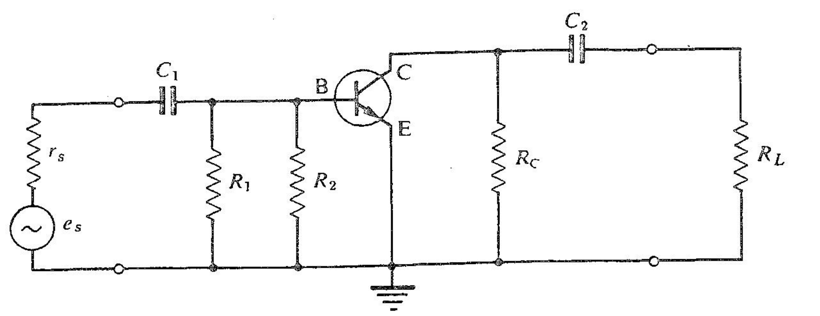


FIGURE 5.17 Ac equivalent circuit of the common emitter amplifier of Fig. 5.14(b).

may be an actual resistance. To be specific, suppose $R_L = 10\ \text{k}\Omega$. Then we must choose

$$\frac{1}{R_L C_2} = \omega_B \ll 2\pi \times 20\ \text{Hz}$$

or

$$C_2 \gg \frac{1}{2\pi 20 R_L} = \frac{1}{(2\pi)(20\ \text{Hz})(10^4\ \Omega)} = 1.6 \times 10^{-6}\ \text{F}$$

or $C_2 \gg 1.6\ \mu\text{F}$. A 10-μF, 25-V capacitor would be adequate. Unless there is some voltage at the top of R_L more positive than $V_C = +11$ V, the polarity of C_2 should be as shown in Fig. 5.14(b) with the positive side connected to the collector.

A glance at Fig. 5.17 shows that if capacitor C_2 is effectively a short circuit, then R_C and R_L will be in parallel. This will be true at all frequencies for which $1/(\omega C_2) \ll R_L$—above the breakpoint of the output high-pass filter formed by C_2 and R_L. Thus, at these frequencies the voltage gain expression (5.27) must be changed by replacing R_C by the parallel combination of R_C and R_L. Thus for the complete circuit, including the load resistance R_L, the voltage gain is

$$A_v = -\frac{\beta(R_C \| R_L)}{R_{BE}} \tag{5.28}$$

The measured gains and impedances for the common emitter amplifier circuit of Fig. 5.14(b) are

Voltage gain	$A_v = -180$
Circuit input impedance	$Z_{in} \cong 1\ \text{k}\Omega$
Circuit output impedance	$Z_{out} \cong 2\ \text{k}\Omega$
3-dB bandwidth	$= 400$ kHz

The resistance R in the input high-pass filter containing C_1 is not just R_1 but the total effective ac resistance (or impedance really) seen by the ac signal. That is, we must draw the ac equivalent circuit, which is obtained by realizing that the power supply terminal at 20 V dc is an ac ground. Thus the ac equivalent circuit is as shown in Fig. 5.17. Notice that R_E is not present in the ac equivalent circuit because we have $X_{C_E} \ll R_E$; the capacitance C_E is an ac short circuit. The input high-pass filter therefore consists of C_1 and the parallel combination of R_1, R_2, and the effective input resistance R_{BE} between the transistor base and emitter terminals. If we assume $R_{BE} = 1\ \text{k}\Omega$, then the effective resistance in the filter is equal to $R_1 \| R_2 \| R_{BE} = 740\ \Omega$, as shown in Fig. 5.18. Thus the input high-pass filter

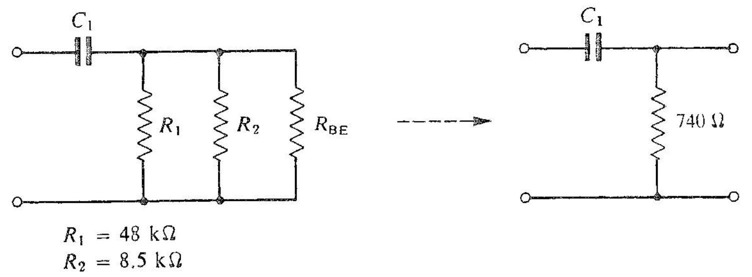


FIGURE 5.18 Input high-pass *RC* filter circuit.

consists of C_1 and a 740-Ω resistor. Therefore to pass frequencies down to 20 Hz we should have

$$\omega_B = 2\pi f_B = \frac{1}{R' C_1} \ll 2\pi(20\ \text{Hz}) \qquad \text{where } R' = R_1 \| R_2 \| R_{BE} = 740\ \Omega$$

which implies

$$C_1 \gg \frac{1}{R' 2\pi(20\ \text{Hz})} = \frac{1}{(2\pi)(20\ \text{Hz})(740\ \Omega)} \cong 10\ \mu\text{F}$$

Hence a 50- or 100-μF capacitor is needed for C_1. Fortunately, the dc voltage C_1 must withstand is not too high, so we may use a low-voltage electrolytic capacitor for C_1, which will be inexpensive. If the dc voltage of the input is more positive than +1.6 V, we hook up C_1 as shown in Fig. 5.14(b). If we expected a normal dc voltage of approximately 5 V to exist across C_1, we would choose a 10-V or a 25-V rating for C_1. The final amplifier circuit is shown in Fig. 5.14 with the dc voltages given for various points in the circuit.

To sum up, the common emitter amplifier has a large voltage gain, 180° phase difference between input and output, a large current gain, and medium input and output impedances. It is the most widely used transistor configuration.

5.8 COMMON COLLECTOR AMPLIFIER DESIGN

In the common collector configuration, shown in Fig. 5.19, the collector terminal is common to both the input and the output. The input is at the base, and the output is taken off the emitter, that is, across the emitter resistor R_E.

The dc bias design is similar to that for the common emitter amplifier. Assume $V_{bb} = 20$ V and $\beta = 100$. A dc load line is chosen and drawn on the I_C-versus-V_{CE} curves. The load line equation is

$$I_E = \frac{V_{bb}}{R_E} - \frac{V_{CE}}{R_E} \qquad (5.29)$$

The dc operating point is chosen (e.g., $V_{CE} = V_{bb}/2 = 10$ V, and $I_E = 10$ mA). R_E is now determined from Ohm's law:

$$R_E = \frac{V_E}{I_E} = \frac{V_{bb} - V_{CE}}{I_E} = \frac{10\text{ V}}{10\text{ mA}} = 1\text{ k}\Omega$$

(R_E could also be calculated from the load line I_E intercept where $V_{CE} = 0$.) $V_E = 10$ V, so $V_B = V_E + 0.6\text{ V} = 10.6$ V. If we now choose $I_D \gg I_B$, R_1 and R_2 are determined. $I_B = I_E/(\beta + 1) = 10\text{ mA}/101 \cong 0.1$ mA, so we choose $I_D = 10 I_B = 1$ mA. Then

$$R_1 = \frac{V_B}{I_D} = \frac{10.6\text{ V}}{1\text{ mA}} = 10.6\text{ k}\Omega$$

$$R_2 = \frac{V_{bb} - V_B}{I_D} = \frac{20\text{ V} - 10.6\text{ V}}{1\text{ mA}} = 9.4\text{ k}\Omega$$

The ac design is simply adding two coupling or blocking capacitors C_1 and C_2. C_1 and the parallel combination of R_1, R_2, and R_{BE} form a high-pass RC filter at the input. But R_{BE} is usually very large because when V_B becomes more positive, so does V_E, thereby making V_{BE} small. If V_{BE} is small, the base draws little current and R_{BE} is large. Thus $R_1 \| R_2 \| R_{BE} \cong R_1 \| R_2$ and we choose C_1 so that the filter breakpoint or knee is less than the lowest signal frequency ω_L: $C_1 \gg 1/(\omega_L R_1 \| R_2)$. Similarly, C_2 and the load form a high-pass RC filter at the output, so we choose C_2 so that the filter breakpoint or knee is less than the lowest signal frequency ω_L: $C_2 \gg 1/\omega_L R_L$.

