

Wave-like properties of particles (continued).

We will slightly re-arrange the order in which the material is presented in the book.

In many experiments particles behave like "real particles" - i.e., objects of very small size, with all its mass enclosed within that size.

Examples of effects, in which particles reveal their "particle nature":

= Electrons, for instance, can be detected by a photographic film - a single electron ~~there~~ produces a tiny dark spot on the film. Or, they can be detected by a fluorescent screen - then a single electron produces a microscopic "flash" on the fluorescent layer.

So, we can determine "where the electron was" and with a screen, "when it was there".

- ~~The~~ Moving ^(charged) particles produce visible "tracks" in cloud chambers

The list can be made longer, of course.

However, we have learned that in other types of experiments the same particles behave like typical waves! (Bragg diffraction from crystals, double-slit interference).

SO, WHAT'S GOING ON? ARE PARTICLES REALLY PARTICLES? PERHAPS THEY ARE WAVES!

No - definitely one cannot say that "particles are waves".

Why? Because particles, as we have said, are "localized object". We will see shortly that a particle position cannot be determined with an infinitely high "precision" - but certainly we can determine with micrometer accuracy "where the particle is".

And how about waves?

Recall - the simplest wave (a plane wave) propagating along certain direction (call it x)

(111)

can be mathematically described as follows:

$$\psi = A \sin\left(\frac{2\pi}{\lambda} x - \frac{2\pi}{T} t\right)$$

where λ is the wavelength, T is the oscillation period, and A is the amplitude.

We usually use a simpler notation, replacing:

$$\frac{2\pi}{\lambda} \equiv k \text{ - called "wavenumber"}$$

$$\text{and } \frac{2\pi}{T} \equiv \omega \text{ - called "the angular frequency"}$$

Hence:

$$\psi = A \sin(kx - \omega t)$$

Now, tell me: where is this wave?

Answer: EVERYWHERE!!! This function spans from $x = -\infty$ to $x = \infty$

A wave is not localized, so a particle cannot be a single wave!

Is ~~this~~ this the only reason why not?

No! Another reason is the velocity.

The wave propagation velocity, you should remember from Ph 212, is $v = \frac{\lambda}{T}$

Let's transform it to the k, ω notation:

$$v = \frac{\lambda}{T} = \frac{2\pi \lambda}{T 2\pi} = \frac{\omega}{k}$$

$$v_{\text{wave}} = \frac{\omega}{k} \quad \text{wave velocity ("phase velocity")}$$

And how about a particle? Let's take a non-relativistic particle:

$$K = \frac{mv^2}{2} \quad \text{and} \quad p = mv \quad \Rightarrow \quad \frac{K}{p} = \frac{\frac{mv^2}{2}}{mv} = \frac{1}{2} v$$

Now, consider the de Broglie Equations:

$$K = h\nu \quad \text{and} \quad p = \frac{h}{\lambda}$$

$$\text{So:} \quad \frac{K}{p} = \frac{h\nu}{\frac{h}{\lambda}} = \nu \cdot \lambda = 2\pi\nu \cdot \frac{\lambda}{2\pi} = \frac{\omega}{k}$$

Comparing, we obtain for the particle

$$v_{\text{particle}} = 2 \frac{\omega}{k}$$

The particle velocity is twice as large as the wave velocity!

CONCLUSION: because of its delocalized character, and its velocity being inconsistent with a particle velocity, a wave definitely cannot be used as a mathematical description of a particle!

At least - a simple plane wave.

So - how to construct a mathematical description of a particle, which provides a proper localization and velocity, but still accounts for the wave-like properties?

ANSWER: let's start playing not with a single wave, but with many waves

Take two waves:

$$\phi_1 = A \sin(k_1 x - \omega_1 t)$$

$$\phi_2 = A \sin(k_2 x - \omega_2 t)$$

For simplicity, let's consider only the moment $t = 0$. So, $\phi_1 = A \sin(k_1 x)$ and $\phi_2 = A \sin(k_2 x)$

