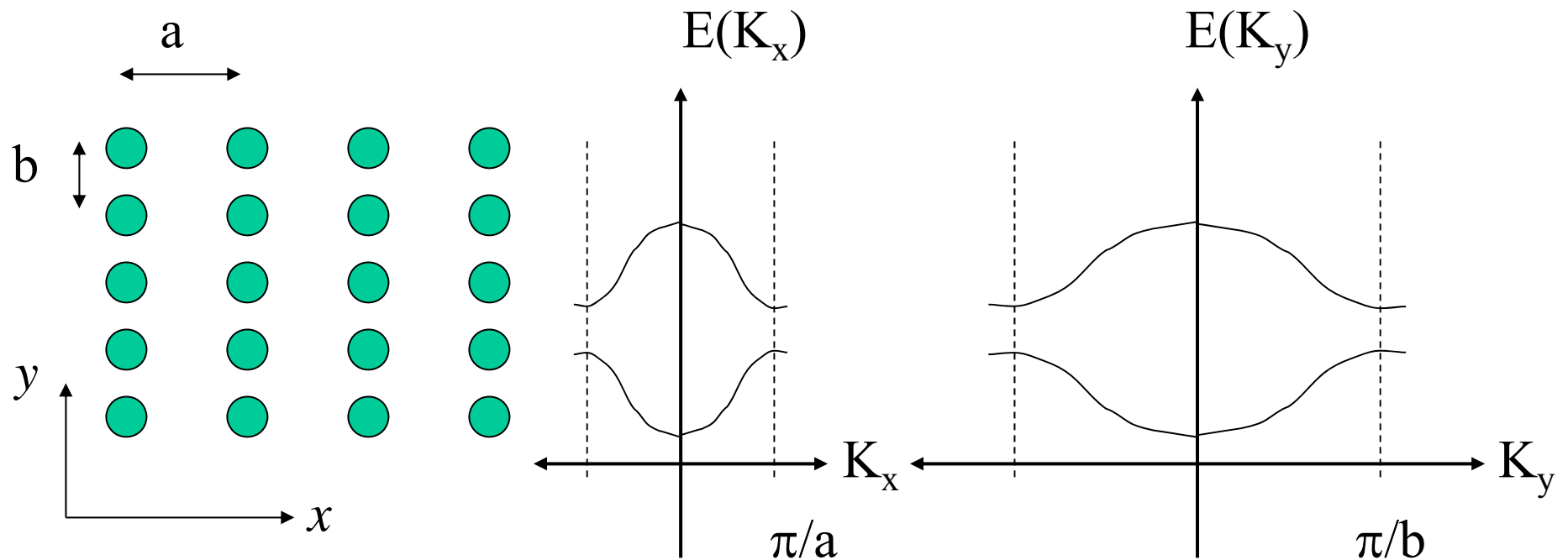


# Generation and Recombination of electron-hole pairs



# Non-cubic lattices:

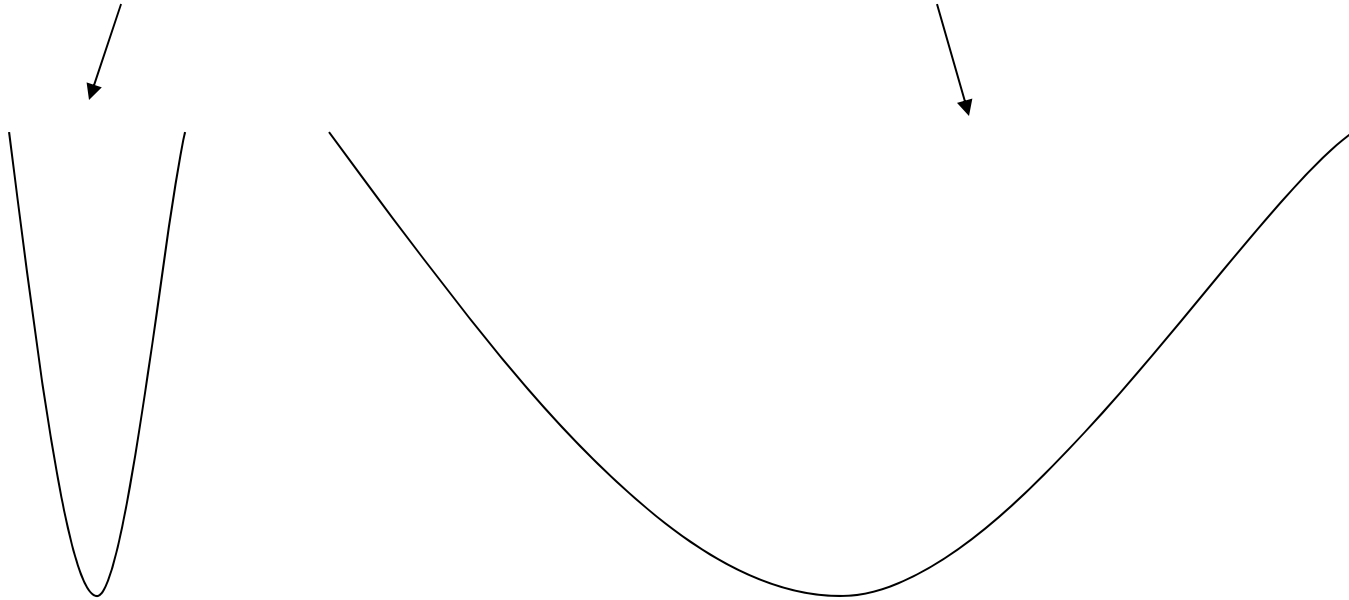
(FCC, BCC, diamond, etc.)