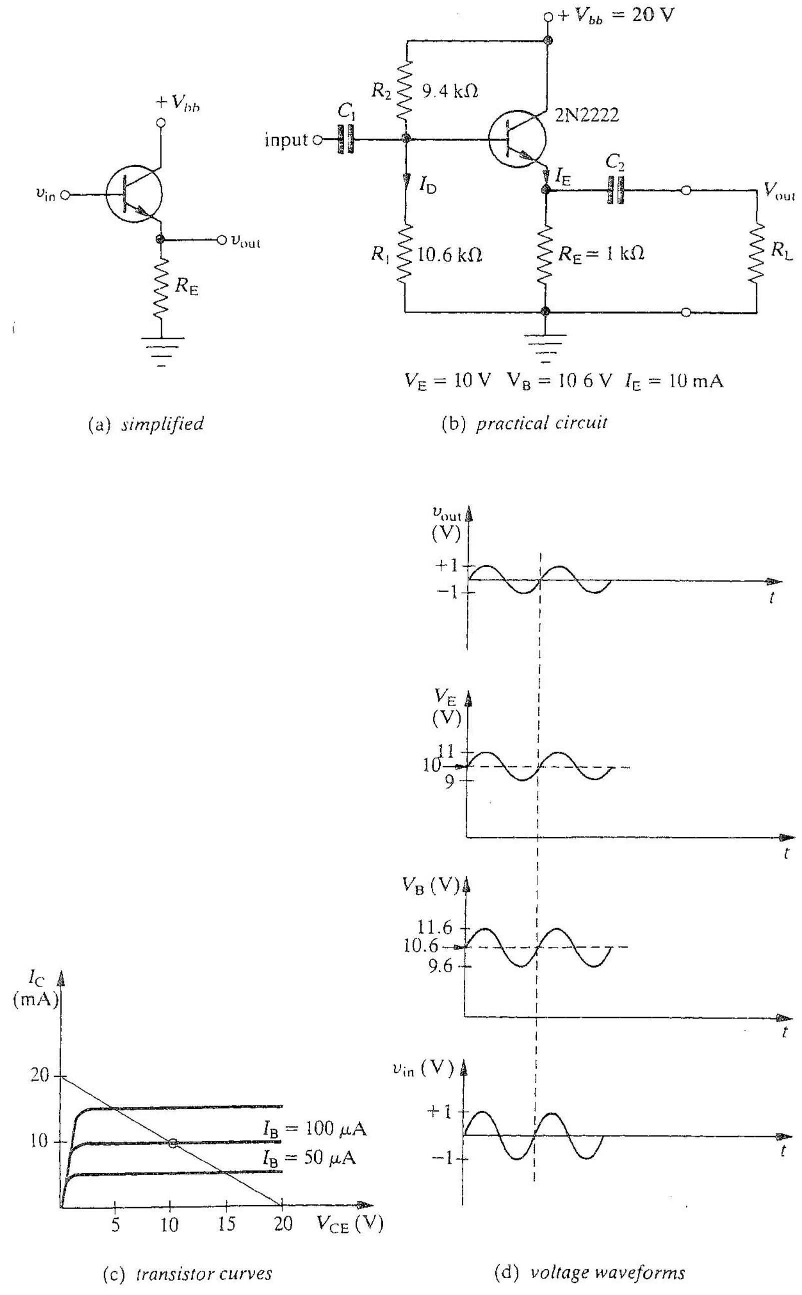
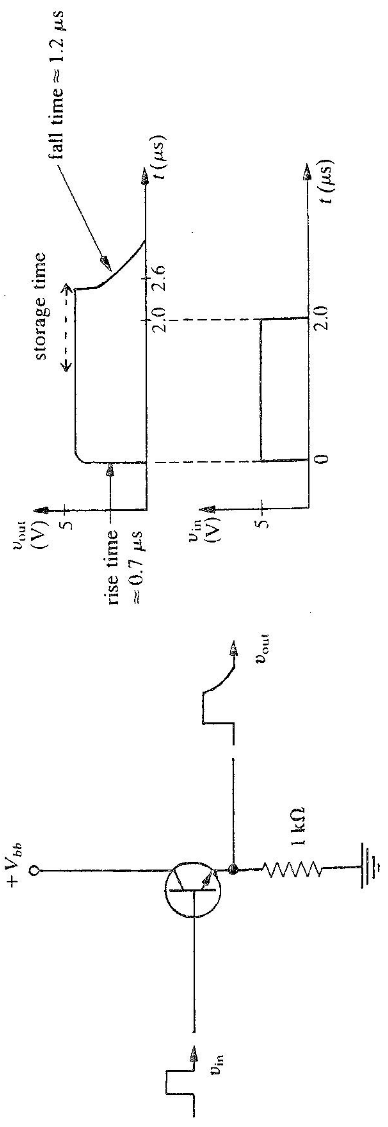


FIGURE 5.19 Common collector amplifier.



(e) *turn-on and turn-off speed*

FIGURE 5.19 Continued.

The measured gains and impedances of the complete circuit of Fig. 5.19 are

Voltage gain	$A_v = 1$
Circuit input impedance	$Z_{in} = 5\ \text{k}\Omega$
Circuit output impedance	$Z_{out} = 5\ \Omega$
3-dB bandwidth	> 10 MHz

Notice that the voltage gain of essentially 1 is reasonable because if the base-emitter junction is forward biased, V_{BE} will remain essentially constant at 0.6 V. Thus, the Kirchhoff voltage law for the input loop $V_{in} - V_{BE} - V_{out} = 0$ implies that $\Delta V_{in} = \Delta V_{out}$, which means a gain of 1.0. Here we have assumed that X_{C_1} is negligible. Also notice that as the input goes more positive, the transistor turns on and I_E increases, which means that the output *also* goes more positive. In other words, the output voltage swing is *in phase* with the input voltage swing; the common collector amplifier is thus often called the "emitter follower" because the output voltage on the emitter "follows" the input voltage at the base.

The large current gain is intuitively reasonable because of the steepness of the I_B (or I_E)-versus-V_{BE} curve; a very small change in V_{BE} (small change in I_B) will produce a very large change in I_E.

The high input impedance is essentially a result of the emitter voltage following the base voltage, thus tending to minimize the base-emitter difference voltage and, hence, the base current. The small base current means, of course, that the input resistance R_{BE} looking into the base is large.

The low output impedance occurs because the ac emitter current swing is much larger than the ac base current swing, and because the input (base) and output (emitter) voltage swings are nearly equal.

All of these properties of the emitter follower can be explained in terms of negative feedback, which we will do in Chapter 8.

An approximate voltage gain expression can be derived by assuming the transistor base presents an effective resistance R_{BE} to the base current (i.e., $V_{BE} = I_B R_{BE}$). Then the KVL implies

$$V_{in} - V_{BE} - V_{out} = 0$$

or

$$V_{in} = V_{BE} + V_{out} = I_B R_{BE} + V_{out}$$

Thus

$$A_v = \frac{\Delta V_{out}}{\Delta V_{in}} = \frac{\Delta V_{out}}{\Delta I_B R_{BE} + \Delta V_{out}} = \frac{i_E R_E}{i_B R_{BE} + i_E R_E}$$

Using $i_B \cong i_E/(\beta + 1)$, we get

$$A_v = \frac{R_E}{\dfrac{R_{BE}}{\beta + 1} + R_E} = \frac{1}{1 + \dfrac{R_{BE}}{(\beta + 1)R_E}}$$

From the transistor curves of Fig. 5.19(c), for the operating point $I_C = 10\text{ mA}$, $V_{CE} = 10\text{ V}$, $I_B = 100\ \mu\text{A}$, so $R_{BE} = V_{BE}/I_B \cong 0.6\text{ V}/100\ \mu\text{A} = 6\text{ k}\Omega$. Thus for $R_E = 1\text{ k}\Omega$ and $\beta = 100$,

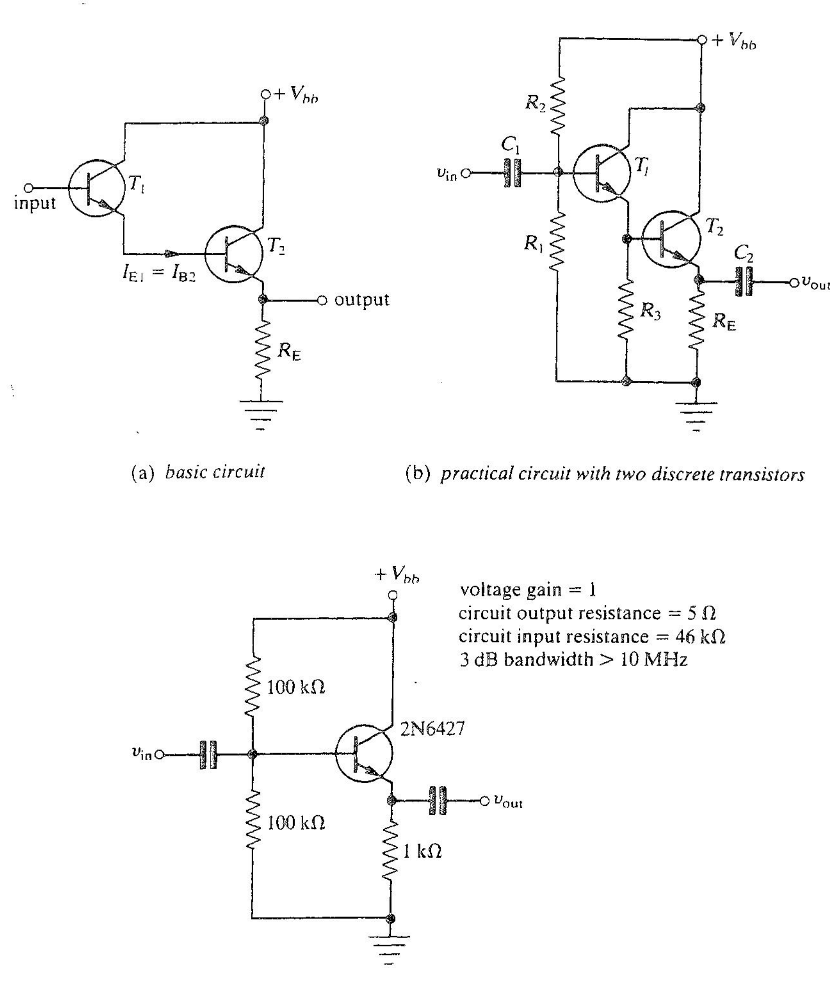
$$A_v \cong \frac{1}{1 + \dfrac{6\text{ k}\Omega}{101\text{ k}\Omega}} = 0.94$$

To sum up, the common collector amplifier has a unity voltage gain, a large current gain, no phase inversion between the input and output, high input impedance, and low output impedance. Its principal use is in driving low-impedance loads such as long cables or loudspeakers.

