PRACTICAL EXAMPLES

Ch. 2, 3: How fast must an object move before its length appears to be contracted to one-half its proper length?

$$L = L_0\sqrt{1 - \frac{u^2}{c^2}}$$

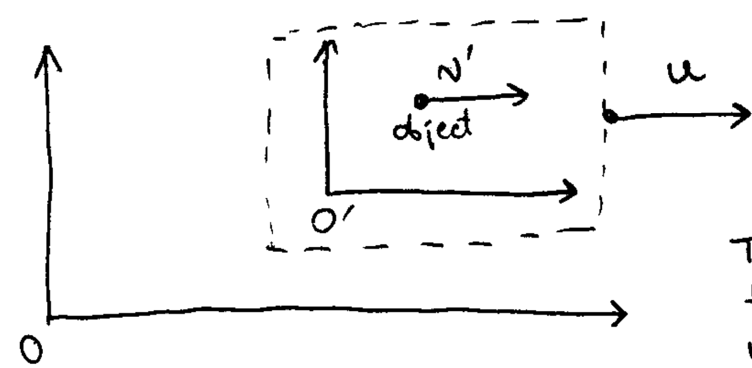
we want L to be $\frac{1}{2} L_0$ $\Rightarrow \sqrt{1-\frac{u^2}{c^2}} = \frac{1}{2}$

$$\Rightarrow 1 - \frac{u^2}{c^2} = \frac{1}{4} \Rightarrow \frac{u^2}{c^2} = \frac{3}{4}$$

$$u = \sqrt{\frac{3}{4}}\, c = \frac{\sqrt{3}}{2} \cdot c = 0.866\, c$$

$c \cong 300\,000$ km/s ; $u = 259{,}808$ km/s

Ch. 2, 8: Two spaceships approach Earth from opposite directions. According to an observer on the Earth, ship A is moving at a speed of 0.753c, and ship B at a speed of 0.851c. What is the speed of ship A as observed from ship B? Of ship B as observed from ship A?



The observer in O' sees an object moving with speed v'

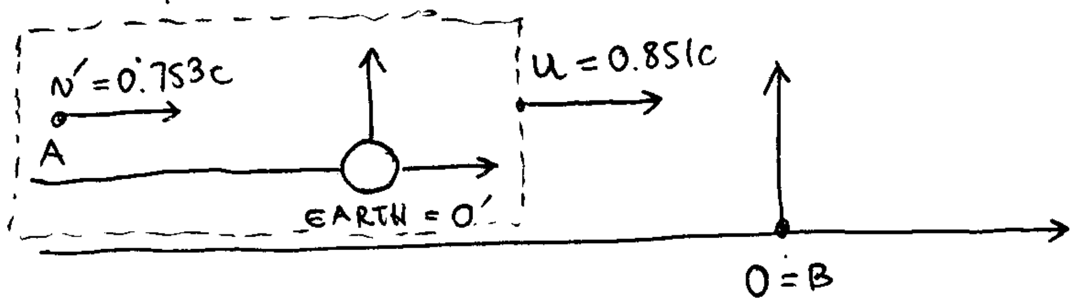
The observer in O sees the frame O' moving with speed u

The observer in O sees the object moving with speed v:

$$v = \frac{v' + u}{1 + \frac{v' \cdot u}{c^2}}$$

Now Earth is the O frame.

For an observer in the ship B, the ship is the O frame, and Earth is the O′ frame:

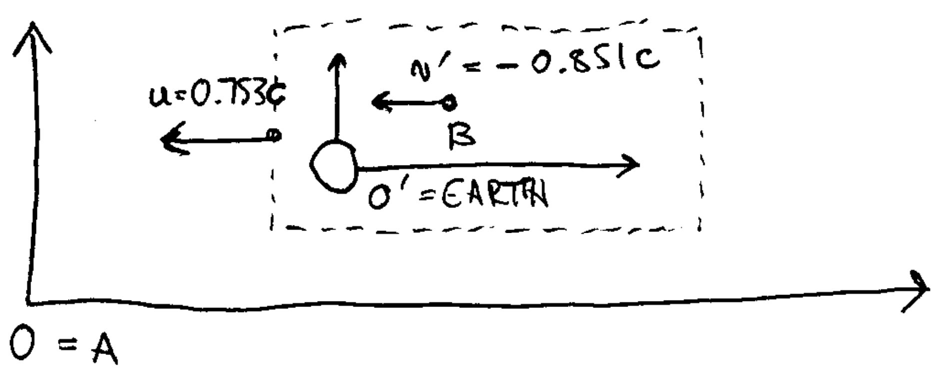


So, the speed of the spacecraft A for an observer in spaceship B is:

$$v = \frac{v' + u}{1 + \frac{v'u}{c^2}} = \frac{0.753c + 0.851c}{1 + \frac{(0.753c)(0.851c)}{c^2}}$$

$$= \frac{1.604}{1.6408}\,c = 0.9776\ c$$

Now, for an observer in A:



So:

$$v = \frac{-0.753c - 0.851c}{1 + \frac{(-0.753c)\cdot(0.851c)}{c^2}} = -0.9776c$$

As expected, the speed values are the same

Chapter 2, Problem 34: An electron and a positron make a head-on collision, each moving at $v = 0.99999c$. In the collision, the electron and the positron disappear, and are replaced by two muons (μ mesons, $mc^2 = 105.7$ MeV), which move off in opposite directions. What is the kinetic energy of each of the muons?

Here, we have to use the law of conservation of the total relativistic energy:

$$K = \underbrace{\frac{mc^2}{\sqrt{1 - v^2/c^2}}}_{\text{Total relativistic energy.}} - \underbrace{mc^2}_{\text{rest energy}}$$

First, we have to know the value of the electron rest mass mc^2

From the book (printed on the back cover) we find that for an electron $mc^2 = 0.511$ MeV

Thus, the total relativistic energy for the electron-positron pair is:

$$E_{TOT}(\text{electron} + \text{positron}) = 2 \times \frac{0.511 \text{ MeV}}{\sqrt{1 - \frac{(0.99999c)^2}{c^2}}}$$

$$= \frac{1.022 \text{ MeV}}{0.004472125} = 228.527 \text{ MeV}$$

The total relativistic energy is conserved. So, for the two muons we can write:

$$K_{\text{TWO MUONS}} = \underbrace{228.527 \text{ MeV}}_{\text{Total relativistic energy}} - 2m_{\text{MUON}}c^2$$

For a muon, $m_{muon}c^2 = 105.7$ MeV

So,

$$K_{\text{TWO MUONS}} = 228.53 \text{ MeV} - 211.4 \text{ MeV} = 17.13 \text{ MeV}$$

$$K_{\text{EACH MUON}} = 8.57 \text{ MeV}$$