Note that for most cases we obtaint the same energy not only for a single pair of n_x and n_y walues.
(it happens only when $n_x = n_y$).

When $n_x \neq n_y$ there are at least two states with the same energy: e.g., $n_x = 1$ and $n_x = 2$ and $n_x = 2$ and $n_y = 1$

For instance, the two states:

$$\psi_{2,5}(x,y) = \left(\frac{2}{L}\right) \sin\left(\frac{2\pi}{L}x\right) \sin\left(\frac{5\pi}{L}y\right)$$

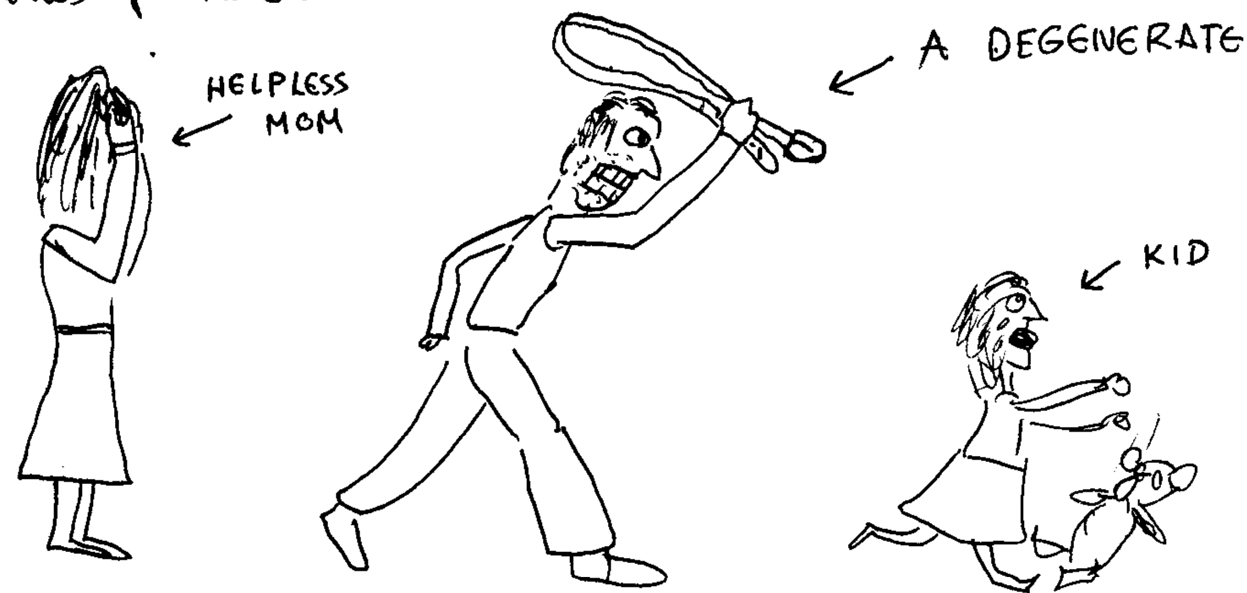
and:

$$\psi_{5,2}(x,y) = \left(\frac{2}{L}\right) \sin\left(\frac{5\pi}{L}x\right) \sin\left(\frac{2\pi}{L}y\right)$$

both have the same energy $E = 29 E_0$, but, from the viepoint of quantum mechanics, they are different states.
STATES in QM can be termed "identical" only if the corresponding wawefunctions are identical. And in the above example they are definitely not!

When two or more different QM-cal states have the same energy, we call such states <u>degenerate</u>. Not that in QM this word has a vary different meaning than in usual life.

We call a DEGENERATE someone as shown in this picture:



But keep in mind that in QM "degeneracy" has a pretty innocent meaning.

In the present case of 2D potential well, most states are <u>doubly degenerate</u>, or we can also say "their degeneracy is <u>twofold</u>"

But sometimes the degeneracy in such a well may be even higher, as in the case of the states:

$n_x = 5$ and $n_y = 5$; $n_x = 1$ and $n_y = 7$; and $n_x = 7$ and $n_y = 1$.

All three have the same energy $E = 50\,E_0$

So, here the degeneracy is <u>threefold</u>.