It should be mentioned that if a rectangular pulse is fed into an emitter follower, the output rise time will differ significantly from the output fall time. For the npn circuit of Fig. 5.19, the negative-going edge of the input pulse will turn the transistor *off*, and the output voltage at the emitter can fall only by the stray capacitance C_{stray} (between the output terminal and ground) discharging through the emitter resistor R_E. Thus, the output time constant for the negative-going edge will be $R_E C_{stray}$. However, for the positive-going edge of the input pulse, the npn transistor will be turned on and the output voltage will rise by the stray capacitance C_{stray} charging through the transistor, which, being turned on, presents a very small output resistance (usually about 100 Ω or less). Thus the time constant for the positive-going edge of the output pulse will be an order of magnitude less than for the negative-going edge. For a pnp transistor the reverse situation holds. The time constant for the negative-going edge will be less than for the positive-going edge of the output. To sum up, it is harder (slower) to turn a transistor off than to turn one on. This is shown in Fig. 5.19(e) along with the storage time which will be covered in 5.11.

An interesting and useful variation of a common collector amplifier or emitter follower is the "double emitter follower" or "Darlington" amplifier shown in Fig. 5.20. The Darlington circuit consists of two transistors connected together such that the emitter current of the first transistor provides the base current of the second transistor. The advantage of this arrangement is that the two transistors may be regarded as one "supertransistor" with a β essentially equal to the *product* of the β's of the two individual transistors. This can be seen from the following approximate argument:

$$I_{E_1} = I_{B_2} \quad \therefore \quad I_{B_1} \cong \frac{I_{E_1}}{\beta_1} = \frac{I_{B_2}}{\beta_1} = \frac{I_{E_2}/\beta_2}{\beta_1} = \frac{I_{E_2}}{\beta_1\beta_2} \tag{5.30}$$



(a) *basic circuit*

(b) *practical circuit with two discrete transistors*

(c) *emitter follower circuit with a 2N6427 single package Darlington transistor*

FIGURE 5.20 Darlington emitter amplifier.

Thus the base current of the input transistor T_1 is the emitter current of the output transistor T_2 divided by $\beta_1\beta_2$, the product of the β's of the two transistors. The same equations hold approximately for the ac signal currents: $i_{B_1} \cong i_{E_2}/\beta_1\beta_2$. An overall β of approximately $100 \times 100 = 10{,}000$ may be achieved, thus producing a circuit with an extremely high input impedance and an extremely low output impedance when connected as an emitter follower.

In practice the principal difficulty is to make the emitter current of T_1

large enough to turn on T_2 and to make the collector current of T_2 small enough to keep T_2 from overheating. Thus we would probably never use the same transistor type for T_1 and T_2 because, assuming $\beta = 100$, if we turn on T_1 by making $I_{E_1} = 1$ mA, then

$$I_{B_2} = I_{E_1} = 1\text{ mA} \quad \text{and} \quad I_{C_2} = \beta_2 I_{B_2} = 100\text{ mA}$$

which might destroy T_2. Or, if we choose $I_{C_2} = 10$ mA to avoid overheating T_2, then $I_{B_2} = I_{C_2}/\beta_2 = 10\text{ mA}/100 = 100\ \mu\text{A} = I_{E_1}$, which would probably not be enough to turn on T_1. And if T_1 is not turned on, its gain is extremely low. Hence, T_1 should be a transistor especially designed to operate at low collector currents of approximately 100 μA, or else T_2 must be capable of handling large collector currents of approximately 100 mA.

A practical circuit including biasing is shown in Fig. 5.20(b). Resistances R_1 and R_2 set the dc operating point: $V_{B_1} = R_1/(R_1 + R_2)V_{bb}$, $V_{E_2} = V_{B_1} - 1.2$ V, and $I_{E_2} = V_{E_2}/R_E$. Resistance R_3 deserves some comment. If the temperature rises, I_{E_1} may increase enough (from thermally generated minority carriers) to turn on T_2. R_3 prevents this by diverting some of I_{E_1} away from the base of T_2.

Darlington transistors are available commercially in a single package with three leads: the emitter (of T_2), the base (of T_1), and the collector (both T_1 and T_2). They are widely used to control large currents (I_{E_2}) with small currents (I_{B_1}). Examples are the TIP-122 (npn) and the TIP-127 (pnp) power Darlingtons at $1.00 each. They will carry up to $I_C = 5$ A and can dissipate up to 65 W. A low-power Darlington is the npn 2N6427 which has a β of 5000 or more and costs $0.35 each.

5.9 COMMON BASE AMPLIFIER DESIGN

In the common base configuration, shown in Fig. 5.21, the base terminal is common to both the input and the output. The input is at the emitter, and the output is taken off the collector, that is, across the collector resistor R_C.

The dc bias design is straightforward. Assume $V_{bb} = 20$ V and $\beta = 100$. The load line equation is

$$I_C = \frac{V_{bb}}{R_C + R_E} - \frac{1}{R_C + R_E} V_{CE} \qquad (5.31)$$

The dc operating point is chosen, say, $V_{CE} = 10$ V and $I_C = 2$ mA. The sum $R_C + R_E$ is determined by the load line I_C intercept where $V_{CE} = 0$ or from

$$I_C(R_C + R_E) = V_{bb} - V_{CE}$$

Thus, $$R_C + R_E = \frac{10\text{ V}}{2\text{ mA}} = 5\text{ k}\Omega$$

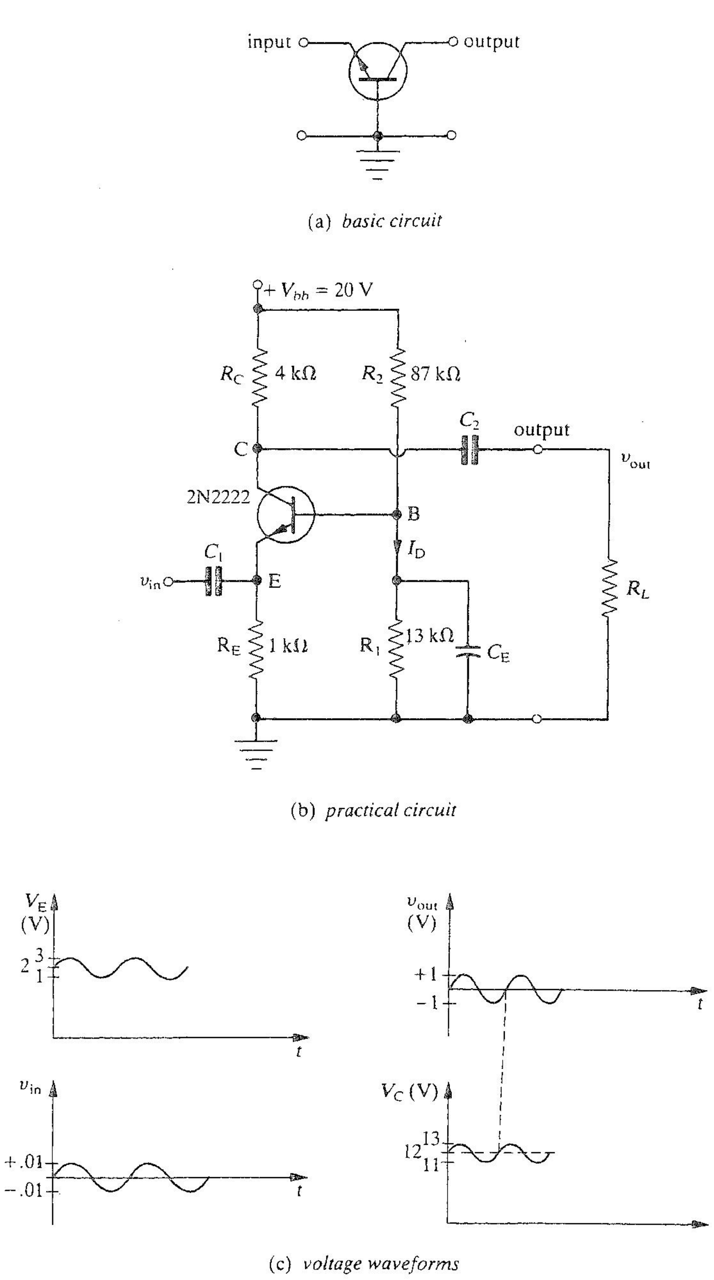


FIGURE 5.21 Common base amplifier.

For high-voltage gain we want a large R_C, so we choose $R_E = 1\ k\Omega$ and $R_C = 4.0\ k\Omega$. Then

$$V_E = I_E R_E = I_C R_E = (2\ mA)(1\ k\Omega) = 2\ V$$

and

$$V_B = V_E + 0.6\ V = 2.6\ V$$

$$I_B = I_C/\beta = 2\ mA/100 = 0.02\ mA, \quad \text{so we choose} \quad I_D = 10 I_B = 0.2\ mA.$$

Then

$$R_1 = \frac{V_B}{I_D} = \frac{2.6\ V}{0.2\ mA} = 13\ k\Omega$$

$$R_2 = \frac{V_{bb} - V_B}{I_D} = \frac{20\ V - 2.6\ V}{I_D} = 87\ k\Omega$$

The ac bias design involves adding the two blocking capacitors C_1 and C_2 in the usual way to make the high-pass RC filters formed by C_1 and R_E and by C_2 and R_L pass the signal frequencies. But we must also add a bypass capacitor C_E to make the base a good ac ground so that the base will be truly common to both the input and the output. This means that the reactance of C_E should be much less than R_1 at the lowest signal frequency ω_L: $1/\omega_L C_E \ll R_E$.