Suppose that k_1 and k_2 do not differ very much: $|k_2 - k_1| \ll k_1, k_2$

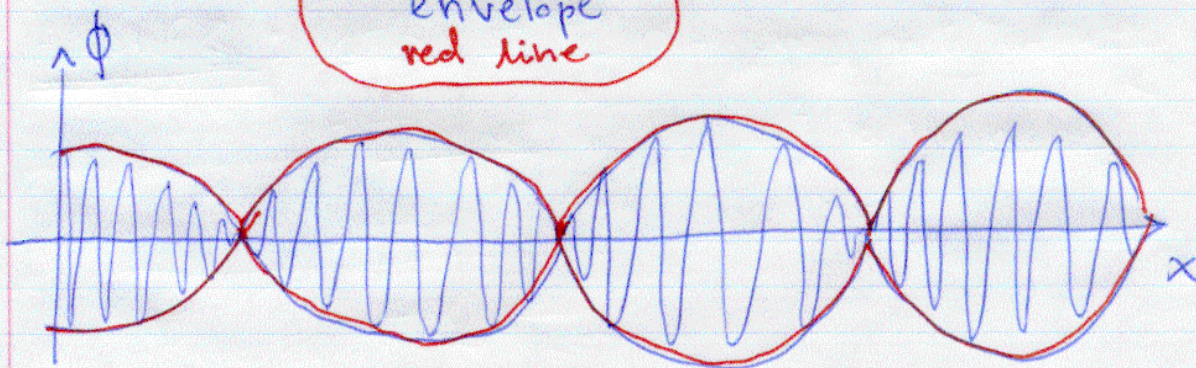
Let's add the two (perform a "superposition" - you should remember that term from Ph 212, too):

$$\phi = \phi_1 + \phi_2 = A \sin(k_1 x) + A \sin(k_2 x)$$

$$= 2A \cos\left(\frac{k_2 - k_1}{2} x\right) \sin\left(\frac{k_1 + k_2}{2} x\right)$$

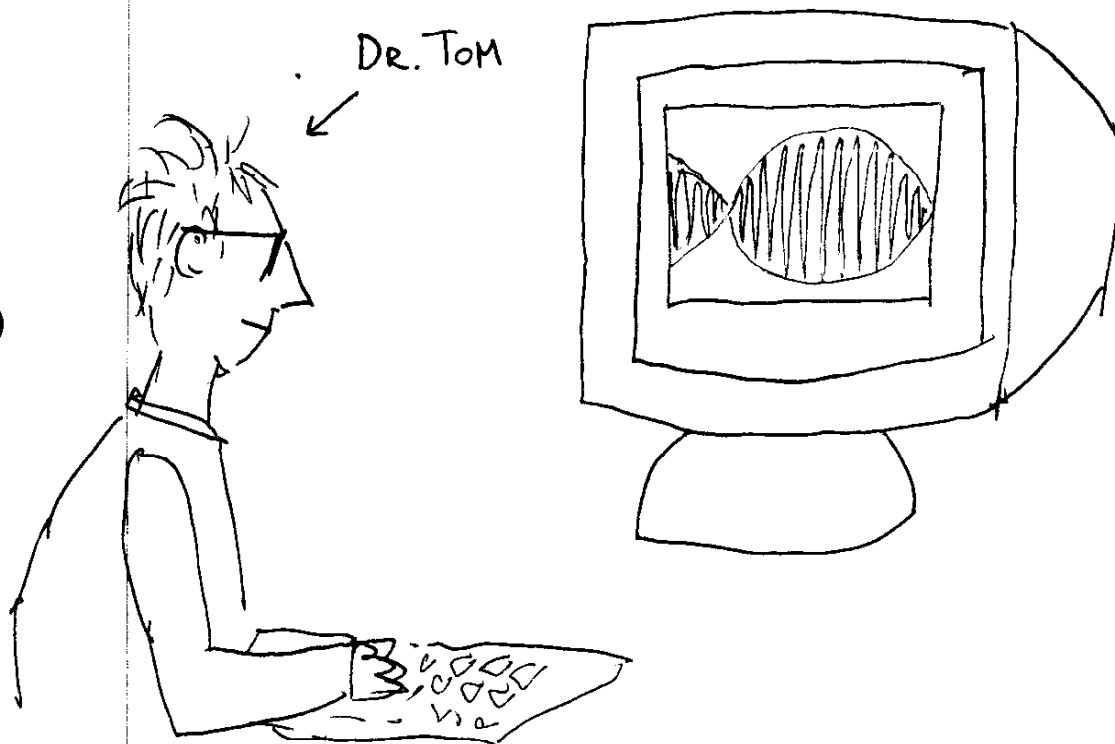
"modulation
envelope"
red line

a wave with average k



The resultant wave forms sort of "groups"...

