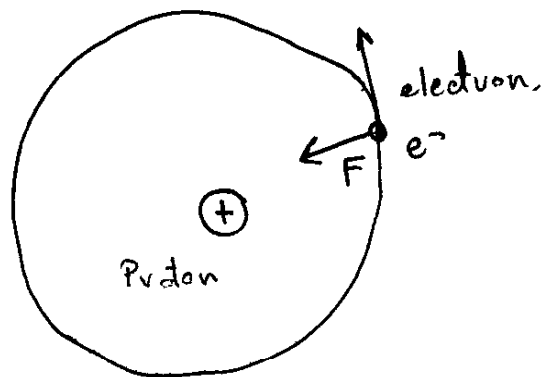


It was the so-called "planetary model" of the hydrogen atom:



In this model, the electron orbits the much heavier proton — just like Earth orbits the Sun.

Earth is attracted by the Sun by the force of gravity: $F_{\text{EARTH-SUN}} = G \frac{M_{\text{SUN}} \cdot M_{\text{EARTH}}}{r^2}$

In Bohr's model, the attractive force is the Coulomb electrostatic force, described by a very similar equation:

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 \cdot q_2}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}$$

(the proton and the electron charge are of the same magnitude: $|q_1| = |q_2| = e$)

On the other hand, as you know from Ph 211, what causes a circular motion of a body of mass m is a centripetal force:

$$F_{\text{centripet.}} = \frac{mv^2}{r}$$

(where v is the speed, and r is the orbit radius).

So, by equating the two forces, we get:

$$\frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} = \frac{mv^2}{r}$$

From this equation, we can obtain the electrons kinetic energy $K = \frac{1}{2}mv^2$:

$$K = \frac{1}{2}mv^2 = \frac{1}{8\pi\epsilon_0} \frac{e^2}{r}$$

Now, again recall from the Ph 213 course: the potential energy of a charge $-e$ in the electric field of charge e is:

$$U = -\frac{1}{4\pi\epsilon_0} \frac{e^2}{r}$$

By adding K and U , we obtain the total energy of the electron:

$$E = K + U = \frac{1}{8\pi\epsilon_0} \frac{e^2}{r} - \frac{1}{4\pi\epsilon_0} \frac{e^2}{r} = -\frac{1}{8\pi\epsilon_0} \frac{e^2}{r}$$

Now, there is a serious difficulty. According to the electrodynamics and Maxwell Equations, an orbiting electron must continuously radiate electromagnetic energy!

As the result, the electron should lose energy, so E becomes smaller and smaller (more negative), meaning that r keeps decreasing, and the electron should eventually fall on the proton!

To solve this problem, Bohr proposed that there are certain "stable" orbits, or stationary states, in which the electron does not radiate energy - i.e., it may stay at such orbits for a long time.

Bohr's hypothesis was that at such orbits the electron ~~orbit~~ angular momentum is an integer multiple of the Planck's constant h :

$$mvr = n\hbar \quad \text{with } n=1, 2, 3, \dots$$

It means that the electron velocity is:

$$v = \frac{n\hbar}{mr}$$

And its kinetic energy is:

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{n\hbar}{mr}\right)^2$$

But we got before: $K = \frac{1}{8\pi\epsilon_0} \frac{e^2}{r}$

Equating the two expressions:

$$\frac{1}{2}m\left(\frac{n\hbar}{mr}\right)^2 = \frac{1}{8\pi\epsilon_0} \frac{e^2}{r}$$

Solving for r , we obtain:

$$r_n = \underbrace{\frac{4\pi\epsilon_0\hbar^2}{me^2}}_{a_0} n^2 = a_0 n^2$$

The allowed values
of the radius
 r

↑
the "Bohr radius".

Plugging the expression for r_n to the equation for the total energy, we obtain the allowed energies of the electron:

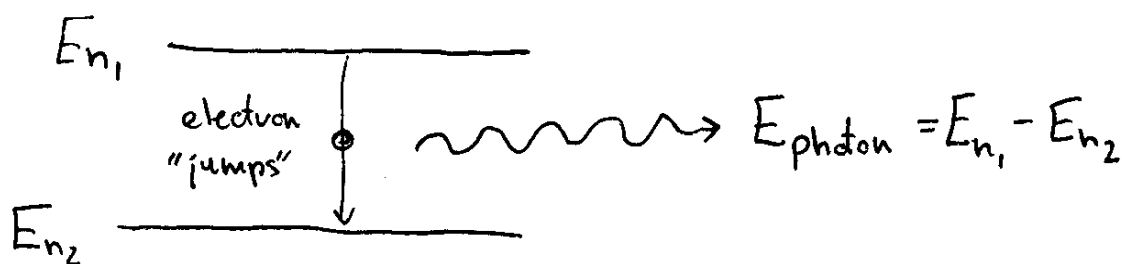
$$E_n = - \underbrace{\frac{me^4}{32\pi^2 \epsilon_0^2 \hbar^2}}_{-13.6 \text{ eV}} \cdot \frac{1}{n^2}$$

$$\text{Or: } E_n = -13.6 \text{ eV} \cdot \frac{1}{n^2} \quad (n=1, 2, \dots)$$

So, we obtained an ~~exp~~ equation that expresses the energy of the allowed electron states in terms of one constant and one integer number!

O.K, but want to know the wavelength of the emitted light. How can we obtain that?

Well, Bohr postulated that the electron can "jump" from one allowed energy state to another, and then it releases a photon whose energy is equal to the difference between the state energies:



But $E_{\text{photon}} = h\nu = 2\pi\hbar\nu$

So: $2\pi\hbar\nu = E_{n_1} - E_{n_2}$

Or, substituting the eq. for E_n :

$$\nu = \frac{me^4}{64\pi^3\epsilon_0^2\hbar^3} \left(\frac{1}{n_2^2} - \frac{1}{n_1^2} \right)$$

The photon's wavelength is $\lambda = \frac{c}{\nu}$, so

$$\lambda = \frac{c}{\nu} = \frac{64\pi^3\epsilon_0^2\hbar^3c}{me^4} \left(\frac{n_1^2 n_2^2}{n_1^2 - n_2^2} \right)$$

Note that if we rewrite it in the form:

$$\lambda = \frac{64\pi^3\epsilon_0^2\hbar^3cn_2^2}{me^4} \left(\frac{n_1^2}{n_1^2 - n_2^2} \right)$$

We obtain exactly the same form as for the experimentally found series, with $\lambda_{\text{min}} = \frac{64\pi^3\epsilon_0^2\hbar^3cn_2^2}{me^4}$