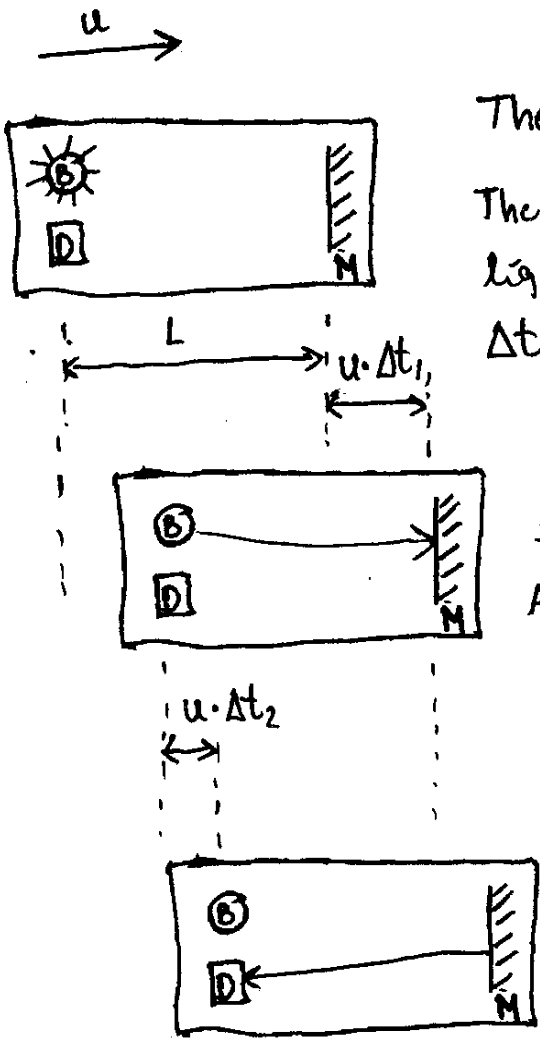
Length contraction

We now turn our "light-clock" sideways. Suppose the clock moves in the horizontal direction with speed u. The clock is now the O' frame. Lets discuss what an observer sees from a stationary frame O.



The bulb flashes

The observer O sees that the light reaches the mirror after Δt_1 from the flash.

But the mirror shifted by $u \cdot \Delta t_1$, so the total distance traveled by light was $L + u \cdot \Delta t_1$

Accordingly: $c\Delta t_1 = L + u \cdot \Delta t_1$

Reflected light reaches the detector after Δt_2

But between the moment of reflection, and the moment of detection, the detector shifted by $u \cdot \Delta t_2$ toward the incoming light.

So, the distance traveled by the light is $L - u \cdot \Delta t_2$

Accordingly, $c \cdot \Delta t_2 = L - u \cdot \Delta t_2$

O – stationary observer

We got two equations:

$$c \cdot \Delta t_1 = L + u \cdot \Delta t_1$$

$$c \cdot \Delta t_2 = L - u \cdot \Delta t_2$$

Solve for Δt_1 and Δt_2:

$$\Delta t_1 = \frac{L}{c-u}$$

$$\Delta t_2 = \frac{L}{c+u}$$

Total time Δt between the flash and the detection is $\Delta t = \Delta t_1 + \Delta t_2$:

$$\Delta t = \frac{L}{c-u} + \frac{L}{c+u} = L \; \frac{c+u+c-u}{c^2-u^2}$$

$$= \frac{2L\,c}{c^2-u^2} = \frac{2L}{c} \; \frac{1}{1-\frac{u^2}{c^2}}$$

But this is the "tick period" as seen by the <u>stationary</u> observer. Previously, we found that the relation between Δt (stationary observer) and Δt_0 (observer in the same frame as the clock) is:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{u^2}{c^2}}}$$

Equating the two:

$$\frac{\Delta t_0}{\sqrt{1-\frac{u^2}{c^2}}} = \frac{2L}{c} \frac{1}{1-\frac{u^2}{c^2}}$$

Here Δt_0 is the "tick time" for an observer in the clock frame, so that $\Delta t_0 = \frac{2L_0}{c}$

Plugging this in, and after some algebra, we obtain:

$$\boxed{L = L_0 \cdot \sqrt{1-\frac{u^2}{c^2}}}$$

This is the clock length seen by the stationary observer. Since $\sqrt{1-\frac{u^2}{c^2}} < 1$, the length apears to be smaller than when the clock does not move (L_0).

This effect is known as "length contraction".