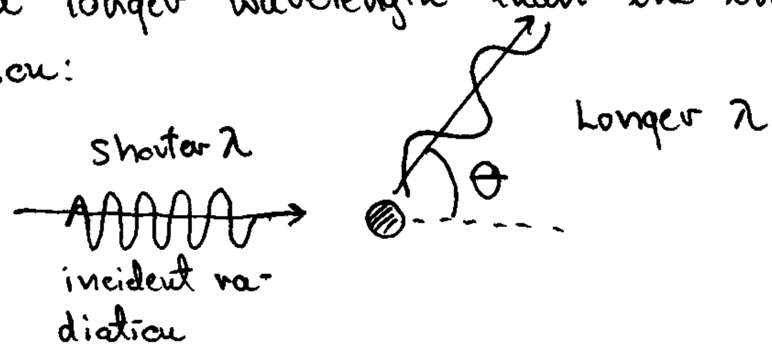
Particlelike properties of EM radiation

Part III: Compton scattering.

In 1923, Arthur H. COMPTON illuminated graphite (a form of carbon) with X-rays.

In 1923, methods of measuring the wavelength λ of X-rays were already well developed. So, since the frequency is related to λ as: $\nu = \frac{c}{\lambda}$, Compton knew the values of λ and ν of the incident radiation.

Compton obseved that the scattered radiation has a longer wavelength than the incident radiation:



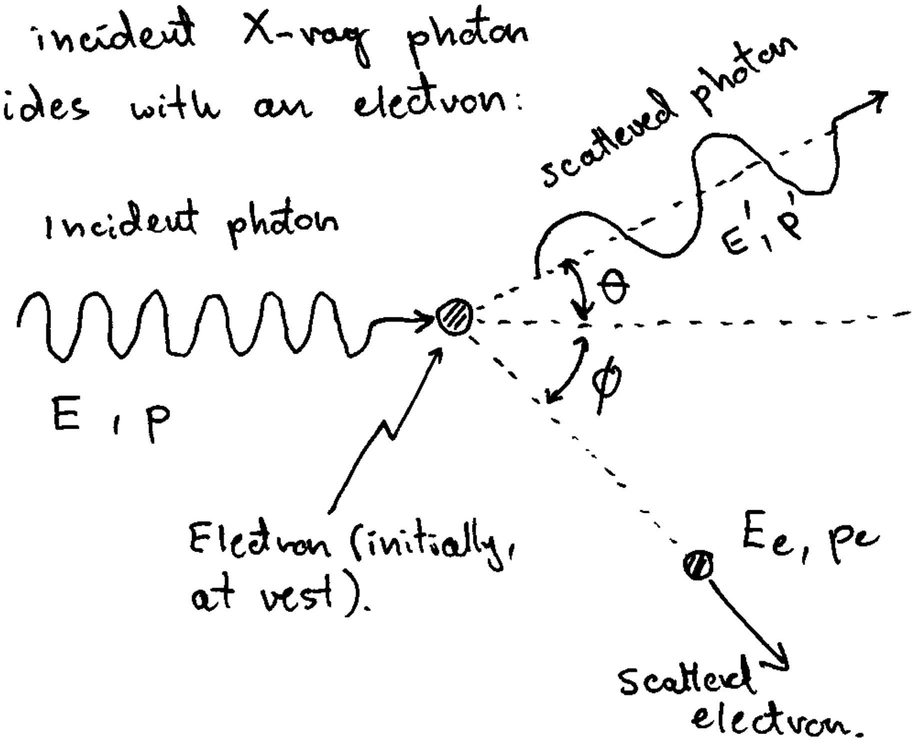
On the grounds of the wave theory, it is impossible to explain the change of the wavelength!

According to the wave theory, in any conceivable scattering process the radiation frequency must be conserved! (and thus λ).

Compton explained the results of his observations in terms of Einstein's photon theory (in 1923, the photon theory was not yet widely accepted — many physicists still expressed serious doubts — so using this theory by Compton was a courageous act!)