Here are the probability densities for these degenerate states:

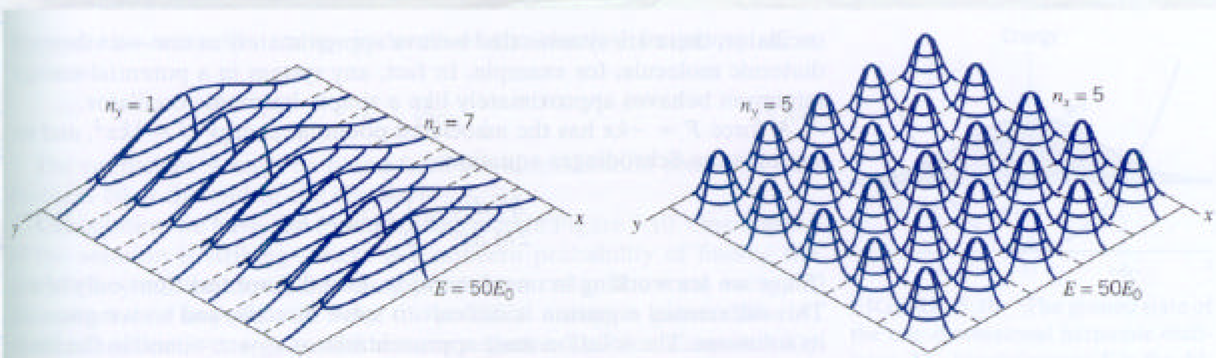
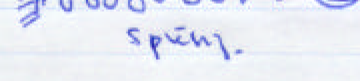


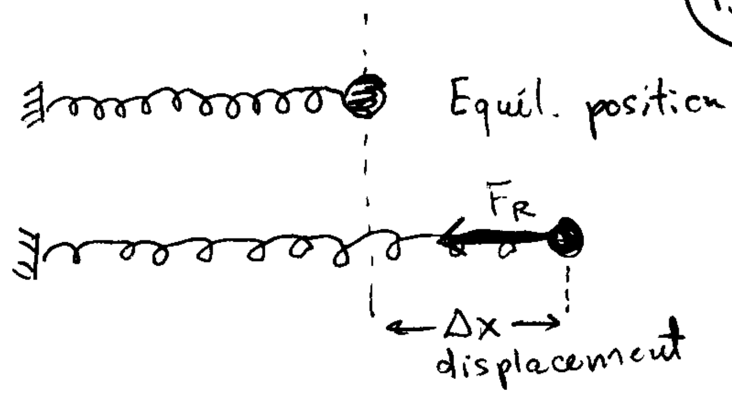
FIGURE 5.9 Two very different probability densities with exactly the same energy.

SIMPLE HARMONIC OSCILLATOR

This is the quantum-mechanical analog of a classical oscillator, i.e., a mass attached to a spring:



Classical:



The restoring force in the classical oscillator is $F_R = -k\,\Delta x$, with k - the "spring constant"

You certainly remember that the potential energy of a stretched spring with constant k is $U = \frac{1}{2}k(\Delta x)^2$

In QM, we call a sytem in which the potential energy is $U = \frac{1}{2}kx^2$ a simple quantum harmonic oscillator (SQHO), or shortly SHO.

Even though SHO is a relatively simple sytem, the SHO theory has an incredible number of applications in various QM-cal theories (e.g., in contemporary quantum theory of solids).

The Schrödinger Equation for the SHO may look pretty <u>user-friendly</u>:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + \frac{1}{2}kx^2\psi = E\psi$$

Unfortunately, solving this equation is pretty challenging...
Therefore, we will skip the procedure and discuss only the energy solution.

We introduce: $\omega_0 = \sqrt{k/m}$, which is nothing else than the <u>classical frequency</u>.

The energy of allowed quantum states is remarkably simple:

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega_0 \quad \text{with } n = 0, 1, 2 \ldots$$

Note! Here $n=0$ is allowed, not like in the potential well!

There is no state with $E = 0$
The lowest possible energy is $E_0 = \frac{1}{2}\hbar\omega_0$, the famous "zero energy".