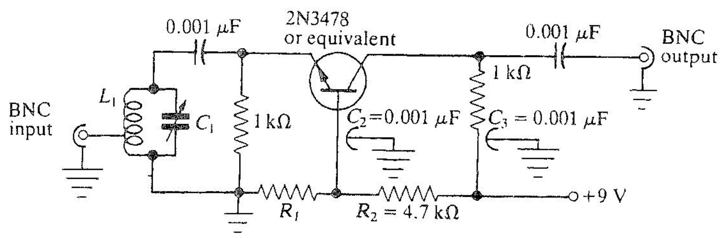
The measured values for the gains and impedances of the common base amplifier of Fig. 5.20 are

Voltage gain	$A_v = 250$
Circuit input impedance	$Z_{in} = 17\ k\Omega$
Circuit output impedance	$Z_{out} = 3.5\ k\Omega$
3-dB bandwidth	$= 150\ kHz$

The high-voltage gain is reasonable because the small input voltage is applied to the emitter, and the base voltage is kept constant (an ac ground). Thus, provided X_{C_1} is negligible, the full input voltage will appear between the base and emitter, just as in the case of the common emitter amplifier. However, in the common base amplifier when the input goes positive, the output *also goes positive*. A positive input makes the emitter more positive than the base, or, equivalently, the base is made more negative than the emitter, thus turning the transistor off more (i.e., the collector current decreases). This decrease in the collector current causes the output voltage at the collector to rise, thus giving a positive-going output. In other words, the output is *in phase* with the input but much larger in magnitude.

To sum up, the common base amplifier has a high voltage gain, no phase inversion between input and output, a relatively low input impedance, and a relatively high output impedance. Its principal use is at very high frequencies (50 MHz and higher), because the transistor base physically separates the input (emitter) and the output (collector). Thus, there is minimal feedback possible from output to input, which minimizes unwanted oscillations in practical circuits.

An example of a practical common base transistor 400-MHz amplifier is shown in Fig. 5.22. The common base configuration provides excellent



C_1: = 1 to 7.5 pF
L_1 : $\frac{1}{4}$ in. O.D. copper tube, 3½ in. long

FIGURE 5.22 Common base 400-MHz amplifier.

isolation between the input and the output, thus producing an amplifier circuit with less tendency to oscillate than the common emitter configuration. The input "coil" L_1 resonates with C_1 at 400 MHz. L_1 is the small inductance of a $\frac{1}{4}$-in. diameter copper tube, approximately 0.05 μH. The base is an ac ground because of C_2. Notice the extra filtering of the supply voltage from C_3. Both C_2 and C_3 are "feed-through" bypass capacitors, so R_1 and R_2 are on the outside of the metal chassis, while all the other components are on the inside where they are well shielded. R_1 is adjusted to set the dc operating point (I_C) for the best measured signal-to-noise ratio.

Finally, for all three transistor configurations—the common emitter, common collector (emitter follower), and common base—the following checks should be made in debugging a circuit designed to amplify a small input signal (i.e., analog amplifiers, as opposed to a "switching" circuit that just turns on or off):

1. Is the base-emitter junction forward biased at approximately 0.6 V? For an npn transistor the base should be positive with respect to the emitter.
2. Is the base-collector junction reverse biased by at least 2 or 3 V? For an npn transistor the collector should be positive with respect to the base.
3. Is the dc collector current at least 1 mA? Check by measuring the dc voltage drop across R_C or R_E.
4. Is the divider current I_D large compared to I_B?

5.10 TRANSISTOR EQUIVALENT CIRCUITS

A transistor equivalent circuit is basically a circuit consisting of ideal voltage and/or current "sources" or "generators" and passive components (R, L, and C), which acts electrically exactly like the transistor. In other words, the transistor can be replaced by an appropriate collection of generators and passive components of the equivalent circuit. The advantage of the equivalent circuit is that one can predict the transistor's exact behavior (gain, etc.) by writing down the Kirchhoff voltage and current laws for the various loops and junctions and solving for the desired quantities by using only algebra and Ohm's law. A simple example will show how the calculations are performed. Suppose a certain transistor's equivalent circuit is simply an ideal voltage generator of magnitude Av_{in} in series with a resistance R as shown in Fig. 5.23. Algebraically, A is a

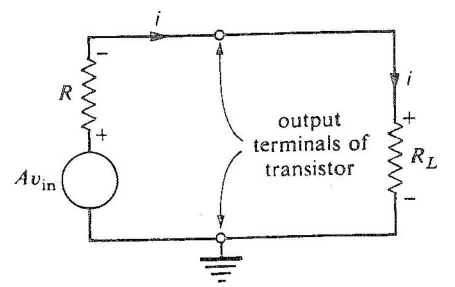


FIGURE 5.23 Simple equivalent circuit.

positive number, and v_{in} is the amplitude of the input voltage to the transistor. The R_L is the load resistance connected to the output terminals. The voltage generator produces a voltage A times as large as the input, so one might intuitively think of A as the voltage gain at this stage of the calculation. However, the voltage gain is

$$A_v = \frac{v_{out}}{v_{in}} = \frac{iR_L}{v_{in}} \tag{5.32}$$

and the current i can be obtained from the Kirchhoff voltage equation for the loop containing the generator, R, and R_L.

$$Av_{in} - iR - iR_L = 0 \quad \text{or} \quad i = \frac{Av_{in}}{R + R_L} \tag{5.33}$$

Thus the voltage gain becomes

$$A_v = \frac{v_{out}}{v_{in}} = \frac{iR_L}{v_{in}} = \frac{Av_{in}R_L}{(R + R_L)v_{in}}$$

or

$$A_v = A\left(\frac{R_L}{R + R_L}\right) \tag{5.34}$$

We see that the voltage gain depends on A, R, and R_L, and only as R_L becomes very large compared to R do we get $A_v \cong A$.

Let us now develop a perfectly general equivalent circuit that will apply to *any* four-terminal device, including a transistor, as shown in Fig. 5.24. I_1 and I_2 are the currents flowing into the input and output, respec-

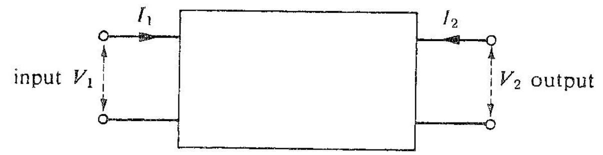


FIGURE 5.24 General four-terminal black box.

tively; and V_1 and V_2 are the voltage differences across the input and the output terminals, respectively. We have four variables: I_1, V_1, I_2, and V_2; they represent the total, instantaneous values of the currents and voltages. There are two Kirchhoff voltage equations we can write—one for the input loop and one for the output loop. In general, they can be expressed as

$$f(I_1, V_1, I_2, V_2) = 0 \tag{5.35}$$

and

$$g(I_1, V_1, I_2, V_2) = 0 \tag{5.36}$$

where f and g are mathematical functions whose exact form depends on the internal structure of the black box. We now choose (arbitrarily) to solve for the input voltage V_1 and the output current I_2 in terms of the other two variables I_1 and V_2. In mathematical language we are treating I_1 and V_2 as the independent variables and I_2 and V_1 as the dependent variables. This can always be done by solving (5.35) for V_1 and substituting the resulting expression for V_1 into (5.36), thus obtaining an equation not involving V_1.

$$g(I_1, I_2, V_2) = 0$$

This equation can then be solved for I_2 in terms of I_1 and V_2:

$$I_2 = I_2(I_1, V_2) \tag{5.37}$$

Similarly, (5.36) can be solved for I_2 and substituted in (5.35), which can then be solved for V_1:

$$V_1 = V_1(I_1, V_2) \tag{5.38}$$

In general, we are interested in the response of the transistor to ac signals, so we will take the differential of (5.37) and of (5.38) to obtain expressions for the change in I_2, dI_2, and the change in V_1, dV_1.

$$dI_2 = \left(\frac{\partial I_2}{\partial I_1}\right)_{V_2} dI_1 + \left(\frac{\partial I_2}{\partial V_2}\right)_{I_1} dV_2 \tag{5.39}$$

$$dV_1 = \left(\frac{\partial V_1}{\partial I_1}\right)_{V_2} dI_1 + \left(\frac{\partial V_1}{\partial V_2}\right)_{I_1} dV_2 \tag{5.40}$$

Let us change to a notation useful for considering ac signals or any change in the currents and voltages. Let $i_2 = dI_2$, $i_1 = dI_1$, $v_1 = dV_1$, and $v_2 = dV_2$. That is, lowercase v's and i's refer to *changes* in the voltages and currents or, equivalently, to *ac* signal *amplitudes*. With this notation,

$$i_2 = \left(\frac{\partial I_2}{\partial I_1}\right)_{V_2} i_1 + \left(\frac{\partial I_2}{\partial V_2}\right)_{I_1} v_2 \tag{5.41}$$

$$v_1 = \left(\frac{\partial V_1}{\partial I_1}\right)_{V_2} i_1 + \left(\frac{\partial V_1}{\partial V_2}\right)_{I_1} v_2 \tag{5.42}$$

We now define the h parameters for our four-terminal black box in terms of the partial derivatives:

$h_{21} \equiv \left(\frac{\partial I_2}{\partial I_1}\right)_{V_2}$ *a pure number* $\qquad h_{22} \equiv \left(\frac{\partial I_2}{\partial V_2}\right)_{I_1}$ *a conductance in mhos or siemens* (1 mho = 1 ohm^{-1} = 1 siemen)

$h_{11} \equiv \left(\frac{\partial V_1}{\partial I_1}\right)_{V_2}$ *a resistance in ohms* $\qquad h_{12} \equiv \left(\frac{\partial V_1}{\partial V_2}\right)_{I_1}$ *a pure number* (5.43)

The h parameters have a variety of dimensions; hence, the name "hybrid" parameters. With this notation we have

$$i_2 = h_{21}i_1 + h_{22}v_2 \tag{5.44}$$

$$v_1 = h_{11}i_1 + h_{12}v_2 \tag{5.45}$$

Equations (5.44) and (5.45), relating the dependent variables i_2 and v_1 to the independent variables i_1 and v_2 via the h parameters, determine the equivalent circuit. The term $h_{21}i_1$ means there is a current generator of magnitude h_{21} times i_1. The term $h_{22}v_2$ means the voltage v_2 appears across a conductance h_{22} (or equivalently across a resistance of $1/h_{22}$ ohms). The term $h_{11}i_1$ means the current i_1 flows through an effective resistance of h_{11} ohms. The term $h_{12}v_2$ means there is a voltage generator of magnitude $h_{12}v_2$. Therefore, we can draw the equivalent circuit of Fig. 5.25. Equation (5.44) is seen to be merely the Kirchhoff current equation for the output, and equation (5.45) is merely the Kirchhoff voltage equation for the input. The important point here is that the perfectly general mathematical treatment that led to equations (5.44) and (5.45) implies the equivalent circuit of Fig. 5.25.