One time, I plotted such a function on my computer screen. The frame was only slightly longer than a single "group". The plot looked like this:

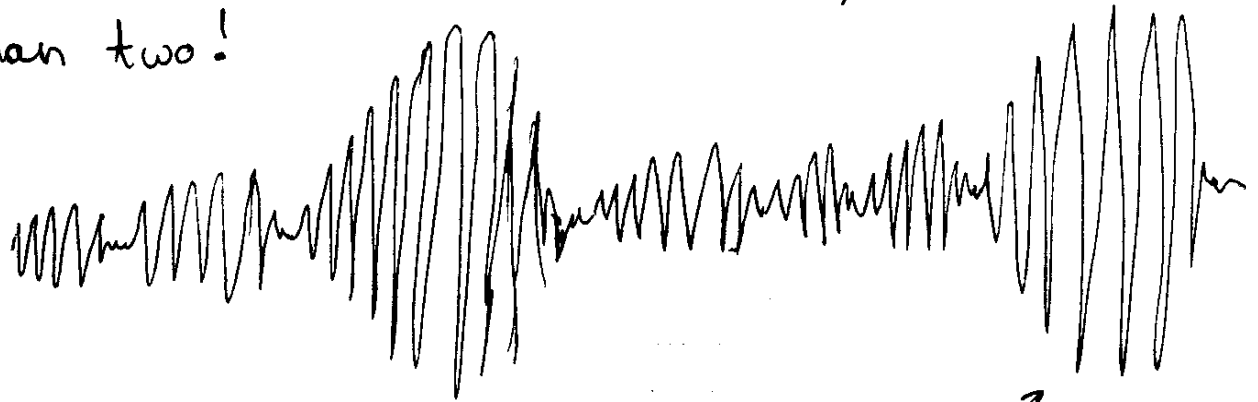


Then my Mom, who was visiting, walked into the room, looked at the screen, and said: "Oh, what a funny FISH!"

I liked that name, and since then I started calling the groups "fishes".

Well, but how can those "fishes" help?
There is plenty of them, and we need a
description of a single particle!

Well, then let's add ("superpose") more waves
than two!



With adding more waves, the "fish-like" features
start shifting apart!

Now it is time for displaying computer
plots!

We will see - the more frequencies we add,
the larger becomes the distance between
two big "fish-like things".

We will now start using the proper term:
WAVE PACKETS.

If we add many waves of different frequencies (k values), but of the same amplitude, we always see smaller packets between the major ones.

How to get rid of them? Well, it turns out that if we use a Gaussian distribution of amplitudes, the "small fish" disappear, and only the large ones stay.

But there are still many packets, and we need a single one!

But again - if we increase the number of frequencies (or, different k values), the packets shift apart.

So, if we go to ∞ with the # of waves, we will have one at the center, and the left one and the right one will shift to $x = -\infty$ and $x = +\infty$ respectively.

What does it mean that something is located at $x = -\infty$ or $x = +\infty$?

IT MEANS "NOWHERE"!

Thus, superposing an ∞ number of waves with a Gaussian distribution of amplitudes produces a SINGLE PACK!

THE localization problem has been solved!

Well, but there is still the velocity problem... Does the fact that we added many waves change the propagation velocity?

The answer is YES! The packet moves with a ~~low~~ higher velocity than the propagation velocity of constituent waves!

It is not a trivial thing, but one can show that if add (or superpose) many waves with different k values, and

with different frequencies $\omega = \omega(k)$,
 then the velocity of the packet is:

$$v_{\text{packet}} = \frac{d\omega}{dk}$$

Let's accept
 without proof...

Now, consider again a non-relativistic particle:
 $p = mv$, and $K = \frac{mv^2}{2}$. And again, let's use
 the de Broglie equations:

$$\omega = 2\pi\nu = 2\pi \underbrace{\frac{K}{h}}_{\substack{\text{from de Broglie} \\ \text{2nd Eq.}}} = \frac{2\pi}{h} \frac{mv^2}{2}$$

$$k = \frac{2\pi}{\lambda} = \underbrace{\frac{2\pi}{h} \frac{p}{v}}_{\substack{\text{from de Broglie} \\ \text{1st Equation}}} = \frac{2\pi}{h} p = \frac{2\pi}{h} mv$$

So:

$$\frac{d\omega}{dk} = \frac{d\left(\frac{2\pi}{h} \cdot \frac{m}{2} v^2\right)}{d\left(\frac{2\pi}{h} m v\right)} = \frac{\frac{2\pi m}{2h}}{\frac{2\pi m}{h}} \frac{dv^2}{dv}$$

$= \frac{1}{2} \cdot 2v = v$. The wave packet moves with
 the same velocity as the particle!