Inductors and Time-Dependent Signals

Concept

The purpose of this lab is to learn about time-dependent (AC) analysis of RL circuits using a function generator and an oscilloscope. The transient response of an RL circuit will be studied in the time-domain using the combination of a square-wave signal from a function generator and an oscilloscope. Frequency-domain behavior will be measured as well, and the response functions of RL circuits will be determined. Complex impedance of inductors will be introduced, and Fourier analysis of waveforms will be presented.

An inductor has very little *DC* resistance, but can have a large *AC* impedance. This is a consequence of Faraday's Law of Induction, which relates the rate of change of the magnetic field within the coil to an electric field. The relationship between the rate of change of current through the coil and the potential across it is DF = LdI / dt, where *L* is the inductance in Henrys. The complex impedance of an ideal inductor when the frequency of the applied signal is *W* is Z = iWL.

Helpful hints and warnings

The "ground symbol" in a circuit implies that the grounds (outer conductors or shields) of the signal generator and the oscilloscope are connected to the circuit at this point. Unlike the DMM, the signal generator and oscilloscope grounds can be connected only to the circuit ground. Thus, in the high-pass RL circuit, the oscilloscope can be used to measure the potential across only the inductor. Conversely, in the low-pass LR circuit, the scope can be used to measure the potential across only the resistor.

To read the inductance on the encapsulated inductors, look for three numbers such as 151. The first two digits are the real first two digits of the inductance. The third digit is the order of magnitude or power of ten. So, 152 means an inductance of $15 \cdot 10^2 = 1500$ something. For the encapsulated inductors in the laboratory, the "something" is nanoHenry or nH. So, 1500 nH = $1.5 \,\mu$ H. Measure both the inductance and resistance of your inductor using the LRC meter.

For your measurements, be sure to vary the frequency of the applied signal over a wide range, such as 1 Hz to 1 MHz, to make sure that you are working in the right range for your choice of R and L.

Experimental Instructions

1. Time-dependent analysis of *RL* circuits

a. Square waves and the *RL* circuit:



Figure 1: *RL* circuit with square-wave input.

- (i) The circuit shown in Fig. 1 has been built with R = 1 kW and L = 22 mH.
- (ii) Apply a square-wave signal and view the output on the oscilloscope.
- (iii) Measure the circuit time constant t by recording the output wave and fitting the appropriate part of the waveform to a model.
- **b**. Square waves and the *LR* circuit:



Figure 2: *LR* circuit with square wave input.

(i) Repeat the same measurements as above for the *LR* circuit.

2. Frequency response of RL circuit configurations



- **a**. Apply a sine-wave signal to each configuration and vary the frequency from ~10 Hz to 1 MHz. Make at least 20 measurements. Because you will be plotting your data versus $\log n$, make at least two measurements per decade of frequency. Determine the transmission function A(n) by dividing the output amplitude by the input amplitude. Be sure to measure the input amplitude from the function generator at each frequency, since the combination of your circuit and the limitations of the generator will lead to a signal that will generally decrease in amplitude with frequency. Measure the phase difference between the output and input signals. A good point on the waveform to use for such measurements is the point at which the trace crosses the 0 Volts line. If the period of the input signal is T and the displacement of the output signal is t, then the phase difference is f = 2pt/T.
- **b.** Plot the data and theoretical curves together. For the amplitude plots, use the decibel (dB) scale $20 \log_{10} A(n)$ for the vertical axis and $\log n$ for the horizontal axis. For the phase plots, use a linear vertical axis for the phase and $\log n$ for the horizontal axis. Determine the *corner* or *characteristic* frequency, f_c from the data plots by identifying the -3dB point on the amplitude plot (also known as a Bode plot) and the 45° point of the phase plot. Compare these to the expected theoretical value. Draw conclusions about the behavior of both circuits.

3. Fourier series analysis on oscilloscope

Spend a square-wave signal to your oscilloscope. Turn on the math FFT function (or use LabView FFT) and plot your square wave and Fouier spectrum on second axes. Record the input signal FFT and use it to verify the behavior of the Fourier coefficients of a square wave that you calculated in Homework 3.

Suppose you wanted to sample your RL circuit this square wave to get Bode plot information, explain why you would want to apply a square wave with a period approximately 100 times longer than the time constant *t* of the circuit. [feel free to try-it – not required!]

4. Fourier transform analysis

- **a.** Apply a pulse signal to the *RL* circuit. The pulse signal frequency is not critical. The frequency used above may work well. The critical aspect here is that the pulse width is about half the circuit time constant or smaller. If the input pulse width is too small, then the output signal may be too small to measure properly. The function generator will not allow the pulse width to be smaller than 0.1% of the pulse period. Adjust the oscilloscope time base so that the time per division is approximately 25 times the pulse width. If needed, adjust the pulse frequency so that there is only one pulse in the oscilloscope trace, but note that this will change the pulse width. These are approximate guidelines. The key aspect to getting good Fourier data in this experiment is to (a) have the FFT spectrum of the input signal include about 5 lobes of the expected *sinc* function and (b) have the first zero of that *sinc* function be larger than the circuit characteristic frequency. Switch both oscilloscope channels to AC coupling to suppress the large DC (zero frequency) component of the Fourier spectrum. Set the trigger level above zero to ensure that the scope captures the signal.
- **b.** Record the input signal FFT and use it to verify the behavior of the Fourier spectrum of a square pulse that you calculated in Homework 3.
- **c.** Adjust the Noise Threshold value in LabVIEW to "clean up" the Output/Input data. The Noise Threshold value ignores (*i.e.* sets equal to zero) any Output FFT values less than the threshold value.

Record Output/Input spectra for the RL (or LR) and compare the results from 2(b) or theory predictions.