Some physical feeling for the h parameters can be obtained from the equivalent circuit of Fig. 5.25. The h_{11} parameter is a resistance in the input circuit, usually called the "input resistance." The term $h_{12}v_2$ is the amplitude of a voltage generator in the input; it represents how much of the output voltage v_2 is transferred or fed back to the input, and h_{12} is called the "reverse voltage transfer ratio." The word "reverse" is used to denote the transfer from the output back to the input. The h_{21} parameter represents how much of the input current i_1 is transferred to the output; h_{21} is called the "forward current transfer ratio." The higher the value of h_{21} is, the larger is the change in output current for a given input current change. We call h_{22} the "output admittance" because it is an admittance or conductance directly across the output terminals.

The preceding development is entirely mathematical and is exact; that is, no approximations have been made except that v and i must refer to small signals because equations (5.41) and (5.42) hold exactly only for infinitesimal i's and v's. However, we have not yet shown that the equivalent circuit of Fig. 5.25 is, in fact, a representation of a *real* transistor. To do so, we must look at the *experimental* input and output curves for a transistor and see if we can accurately represent the transistor with the equivalent circuit by choosing numerical values for the h parameters. For a useful equivalent circuit we would like a constant set of h parameters to represent the experimental curves over a *wide* range of currents and

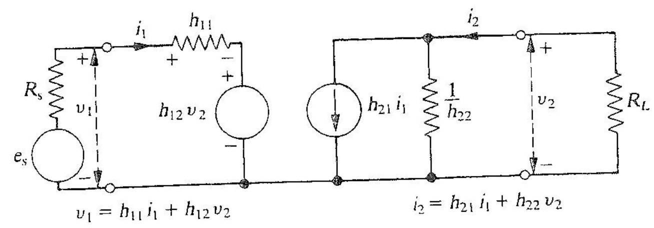


FIGURE 5.25 Transistor h parameter equivalent circuit.

voltages. The equivalent circuit is simply not useful if the h parameters are strong functions of current and voltage.

A transistor has three terminals, whereas our black box from which the equivalent circuit was developed has four. Hence, for the equivalent circuit to be applied to a transistor, one transistor terminal must be common between the input and the output. This can be either the emitter, the collector, or the base, called, respectively, the "common emitter" (CE), the "common collector" (CC), or the "common base" (CB) configuration.

For the widely used common emitter configuration, we have:

$$h_{21} = \left(\frac{\partial I_2}{\partial I_1}\right)_{V_2} = h_{fe} = \left(\frac{\partial I_C}{\partial I_B}\right)_{V_{CE}} = \beta \tag{5.46}$$

The h_{fe} parameter is often loosely called the "current gain." Capital letter subscripts refer to dc values; thus

$$h_{FE} = \frac{I_C}{I_B}$$

In most cases, $h_{fe} \cong h_{FE}$.

$$h_{11} = \left(\frac{\partial V_1}{\partial I_1}\right)_{V_2} = h_{ie} = \left(\frac{\partial V_{BE}}{\partial I_B}\right)_{V_{CE}} \tag{5.47}$$

But $I_E = I_0(e^{eV_{BE}/kT} - 1)$ for the base-emitter junction from Chapter 4, and $I_E = (\beta + 1)I_B$, so

$$I_B = \frac{I_0}{(\beta+1)}(e^{eV_{BE}/kT} - 1) \cong \frac{I_0}{(\beta+1)} e^{eV_{BE}/kT}$$

Thus $$h_{ie} = \left(\frac{\partial V_{BE}}{\partial I_B}\right)_{V_{CE}} = \left(\frac{\partial I_B}{\partial V_{BE}}\right)^{-1} = \left[\frac{\partial}{\partial V_{BE}}\left(\frac{I_0}{(\beta+1)} e^{eV_{BE}/kT}\right)\right]^{-1}$$

$$h_{ie} = \frac{(\beta+1)kT}{eI_0} e^{-eV_{BE}/kT}$$

or for $T = 300$ K and $\beta = 100$

$$h_{ie} \cong \frac{\beta kT}{eI_C} \cong \frac{2.6\ \text{V}}{I_C} \tag{5.48}$$

Thus, if $I_C = 100\ \mu$A, then $h_{ie} \cong 26$ kΩ. If $I_C = 1$ mA, then $h_{ie} \cong 2.6$ kΩ. A useful formula is

$$h_{ie}(\text{in k}\Omega) \cong \frac{2.6\ \text{k}\Omega}{I_C(\text{in mA})} \tag{5.49}$$

The effective ac signal resistance of the base-emitter junction R_{BE} which we introduced earlier in this chapter is equal to h_{ie}.

The h parameters will be different for the three configurations, and letter subscripts are usually used to distinguish one from another according to Table 5.2. The second letter of the subscript denotes the common terminal of the configuration, e.g., h_{fe} is h_{21} for the common emitter configuration and h_{fc} is h_{21} for the common collector configuration.

TABLE 5.2. *h* Parameter Values for the Three Transistor Configurations

h parameter	CE	CC	CB
h_{11}	$h_{ie} \approx 1$ kΩ	$h_{ic} \approx 1$ kΩ	$h_{ib} \approx 10\ \Omega$
h_{12}	$h_{re} \approx 10^{-4}$	$h_{rc} \approx 1$	$h_{rb} \approx 2 \times 10^{-4}$
h_{21}	$h_{fe} \approx 100$	$h_{fc} \approx -100$	$h_{fb} \approx -0.99$
h_{22}	$h_{oe} \approx 2 \times 10^{-5}$ siemens	$h_{oc} \approx 2 \times 10^{-5}$ siemens	$h_{ob} \approx 2 \times 10^{-7}$ siemens
Δh	$\Delta_e h \approx 2 \times 10^{-2}$	$\Delta_c h \approx 100$	$\Delta_b h \approx 4 \times 10^{-4}$
$\Delta h \equiv h_{11}h_{12} - h_{12}h_{21}$			

CE = common emitter
CC = common collector
CB = common base

From the general h parameter equivalent circuit of Fig. 5.25 and (5.44) and (5.45), we can calculate the voltage gain, the current gain, the input impedance, and the output impedance of the transistor. The results are

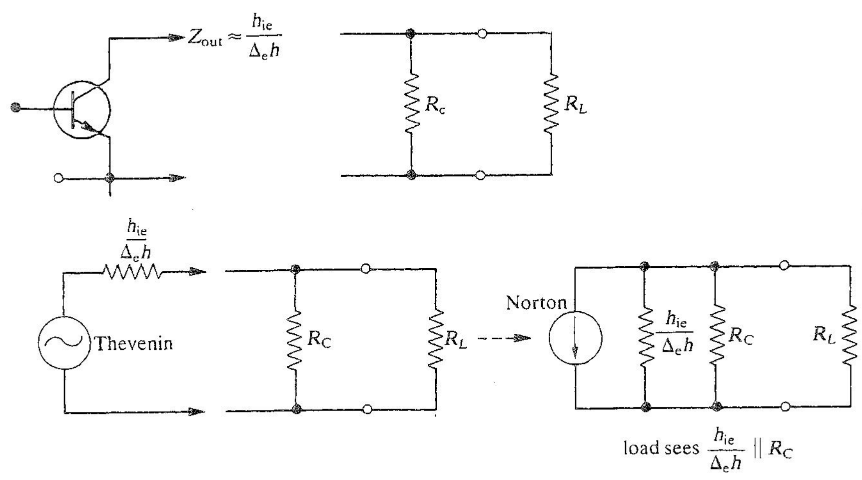


FIGURE 5.26 Transistor "sees" $R_C \parallel R_L$.

voltage gain: $$A_v = \frac{-h_{21}R_L}{\Delta h R_L + h_{11}} \tag{5.50}$$

current gain: $$A_i = \frac{h_{21}}{1 + h_{22}R_L} \tag{5.51}$$

input impedance: $$Z_{in} = \frac{\Delta h R_L + h_{11}}{1 + h_{22}R_L} \tag{5.52}$$

output impedance: $$Z_{out} = \frac{h_{11} + R_s}{\Delta h + h_{22}R_s} \tag{5.53}$$

where $$\Delta h = h_{11}h_{22} - h_{12}h_{21}$$

Equations (5.50)–(5.53) apply to the *transistor alone*, with no input biasing resistors, collector resistor, or load resistor.

For the common emitter configuration, R_C and R_L (see Fig. 5.26) are in parallel in terms of ac, so R_L in (5.50)–(5.52) must be replaced by $R_C \| R_L$. The output impedance expression (5.53) for the transistor alone refers to the impedance looking into the transistor collector terminal. Therefore the load R_L sees the *transistor* output impedance in parallel with R_C. The output impedance can never be larger than R_C.

The input impedance (5.52) is that of the transistor alone. A signal source sees $R_1 \| R_2$ in parallel with (5.52). Thus the source sees $Z_{in} \| R_1 \| R_2$. The input impedance can never be larger than $R_b = R_1 \| R_2$.

Similar arguments apply to the CB and CC configurations.