Different lattice spacings lead to different curvatures for  $E(K)$  and effective masses that depend on the direction of motion.

# Memory Aid

“a hairpin is lighter than a frying pan”

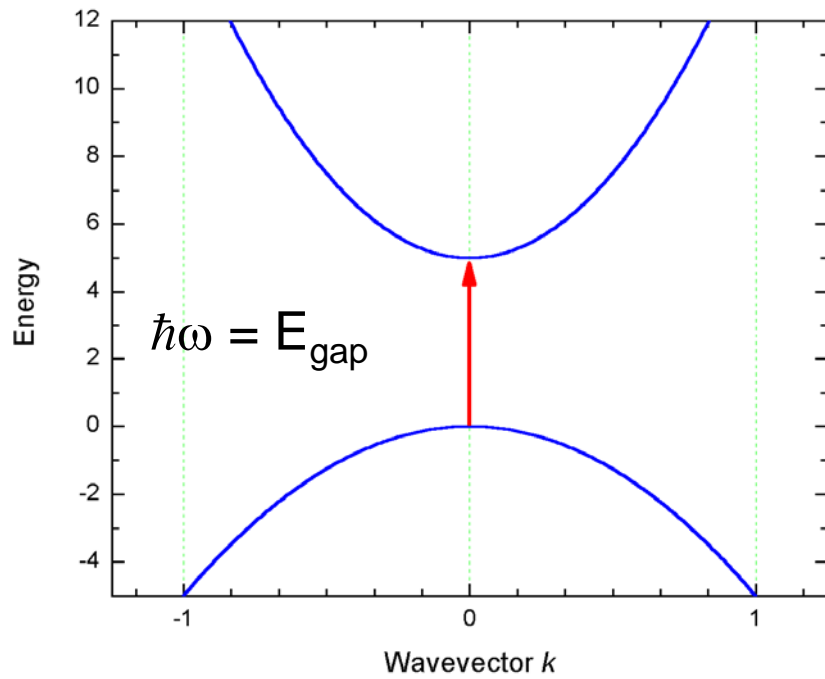


light  $m^*$   
(larger  $d^2E/dK^2$ )

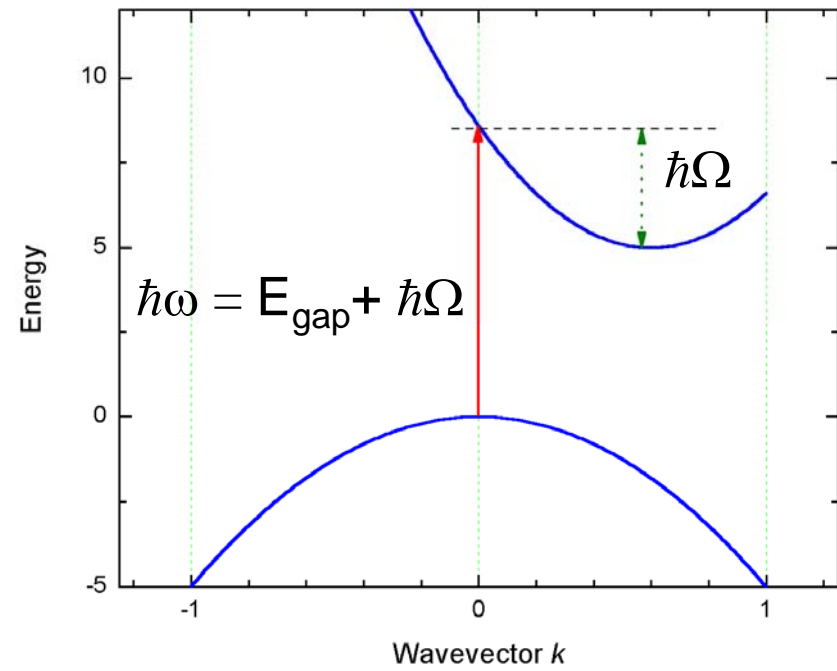
heavy  $m^*$   
(smaller  $d^2E/dK^2$ )