Compton's reasoning: in graphite, there is an abundance of weakly coupled electrons – one can think of them as of "free electrons".

An incident X-ray photon collides with an electron:



The photon passes some part of its energy to the electron, and flies away with less energy

The energy has to be conserved, so the remaining part of the energy is the kinetic energy of the scattered electron.

The energies involved:

Photon: E - initial photon energy

E' - scattered " "

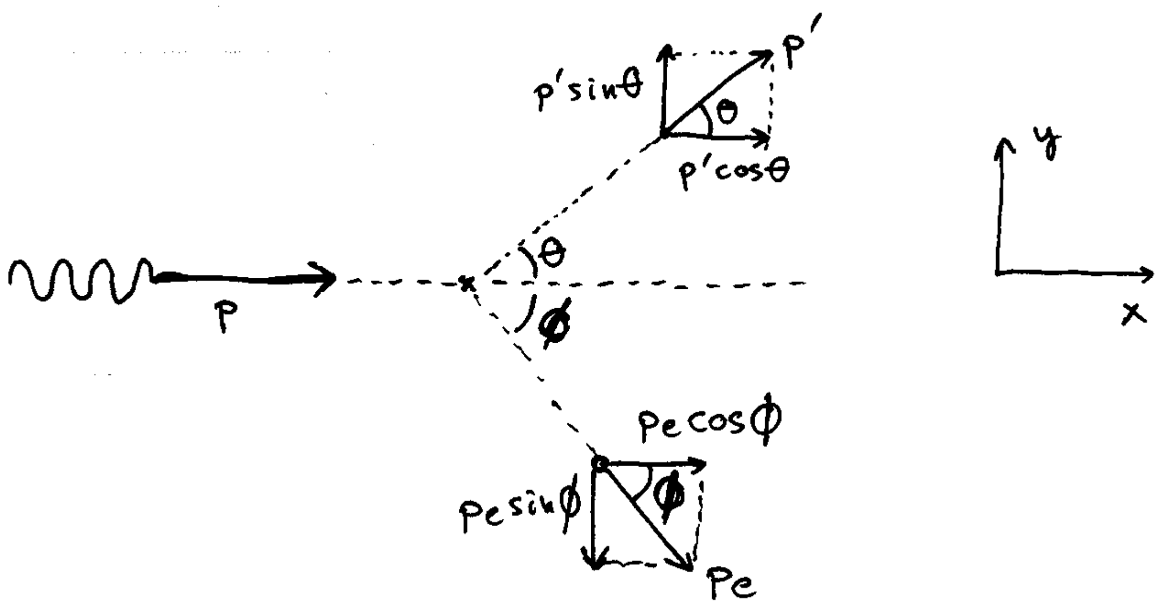
Electron: initially, only the rest energy, $m_e c^2$

final energy: E_e, which is the total relativistic energy

From energy conservation:

$$E + m_e c^2 = E' + E_e$$

Now, momentum. It also must be conserved!



Conservation of momentum components in the x direction:

$$p = p' \cos\theta + p_e \cos\phi$$

And in the y direction: $p' \sin\theta = p_e \sin\phi$

Which can be rewritten as:

$$p_e \cos\phi = p - p' \text{sos}\,\theta$$
$$p_e \sin\phi = p' \sin\theta$$

Let's square both equations, and add them:

$$p_e^2 \cos^2\phi = p^2 - 2pp' \cos\theta + p'^2 \cos^2\theta$$
$$p_e^2 \sin^2\phi = p'^2 \sin^2\theta$$

since $\sin^2\alpha + \cos^2\alpha = 1$, we obtain:

$$\boxed{p_e^2 = p^2 - 2pp'\cos\theta + p'^2}$$

Now, recall the energy conservation formula we obtained a moment ago:

$$E + m_e c^2 = E' + E_e$$

Rewrite: $\boxed{E_e = E + mc^2 - E'}$

Now we take the general formula for the <u>total relativistic energy</u> of an electron and square it:

substitute → $E_e^2 = c^2 p_e^2 + m_e^2 c^4$

↑ substitute

So:

$$(E - m_e c^2 - E')^2 = c^2(p^2 - 2pp'\cos\theta + p'^2) + m_e^2 c^4$$

$$E^2 + \cancel{m_e^2 c^4} + E'^2 - 2Em_ec^2 - 2E'E + 2m_ec^2E'$$

$$= c^2p^2 - 2c^2pp'\cos\theta + c^2p'^2 + \cancel{m_e^2 c^4}$$

Pretty complicated, yes? But we still have not used the relation between the photon momentum and energy, $p = E/c$.