The approximate gains and impedances for the transistor *circuits* are given in Table 5.3 in terms of the widely used common emitter parameters.

TABLE 5.3. Approximate Gains and Impedances for Transistor Circuits

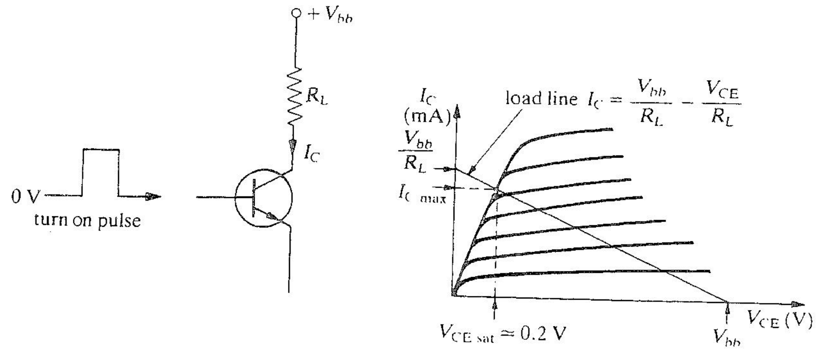
	CE	*CC*	*CB*
A_v	$-\frac{h_{fe}(R_C \| R_L)}{h_{ie}} \approx -h_{fe} \approx 100$	1	$\frac{h_{fe}R_L}{h_{ie}} \approx h_{fe} \approx 100$
A_i	$h_{fe} \approx 100$	$-h_{fe} \approx -100$	1
Z_{in}	$(h_{ie} \| R_1 \| R_2)$	$\sim h_{fe}(R_E \| R_L \| R_1 \| R_2)$	$\left(\frac{h_{ie}}{h_{fe}} \| R_1 \| R_2\right)$
Z_{out}	$\frac{h_{ie}}{\Delta_e h} \| R_C \cong 5\ k\Omega \| R_C$	$\sim \frac{h_{ie}}{h_{fe}} \| R_E$	$\frac{h_{fe}R_s}{\Delta_e h} \| R_C$

5.11 TRANSISTOR SWITCHES

In the previous sections we have assumed that the transistors are used to amplify relatively small sinusoidal signals or other small pulses. In other words, the purpose of the circuit was to produce an enlarged replica of the

input with higher voltage, current, and power. Such an amplifier is often called a "linear" amplifier. But in some applications the transistor is merely used to turn something completely on or off; that is, it acts like a switch. Thus the two states of the transistor are "full on" or "full off."

Figure 5.27 shows a transistor, with a load R_L in series with its collector, and the dc load line. As long as the base voltage is kept below



(a) *basic circuit* (b) *load line*

FIGURE 5.27 Transistor switch.

approximately 0.5 V (ground is fine), the transistor is "full off," $I_C = 0$, and $V_{CE} = V_{bb}$. If the base voltage is raised to 0.6 V or 0.7 V, the transistor will turn on, and I_C will depend on I_B: $I_C = \beta I_B$. But the load line implies that regardless of how large I_B is, V_{CE} cannot fall below the value $V_{CE\,sat}$ and I_C cannot exceed $I_{C\,max}$. $V_{CE\,sat}$ is the "saturation" collector emitter voltage and is typically 0.2 V for $I_C = 10$ mA, increasing for larger collector currents. $V_{CE\,sat}$ may be as high as 1 V for $I_C = 5$ A for a high-power transistor. From Kirchhoff's voltage law and Ohm's law,

$$I_{C\,max} = \frac{V_{bb} - V_{CE\,sat}}{R_L} \cong \frac{V_{bb}}{R_L}$$

When the (npn) transistor is "full on" ($I_C = I_{C\,max}$), its collector will be at approximately 0.2 V and its base at 0.6 or 0.7 V. Thus, the base-collector junction is *forward* biased instead of reverse biased, which was true for all the linear amplifier circuits we considered in Sections 5.7–5.9. When the base-collector junction is forward biased, the transistor is said to be in "saturation"—increasing the base current will no longer increase the collector current. In other words, $I_B > I_{C\,max}/\beta$ for saturation. The situation for linear amplifiers is $I_B = I_C/\beta$, with the base collector junction reverse biased.

Let us now go through a quick switching design problem. Suppose we

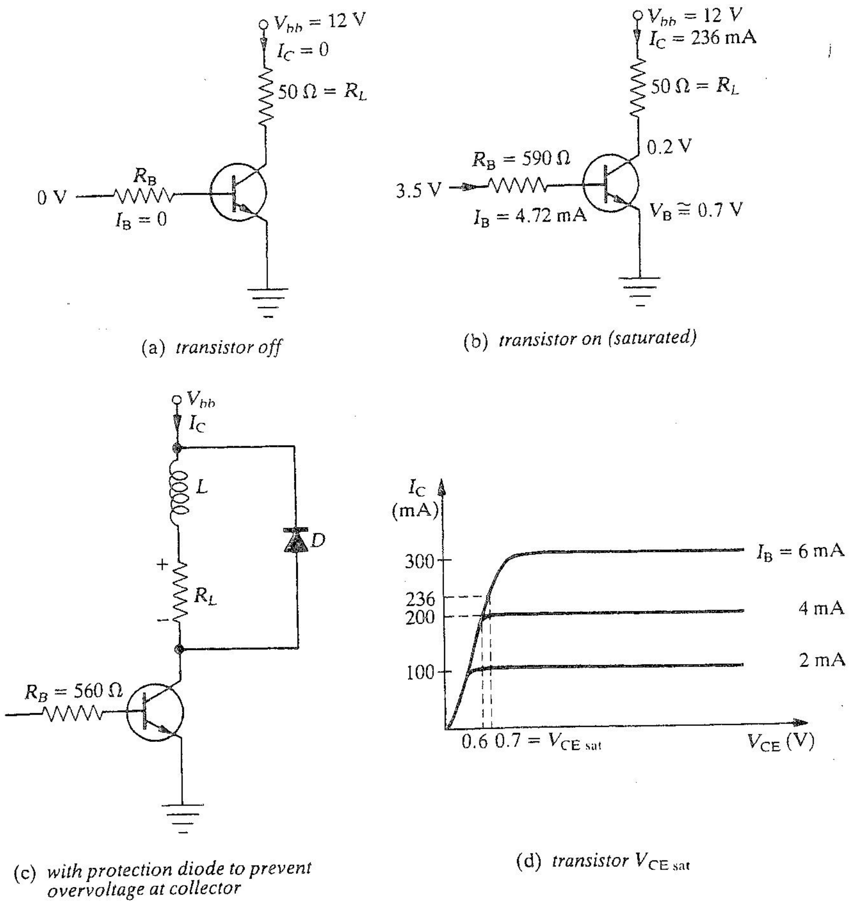


FIGURE 5.28 Switching problem circuit.

have a solenoid-operated valve with 50 Ω resistance that requires a current of 200 mA to open, and we wish to hold this valve open with a positive 3.5-V pulse coming from a microcomputer that can supply up to 5 mA output current when its output is +3.5 V. The circuit is shown in Fig. 5.28. We clearly must choose a transistor capable of carrying a 200-mA collector current. Assume the transistor $\beta = h_{FE} = 50$. The voltage drop across the solenoid valve will be (50 Ω)(200 mA) = 10 V, so the supply voltage must be at least 10 V. Let us choose $V_{bb} = 12$ V.

When the input signal from the microcomputer is 0 V, the base current will be zero and the collector current through the 50-Ω solenoid coil will be essentially zero. (It will equal $I_{CO} \approx \mu$A due to the thermally generated minority carriers in the transistor and leakage current.)

When the input signal is 3.5 V, base current will flow into the base, and the transistor will be on with $V_{BE} \cong 0.7$ V. The base current will be

$$I_B = \frac{3.5\text{ V} - V_B}{R_B} \cong \frac{3.5\text{ V} - 0.7\text{ V}}{R_B}$$

The smaller R_B the larger I_B, and if the transistor is not saturated $I_C = h_{FE}I_B$ will also be larger. If the transistor is saturated, $V_{CE} = 0.2$ V, so $V_C = 0.2$ V and $I_C = I_{C\,max}$ is fixed by Ohm's law *regardless* of I_B:

$$I_{C\,max} = \frac{V_{bb} - V_C}{R_L} = \frac{12\text{ V} - 0.2\text{ V}}{50\ \Omega} = 236\text{ mA}$$

If we assume $V_{BE} = 0.7$ V for a saturation (this value depends upon the transistor type), then for the transistor to be at the edge of saturation,

$$I_B = \frac{I_{C\,max}}{h_{FE}} = \frac{236\text{ mA}}{50} = 4.72\text{ mA}$$

The microcomputer input can supply up to 5 mA, so this value of I_B is okay. Thus, R_B must be

$$R_B = \frac{3.5\text{ V} - 0.7\text{ V}}{4.72\text{ mA}} = 590\ \Omega$$

In other words, $R_B = 590\ \Omega$ will allow the 3.5-V input to drive the transistor to the edge of saturation.

If we use a lower value of R_B, I_B will be increased but I_C will remain locked at 236 mA because the transistor is saturated. For example, if $R_B = 560\ \Omega$, then

$$I_B = \frac{3.5\text{ V} - 0.7\text{ V}}{560\ \Omega} = 5.0\text{ mA}$$

This is the maximum current the microcomputer can supply at 3.5 V, so R_B must be greater than 560 Ω. For R_B between 560 Ω and 590 Ω the transistor will be saturated and I_C will be 236 mA. If R_B is greater than 590 Ω, the transistor will not be saturated, V_C will be above 0.2 V, and $I_C = h_{FE}I_B$ will hold. If we assume $V_{BE} = 0.6$ V for the nonsaturated transistor, R_B = 725 Ω will produce $I_B = 4$ mA and $I_C = 200$ mA. In this nonsaturation condition any change in the 3.5-V input will produce a change in I_C, which is bad.