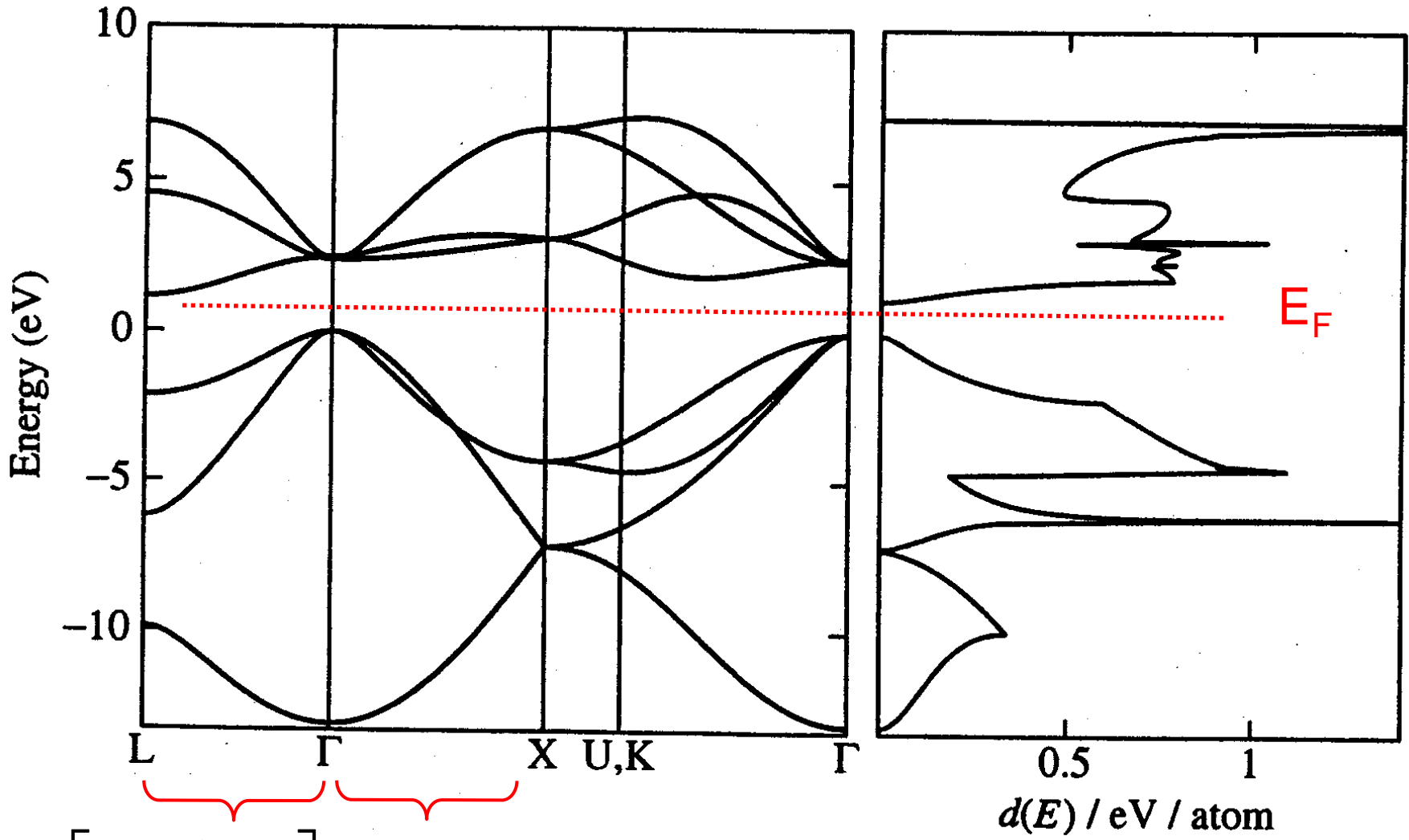
## Direct Energy Gap



## Indirect Energy Gap



# Silicon



$$\vec{k} = \left[ \frac{k}{\sqrt{3}}, \frac{k}{\sqrt{3}}, \frac{k}{\sqrt{3}} \right] \quad \vec{k} = [k, 0, 0]$$

# Light Emission

- energy (E) and momentum ( $\hbar\mathbf{K}$ ) must be conserved
- energy is released when a quasi-free electron recombines with a hole in the valence band:

$$\Delta E = E_g$$

- does this energy produce light (photon) or heat (phonon)?
- indirect bandgap:  $\Delta\mathbf{K}$  is large
  - but for a direct bandgap:  $\Delta\mathbf{K}=0$
- photons have very low momentum
  - but lattice vibrations (heat, phonons) have large momentum
- Conclusion: recombination ( $e^-+h^+$ ) creates
  - *light* in direct bandgap materials (GaAs, GaN, etc)
  - *heat* in indirect bandgap materials (Si, Ge)