So, we substitute: $p = E/c$, and $p' = E'/c$, to get:

$$\cancel{E^2} + \cancel{E'^2} - 2Em_ec^2 - 2E'E + 2m_ec^2E'$$

$$= \cancel{E^2} - 2EE'\cos\theta + \cancel{E'^2}$$

$$2m_ec^2(-E + E') = 2EE'(1 - \cos\theta)$$ Divide by $EE' \cdot 2m_ec^2$

$$\frac{1}{E'} - \frac{1}{E} = \frac{1}{m_ec^2}(1 - \cos\theta)$$

Finally, using: $E = h\nu = h \cdot \frac{c}{\lambda}$, we obtain:

$$\boxed{\lambda' - \lambda = \frac{h}{m_ec}(1 - \cos\theta)}$$

If one uses a fixed ~~energy~~ wavelength of the incident beam, and measures scattered wavelength as a function of θ:

$$\lambda'(\theta) = \lambda_{fixed} + \frac{h}{m_e c}(1 - \cos\theta)$$

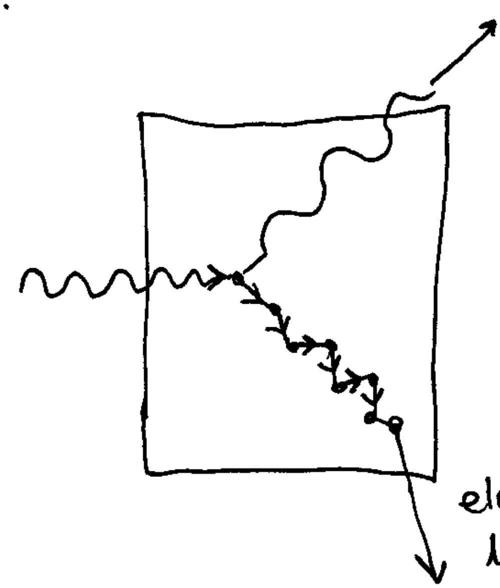
$\lambda'(\theta)$ was exactly what Compton measured, and he obtained a spectacular agreement with the above formula!

Compton's experiments offered an extremely strong support for the Einsteins photon theory. After the results became widely known, nobody could any more express doubts that photons really exist!

But you may ask: even a better confirmation of the photon theory would be obtained if Compton also measured the energy and momentum of the scattered electrons, and showed that they also agree with the theory.

Why didn't he measure the electron energy and flight direction?

Answer: such results would be meaningless. In condensed matter, fast electrons very quickly loose their energy due to collisions with atoms. Also, their pathes get distorted:



However, with the progress of experimental techniques, also reliable measurements of the scattered electron energy and momentum became possible, and such studies provided further confirmation of the Compton's theory.

Compton experiments offered the final proof for the particlelike nature of EM radiation.

Does it mean that the wave theory of EM radiation was "killed"? NO!!!
Compton experiments did not change the fact that EM radiation manifests its wave-like nature in many other experiments! (double-slit experiment, Bragg scattering, ...)

SO WHAT'S GOING ON?!!

Well – all those experiments and facts we reviewed point to the DUAL NATURE of EM radiation: in some circumstances light manifests its wave-like nature, and in some other circumstances, it behaves as if it consisted of particles...
ABSURD? No! Such is the microworld
And next week we will see that not only light, also particles such as electrons, protons, neutrons exhibit a similar "dual nature". That's simply how the microworld is organized!