A comment about the load: If the load inductance is appreciable, as would be the case for a coil of many turns, any sudden change in the load

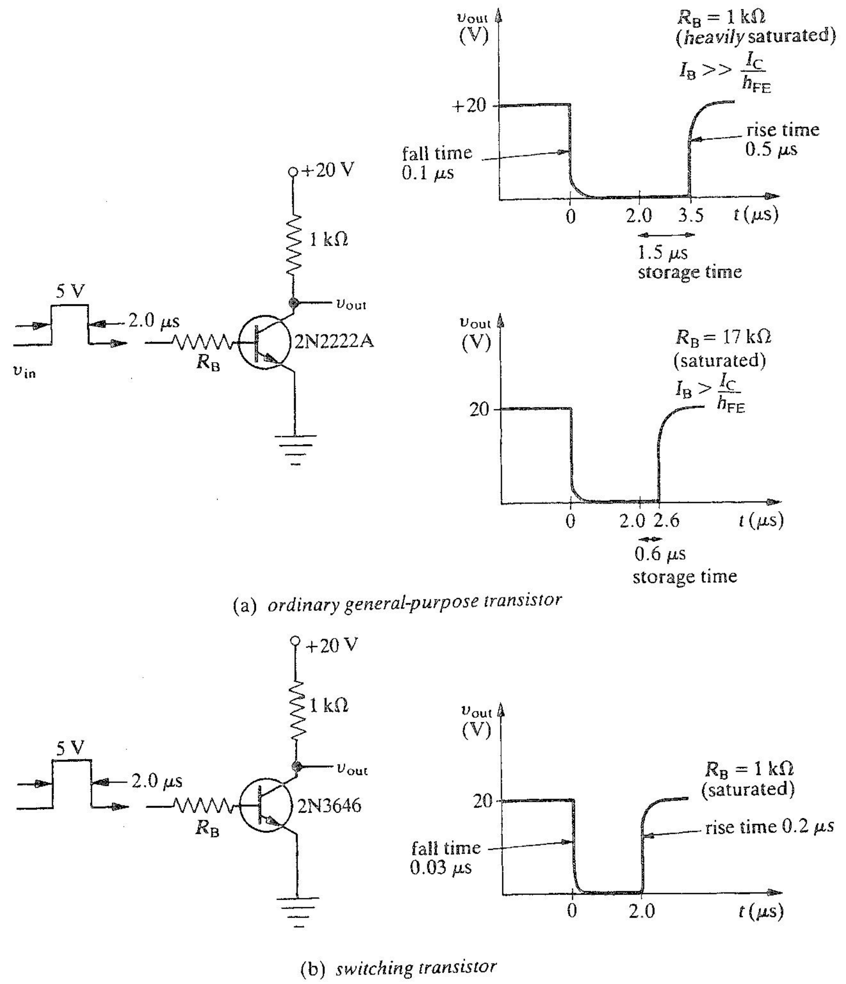


FIGURE 5.29 Storage time in standard transistor.

current would produce a large-amplitude voltage oscillation due to Faraday's law, $V_L = -L(dI/dt)$. The voltage swing might be enough to damage the transistor. To prevent this, connect a diode across the load, as shown in Fig. 5.28(c). The diode is reverse biased for normal circuit operation, but if a voltage oscillation has occurred, the diode will conduct on the reverse polarity oscillation and will limit the voltage across the load to 0.6 V. The oscillation will also be quickly damped out.

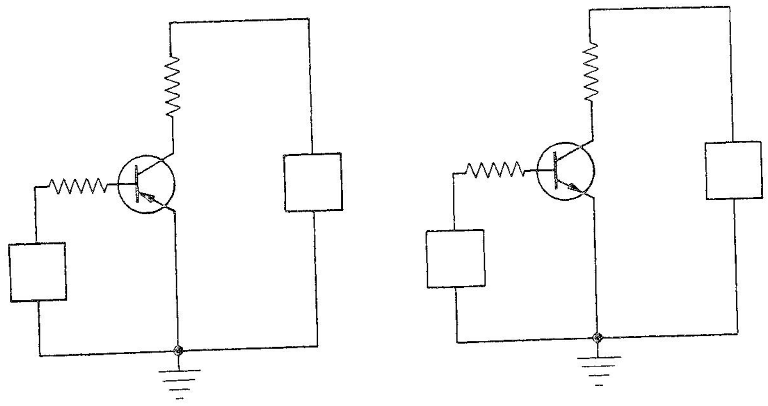
Finally, we should mention that although saturating the transistor provides a margin for error in I_B to keep the transistor fully on, a saturated transistor is slower to turn off than an unsaturated one with the same collector current, because in a saturated transistor the lack of reverse bias

at the base collector junction means there is no depletion region there. Thus, the base is thicker, and because there is no electric field in the base the excess charge carriers in the base (due to the excess I_B) must slowly *diffuse* out of the base before the transistor can turn off. During the time it takes the excess electrons to diffuse out of the base, the transistor is still on, and this results in a "storage time" which lengthens the time the transistor is on. This is shown in Fig. 5.29(a). Notice the larger the base current the more saturation and the longer the storage time. Using a special switching transistor will eliminate the storage time as shown in Fig. 5.29(b).

To sum up, if it is important to make sure the transistor is fully on, use excess I_B to achieve saturation. Then small changes in I_B will not change I_C. The disadvantage is that the saturated transistor will be slow to turn off. If it is important to turn the transistor off quickly, then the transistor should not be saturated when it is conducting; this is achieved by using less base current. The disadvantage is that if the base current even slightly decreases, the collector current will decrease also.

PROBLEMS

1. (a) Describe the construction of a pnp transistor and an npn transistor. (b) What is the physical meaning of α?
2. Describe how to test an npn transistor with an ohmmeter.
3. Label the polarities of the two voltage supplies:



4. (a) The base voltage of an "on" silicon pnp transistor is always approximately ______V more ______ than the emitter. (b) The base voltage of an "on" silicon npn transistor is always approximately ______V more ______ than the emitter.
5. Carefully sketch the depletion region in a properly biased npn transistor. Show the mobile charge carriers, the fixed ionized impurity atoms, and any electric field vector present.

6. (a) How are I_C, I_B, and I_E related? (b) How are I_C and I_B related in terms of α? In terms of β? (c) How are I_C and I_E related in terms of α? In terms of β? (d) How are I_B and I_E related in terms of α? In terms of β?

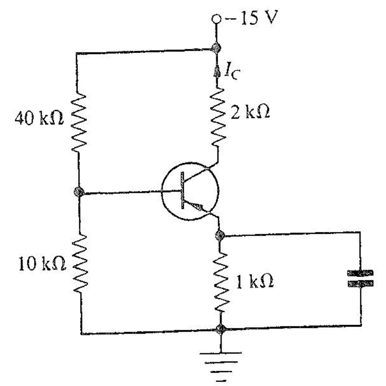
7. Explain why the base of a transistor is purposely made thin. Would increasing the doping concentration of the base tend to increase or decrease the α? Explain.

8. Sketch a graph of the collector current versus base-emitter voltage for a silicon transistor. Repeat for the base current versus base-emitter voltage. Include approximate numerical values for the voltages and currents. Assume the transistor $\beta = 50$.

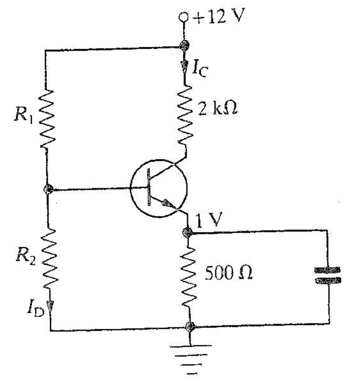
9. Explain briefly why a transistor amplifies when connected in the common emitter configuration.

10. Consider a transistor with a maximum power dissipation of 200 mW and a 20-V power supply. On a graph of I_C versus V_{CE} sketch the maximum power curve and shade in the forbidden region of operation. Also draw the dc load line for a 2-kΩ collector resistor. Is this a safe load line? Repeat for a 400-Ω collector resistor. Is this load line safe?

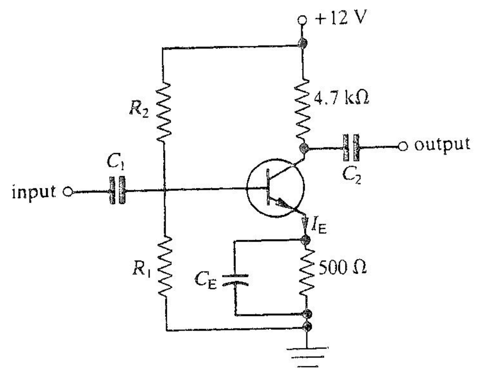
11. Calculate I_C and V_{CE}. The transistor is silicon and has a β of 100.



12. Calculate R_1 and R_2 if $I_D = 20I_B$. The transistor is silicon and has a β of 100. $V_E = 1$ V.

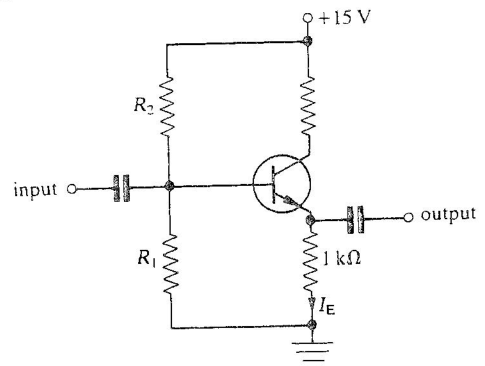


13. (a) Calculate R_1 and R_2 if $I_E = 1.5$ mA, $\beta = 100$. (b) Calculate the voltage gain. (c) Estimate the input and output impedances.

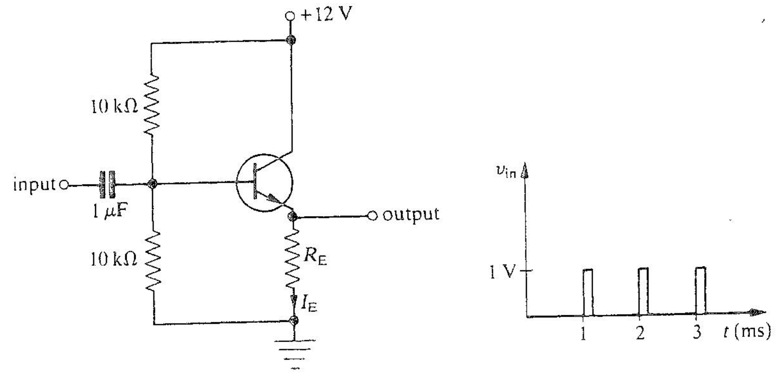


14. Calculate C_1 and C_2 if the desired bandwidth is (a) 20 Hz to 20 kHz, (b) 300 Hz to 3 kHz (R_L = 100 kΩ).

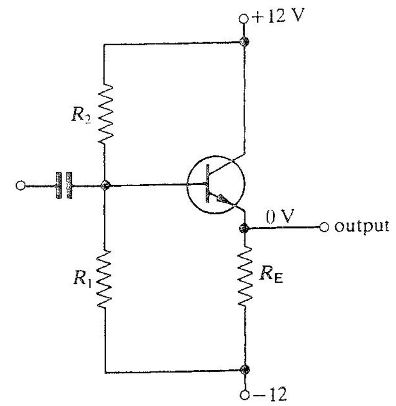
15. (a) Calculate R_1 and R_2 if $I_E = 5$ mA, $\beta = 100$. (b) Estimate the input and output impedances.



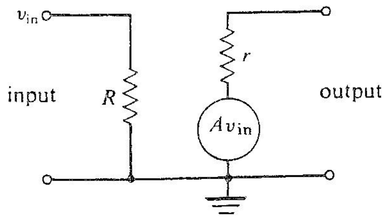
16. (a) Calculate R_E if $I_E = 4$ mA, $\beta = 100$. (b) Carefully sketch the output voltage to scale, for the 1-V input pulses shown.



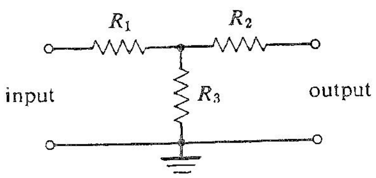
17. The output dc voltage (with no input) is exactly zero, and $I_E = 6$ mA. Calculate (a) R_E and (b) R_1 and R_2.



18. In the simple equivalent circuit shown calculate (a) the current gain, (b) the voltage gain, (c) the input impedance, and (d) the output impedance.

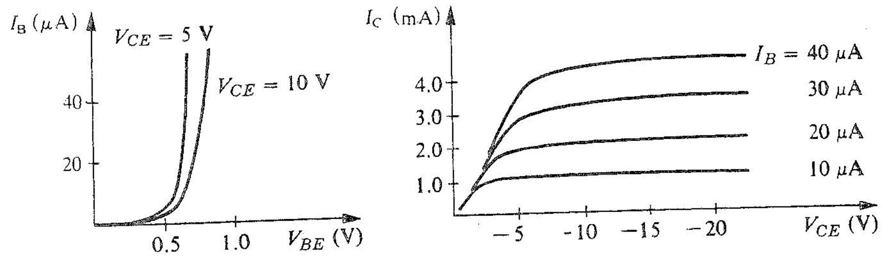


19. Explain why the h parameter equivalent circuit can be applied to any four-terminal device so long as small signals are concerned.
20. State the units of the four h parameters.
21. Derive the h parameter equivalent circuit of the following resistive "black box."

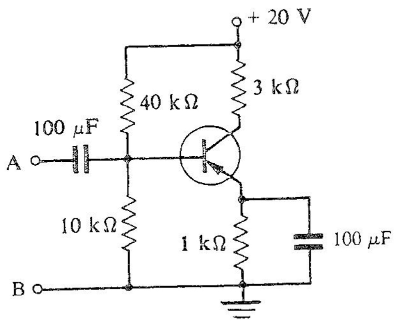


22. Give typical values for h_{ie}, h_{re}, h_{fe}, and h_{oe}. What would be the values for an ideal transistor?

23. (a) Calculate the four h parameters and β for the transistor whose input and output characteristic curves are shown below if the dc operating point is $I_C = 2$ mA, $V_{CE} = 10$ V. (b) As the collector current is increased, how do h_{fe} and h_{oe} change? (c) Compare h_{fe} with $\beta = \alpha/(1-\alpha)$. (d) Explain why V_{CE} at the operating point should be greater than (1 mA, 1 V).

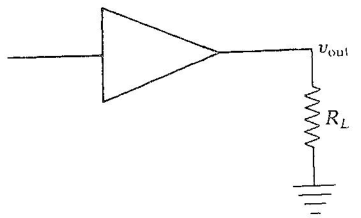


24. Show that the input impedance for a transistor connected in the common emitter configuration is equal to h_{ie} if $h_{re} = 0$. [*Hint:* Use the h parameter equivalent circuit.]
25. Calculate an approximate value for h_{ie} for a transistor at room temperature with $h_{fe} = 100$ and an operating point $V_{CE} = 10$ V, $I_C = 1$ mA. Repeat if the operating point is changed to $V_{CE} = 5$ V, $I_C = 100$ μA.
26. From the definition of h_{ie} and the current–voltage graph for the emitter-base junction, show (by a graphical argument) that the value of h_{ie} decreases for increasing emitter current. Sketch a rough graph of h_{ie} versus I_E.
27. Calculate the approximate 1-kHz ac input impedance to the *circuit* (not the transistor alone) between terminals A and B. $h_{ie} = 2$ kΩ, $h_{re} = 10^{-4}$, $h_{fe} = 100$, $h_{oe} = 10^{-5}$ mhos.



28. Calculate the voltage and current gains for the amplifier of Problem 27.
29. Calculate the output impedance for the amplifier of Problem 27.
30. Explain why R_1 should not be much less than h_{ie} for a common emitter amplifier. (R_1 is the external resistance from base to ground.)

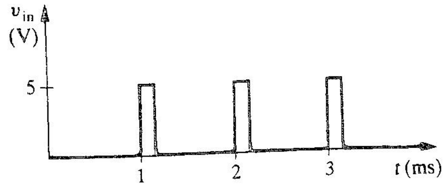
31. What is the effective output impedance of an amplifier driven by a constant input voltage whose output versus load resistance is given in the following table?



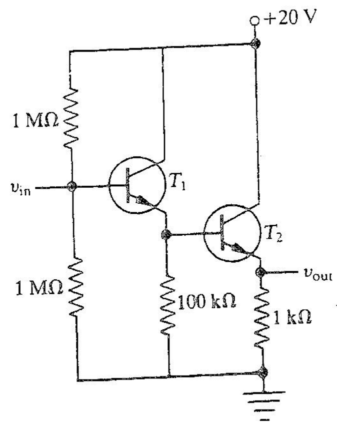
R_L	v_{out}
1 MΩ	500 mV
100 kΩ	500 mV
10 kΩ	450 mV
1 kΩ	250 mV
100 Ω	50 mV

32. Explain briefly, without using h parameters, why one would expect the voltage gain of an emitter follower or common collector amplifier to be less than unity.

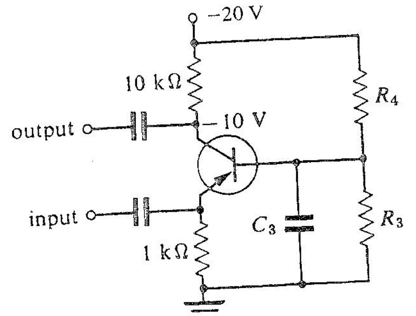
33. Design an emitter follower for the input pulses shown. The transistor should have $I_E = 1$ mA *only* when the input pulse is at 0 V. [*Hint*: This involves choosing an appropriate dc operating point for the transistor.]



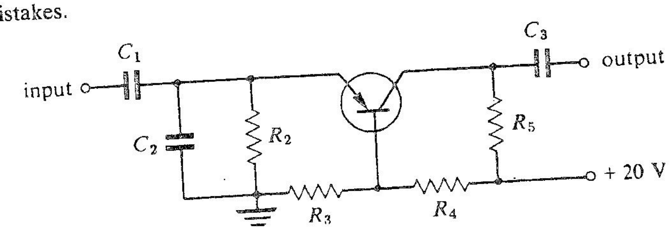
34. For the Darlington circuit shown, calculate (a) V_{E2}, (b) I_{E2}, and (c) I_{E1}. The transistors are silicon and $\beta_1 = 100$, $\beta_2 = 50$.



35. Calculate C_3, R_3, and R_4. The transistor is silicon. What would you estimate the output impedance to be?



36. What is wrong with the common base amplifier shown below? Why? Correct the mistakes.



37. Calculate (a) V_{bb} if I_L must be 400 mA when the transistor is saturated "on," (b) the base current that must be supplied to turn the transistor on, and (c) R_B so that the transistor is just saturated when it is "on." $V_{CE\,sat} = 0.3$ V and $h_{FE} = 100$ for $I_C < 0.4$ A for the 2N3055.